
S. Kayalvizhi
Electrical and Electronics Engineering
Vaaqdevi College of Engineering
Warangal, India.
k_selvam@vaagdevi.edu.in

D. M. Vinod Kumar
Electrical and Electronics Engineering
National Institute of Technology
Warangal, India
dmvk@nitw.ac.in

Abstract—This paper proposes solution to stochastic optimal power flow problem in presence of wind in the power systems. The wind speed is modelled using Weibull probability distribution and Monte-Carlo simulations are applied to reflect the stochasticity of the wind power. The objective of formulated stochastic OPF problem is to minimize the total production cost which includes fuel cost of the conventional generators and the expected uncertainty cost associated with wind power. The uncertainty cost of wind power is evaluated for two cases: i) underestimated case ii) overestimated case. Along with the total cost, the system losses are also minimized. Harmony Search algorithm is employed for solving stochastic OPF and it is tested on IEEE 30-bus system. The system is modified by adding wind farm at one of the load buses. The results obtained by the harmony search algorithm are compared with Cuckoo search algorithm and modified PSO. The comparison has shown the effectiveness of harmony search in achieving minimum production cost as well as active power loss in the system.

Keywords—Optimal power flow, uncertainty cost, stochastic OPF, underestimation and overestimation.

I. INTRODUCTION

The effective operation of power system counts critically on the ability of system to maintain economic efficiency in the unforeseen events. This uncertain system operation not only considers system abnormalities but the intermittent generations also. In past two decades, there is a tremendous growth in the integration of clean and cheap power generating sources such as wind and solar in the power system. So, system studies on economic and optimal operation in the presence of such intermittent generations are inevitable. The stochastic nature of wind speed and solar irradiation results in intermittent power production and this uncertain behaviour impacts the system operation cost. Minimising such additional costs introduced due to intermittency will extract maximum economic benefits to the system operation.

There are substantial amount of work on optimal power flow studies with wind farms and solar integration in the power system. But different ways in which the studies dealing with the incorporation of wind farms are relatively distinct in its own perspective [1]. A precise optimal power flow formulation with wind generation has been outlined in [2]. Few authors attempted OPF by incorporating wind farms with conventional methods [3]-[4]. In [3], gradient method is applied to model a dynamic OPF to incorporate wind farm where the cost associated with the wind power is excluded whereas in the latter [4], Newton method and interior point algorithm have been applied to evaluate the solution to OPF model in the presence of wind farm. A respective penalty and reserve cost for over and under estimation of wind power has been included in the overall cost function of the problem. Mostly the modification in OPF problem formulation with renewable energy sources such as wind and solar are broadly incorporated in two ways: 1. Incorporation of intermittency of wind speed 2. Solution methodology. Since, the Weibull probability distribution resembles the practical wind speed distribution; the wind speed is always modelled with Weibull probability distribution. Most of the research works utilized Weibull probability density function to model inherent stochastic nature of wind speed and has been included in OPF to account for uncertainty cost associated with wind generation. The cost of surplus and deficit in wind power generation is reflected as reserve and penalty cost in the overall operation cost of OPF [5]-[8]. The wind power uncertainty scenarios have been generated using Monte-Carlo simulations. The basic cuckoo search algorithm has been modified and proposed for computing the optimal active power generations [8]. In this work, the actual IEEE 30-bus system has been modified with wind generation at one of the load buses. The active power generations of conventional generating sources along with wind energy source have been optimised and the results are compared with the modified PSO.

