Influence of Ground Impedance on Unbalanced Analysis of Grid

Raghunatha R, Muralidhara V and Thukaram D
BNMIT, BNMIT, IISc(retired)

Abstract—Renewable energy utilization and projects are being used as green energy initiatives worldwide. There are smallest possible renewable energy sources of the order of few KW to large wind mill and PV farms of the capacity of several 100 MW in India. Current systems are hybrid systems having combination of AC, DC, single phase, 3 phase, grounded and ungrounded systems with significant converters for interface with AC grid. Under these conditions the assumption that the transmission, distribution and loads are balanced is perhaps not acceptable. In this respect the influence of ground impedance modeling or more specifically the intentional impedance grounding of the neutral of the 3 phase network must be considered in the analysis. A common assumption made is “Ground potential is always at zero (0)” and neutral voltage can be eliminated in the formulation of analysis. IEEE standards however specify that ground resistance is neither zero nor uniform. Mathematical models are needed that consider the ground impedances and non zero neutral potentials of star connected windings. In this article hypothetical grounding impedances are considered and its influence on power system balanced and unbalanced