Apart from OPF carried out on a healthy system, the contingency included OPF were also attempted by various researchers to assess the security of the system. This type of Security Constrained OPF (SCOPF) in the presence of stochastic renewable sources is currently on the research fronts. SCOPF is mainly carried out to study the system security during various system contingencies such as line outage and generator outages. When it comes to stochastic SCOPF problem formulation it is in need of the details of stochastic renewable power generations such as wind speed and solar irradiation distributions. To avoid such kind of dependencies on actual details of those sources, several prediction techniques and probable distributions were explored for their modelling. Most of these techniques were proposed by using time series methods, fuzzy logic and neural networks. Kiran et al [10] proposed new fuzzy based artificial physics optimisation algorithm to solve SCOPF in the presence of wind power generation. The uncertainty factor has been incorporated as the cost function for underestimated and overestimated wind power. The respective penalty and reserve cost is added to the fuel cost.
of the system. Several such security constrained OPF problem formulations have been proposed by authors in [11]-[13]. Marcelino et al. [11] proposed a hybrid evolutionary approach for SCOPF whereas authors in [12]-[13] applied conventional methods for SCOPF in the presence of stochastic wind generators.

The rest of the paper is organised as follows. A brief mathematical modelling of wind speed, wind power and respective cost of overestimation and underestimation are presented in section 2. The problem of Stochastic OPF in the presence of wind power has been formulated and the proposed solution methodology has been discussed in section 3. Section 4 presents the simulation results, comparison and discussion on the solution methodology proposed. Finally section 5 concludes the paper.

II. MATHEMATICAL MODELLING OF WIND POWER

A. Wind speed modelling

There are various methods to model the wind speed behavior but very often, the recommended expression for wind speed modeling is the Weibull probability density function, which resembles closely to the actual distribution of wind speed at different sites. The wind speed profile estimated using Weibull pdf \( f_w(v) \) is given by

\[
f_w(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp\left[ -\left( \frac{v}{c} \right)^k \right]
\]

Where, \( k \) is called the shape factor and \( c \) is called the scale factor. Scale and shape factor are calculated using the mean (\( \bar{v} \)) and standard deviation (\( \sigma \)) of wind speed data. The expressions for \( k \) and \( c \) are as follows:

\[
k = \left( \frac{\bar{v}}{\sigma} \right)^{-1.186}
\]

\[
c = 1.12\bar{u}
\]

B. Calculation of wind power output

The output power of the wind generator highly depends on two main factors: one is the wind speed of that particular site and the other is the parameters of power performance curve. Therefore, once Weibull pdf is generated, the wind power output can be calculated using the following relation:

\[
P_w(v) = \begin{cases} 0 & 0 \leq v \leq v_{ci} \\ P_r \cdot \left( \frac{v - v_{ci}}{v_r - v_{ci}} \right) & v_{ci} \leq v \leq v_r \\ P_r & v_r \leq v \leq v_{co} \\ 0 & v_{co} \leq v \end{cases}
\]

Here \( v_{ci} \) and \( v_{co} \) are cut-in and cut-out wind speeds whereas \( v_r \) is the rated wind speed and \( P_r \) is the rated power output of the wind generator. If the wind speed is below \( v_{ci} \) and above \( v_{co} \), the wind generator power output is zero. The wind generator power output corresponds to its rated power if the wind speed is within \( v_r \) and \( v_{co} \). It varies linearly if the wind speed is between \( v_{ci} \) and \( v_r \).

The linear transformation of wind power distribution from Weibull distribution of wind speed is given by:

\[
f(P_w) = f_w(v)\left( \frac{dP_w}{dP_w} \right) = f_w(v)\left( \frac{v_r - v_{ci}}{P_r} \right)
\]

From the above eqn. (5), the wind power distribution for linear portion is given by:

\[
f(P_w) = \frac{k(v_r - v_{ci})}{CP_r}\left( v_r P_r + P_w(v_r - v_{ci}) \right)\exp\left( -\left( \frac{v_r P_r + P_w(v_r - v_{ci})}{CP_r} \right) \right) - \frac{1}{CP_r}
\]

The probabilities of getting zero and rated power output from wind turbine for discrete portions are as follows:

\[
f(P_w = 0) = 1 - \exp\left( -\left( \frac{v_{ci}}{c} \right)^k \right) + \exp\left( -\left( \frac{v_{co}}{c} \right)^k \right)
\]

\[
f(P_w = P_r) = \exp\left( -\left( \frac{v_r}{c} \right)^k \right) + \exp\left( -\left( \frac{v_{co}}{c} \right)^k \right)
\]

C. Modelling of expected uncertain cost of wind generator

The wind speed is highly stochastic in nature and so is the power output of the wind generator. The actual generated output of wind turbine may not be equal to the forecasted and scheduled power at all the time. Due to this uncertain nature of wind power output, there may be an unexpected increase in the overall system operating cost to supply the demand. So, these uncertain factor involved in the wind turbine output has to be accounted in the system operating cost. Thus, overestimation and underestimation of wind power output with respect to the scheduled power is to be reflected in the overall objective function. In case of overestimation, the actual power available from the wind generator is less than the scheduled power of that particular generator, so the particular case has to be imposed with a penalty for the shortcoming of the scheduled power. This penalty accounts either for power purchase from the other sources or load shedding. Whereas in case of underestimation, the actual power generated from wind turbine is more than that of the scheduled wind power. The excess power generated is either used to charge the batteries or simply wasted in dummy resistors. The extra power generated is encoded as a reserve cost in the objective function. The uncertain costs incurred for both underestimation and overestimation of wind power generation is given as follows:

\[
C_{UE}(P_{scheduled}) = K_w \int_0^{P_{sch}} (P_{actual} - P_{scheduled}) \cdot f(P_w(v)) \cdot dv
\]

\[
C_{OE}(P_{scheduled}) = K_w \int_0^{P_{sch}} (P_{scheduled} - P_{actual}) \cdot f(P_w(v)) \cdot dv
\]
III. PROBLEM FORMULATION AND SOLUTION METHODOLOGY

A. Problem formulation

The objective of this stochastic OPF problem is to minimize the total cost of operating conventional and wind generators. Thus, the objective function includes the fuel cost of conventional units and expected uncertainty costs of wind generation. Mathematically, it is formulated as:

\[ F_T = \sum_{i=1}^{N_p} a_i P_{Gi}^2 + b_i P_{Gi} + c_i + \sum_{i=1}^{N_w} C_{OE}(P_{scheduled,i}) + \sum_{i=1}^{N_w} C_{UE}(P_{scheduled,i}) \]  

Subject to:

1. Equality constraints:

\[ P_p - V_p \sum_{q=1}^{N} Y_{pq} V_{pq} \cos(\delta_{pq} - \theta_{pq}) = 0 \]  
\[ Q_p - V_p \sum_{q=1}^{N} Y_{pq} V_{pq} \sin(\delta_{pq} - \theta_{pq}) = 0 \]

2. Inequality constraints:

\[ V_{i}^{\text{min}} < V_i < V_{i}^{\text{max}} \quad i \in N \]  
\[ P_{w,i}^{\text{min}} < P_{w,i} < P_{w,i}^{\text{max}} \quad i \in N_w \]  
\[ Q_{G,i}^{\text{min}} < Q_{G,i} < Q_{G,i}^{\text{max}} \quad i \in N_g \]  
\[ Q_{cap,i}^{\text{min}} < Q_{cap,i} < Q_{cap,i}^{\text{max}} \quad i \in N_{cap} \]  
\[ T_{i}^{\text{min}} < T_i < T_{i}^{\text{max}} \quad i \in N_T \]

The first term of the objective function corresponds to the summation of fuel cost of all thermal units. The next two consecutive terms indicate the uncertainty cost associated with wind generators. The uncertain cost is decomposed into two terms, one for the overestimation and other for the underestimation of wind power output. 

For a solution methodology of stochastic OPF procedure is implemented in following steps:

1. Set initial values all system data and algorithmic parameters
2. Definition of various variables used in the OPF
   a. Control variables: These are the independent input variables to power flow. A total of 18 control variables are considered. They are: \( P_g \) (Active power generations at all PV bus except slack bus), \( P_{scheduled} \) (Active power generation of wind generator buses), \( V \) (specified voltage at all PV buses including slack bus), \( Q_{cap} \) (switched shunts at compensated load buses) and \( T \) (Taps of all off-nominal tap changing transformers). Out of all 5 types of control variables, first 3 are of continuous variables and rest 2 are discrete variables.
   b. State variables: These are the solutions obtained from power flow with control variable as inputs. 
   c. Output variables: \( P_T, Q_s \) (at all generator and wind bus) and \( P_p \) (at all lines).
3. Initialize harmony memory, HM (the population of control variables within their limits) according to (12)-(19).
4. Evaluation of objective function using (11).
   a. Fuel costs of thermal generators are calculated using their cost coefficients.
   b. Expected uncertainty cost calculation of wind generator utilizes Monte-Carlo simulations. The following steps summarize the approach:
      i. Generate random wind speed which follows Weibull probability distribution.
      ii. Calculate actual real power available from wind generator \( (P_{actual}) \) using (4).
      iii. Verification of underestimated \( (P_{scheduled} < P_{actual}) \) or overestimated \( (P_{scheduled} > P_{actual}) \) conditions. Where \( P_{scheduled} \) corresponds to decision variable for wind generator.
      iv. Calculation of uncertainty cost \( C_{OE} \) or \( C_{UE} \) for wind generator using (9) and (10) based on step (iii).
      v. Repeat steps (i) to (vi) for N times. (N- number of Monte-Carlo simulations).
      vi. Calculate the expected cost of the uncertainty cost function for wind generator N Monte-Carlo simulations.
   c. While (iter<itermax)
   d. Harmony search improvisation process on the decision variables
      For all \( p \in P \)
      For all \( d \in D \)
      Generate random number rand ()
      If rand () <HMCR
      Select a solution \( x_i \) from the existing population
      If rand () <PAR
      \[ x_i^{new} = x_i + \text{rand}() \times bw \]
      End if
else
    \( x_{\text{new}} \) is randomly generated
End if
End for

Evaluate objectives for the new updated population (step b).

e. Combine the old and new population, best solutions of initial population size is taken to next iteration, step d is repeated till convergence is met. Best solution corresponds to minimum cost function of the problem.

f. Print the results for validation.

IV. SIMULATION AND RESULTS

In this paper, the stochastic Optimal Power Flow (OPF) problem is tested on IEEE 30-bus system using harmony search algorithm. A wind farm consisting of 20×2MW identical wind generators are assumed to be located at 22nd bus in the system. Wind speed modelled with Weibull probability distribution with shape factor, k=2 and scale factor, c=10. To account for the uncertainty, Monte-Carlo simulations (N=1000) are evaluated to calculate the wind power cost. The expected uncertainty cost of wind farm is reflected in two ways: overestimated and underestimated wind power from the scheduled wind power. In this work, the cost coefficients for underestimation (\( K_{\text{ue}} \)) is taken as small compared to overestimation (\( K_{\text{oe}}=10 \)) to show the relative significance of the particular case i.e. in case of underestimation of wind power, the extra unused power is wasted power but in real time power system application, such power can be stored in energy storage systems, thus considered under reserve cost. In case of overestimation case, the cost for overestimating the wind power which is less than the scheduled wind power is treated as penalty.

The generator active power generations limits and cost coefficients of IEEE 30-bus system is referred from [8]. The limits on specified voltage in the generator buses are taken as 0.9-1.1 p.u. There are two switching shunts located at buses 10 and 24 with a value of 19 and 5 MVAR, respectively. These two shunts are discretized with step size of 1MVAR and treated as a set of control variables. Off-nominal tap transformers located at lines connecting buses 6–9, 6–10, 7–12 and 27–28 are also considered as one set of control variables in this paper. For this reason, the off-nominal taps of the transformers are also discretized with the step size of 0.025 p.u. within the range of 0.95–1.05 p.u. Population of 30 with 300 maximum iterations are considered in this work.

Fig.1. presents the stochastic nature of randomly generated wind speed and respective wind power obtained from Monte-Carlo simulation. It depicts the uncertainty in the wind power generation at bus 22. It is further demonstrated by histogram for wind speed and wind power from Monte-Carlo simulations in Fig.2 It is seen from the distribution of wind speed that it follows Weibull probability distribution. Similarly, Fig. 3 shows the histogram of expected uncertainty cost of wind power for underestimated and overestimated cases. Fig.4 shows the voltage profile of the IEEE 30-bus system with wind generator located at bus 22.
The results obtained by the harmony search algorithm have been tabulated in Table I. The results are compared with Cuckoo search algorithm and modified PSO [8]. The optimal values of the control variables obtained by harmony search algorithm are within their limits. Total cost of production is also same as that of the other methods. The total fuel cost of all thermal units is less in case of the proposed methodology compared to cuckoo search and modified PSO. Moreover, the line loss reduction is also appreciable and total loss obtained is 7.1783MW in case of harmony search algorithm but its 7.8MW in cuckoo search and 8.09MW in MPSO.

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<tr>
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<tbody>
<tr>
<td></td>
<td>Voltage (p.u) P_g (MW)</td>
<td>Voltage (p.u) P_g (MW)</td>
<td>Voltage (p.u) P_g (MW)</td>
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<tr>
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<td>1.1000 163.8355</td>
<td>1.1000 166.6133</td>
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<tr>
<td>2</td>
<td>1.1000 45.9758</td>
<td>1.0886 45.5225</td>
<td>1.0867 46.1629</td>
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<td>5</td>
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<td>1.0629 20.4511</td>
<td>1.0593 20.5314</td>
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<td>8</td>
<td>1.1000 11.2113</td>
<td>1.0731 13.3914</td>
<td>1.0639 10.0000</td>
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<td>11</td>
<td>1.1000 13.6437</td>
<td>1.1000 10.0000</td>
<td>1.1000 10.0000</td>
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<tr>
<td>13</td>
<td>1.0900 12.5044</td>
<td>1.1000 12.0000</td>
<td>1.1000 12.0000</td>
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<tr>
<td>22(wind)</td>
<td>1.0587 27.3478</td>
<td>1.0592 25.9991</td>
<td>1.0738 26.1662</td>
</tr>
<tr>
<td>10</td>
<td>10 19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>24</td>
<td>3 5</td>
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Switching Shunt values, MVAR

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<th>Transformer tap</th>
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<td>6-9 1.05 1 0.95</td>
</tr>
<tr>
<td>6-10 1.05 0.95 0.95</td>
</tr>
<tr>
<td>4-12 1.05 1 0.95</td>
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<td>28-27 1.025 0.975 0.95</td>
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Costs and convergence time

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<th>787.8148</th>
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<td>Cost of wind uncertainty ($/h)</td>
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<td>Number of iterations</td>
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<td>300</td>
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<tr>
<td>Losses (MW)</td>
<td>7.1783</td>
<td>7.81</td>
<td>8.09</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper, stochastic OPF problem formulation in the presence of wind farm has been discussed and solution methodology using harmony search algorithm has been proposed. The fuel cost of thermal generators and uncertainty cost associated with wind generators are minimized along with line losses. The line losses are evaluated after incorporating the optimal generations, voltage set points at the generator buses and optimally discretized switching shunts and tap changing transformers in the conventional load flow. The uncertainty evaluations of wind power are carried out using 1000 Monte Carlo simulations and the respective over and underestimation of available wind power is reflected in terms of cost function included in the objective function. The comparison of results has been presented to show the effectiveness of the proposed solution methodology using harmony search algorithm.

REFERENCES


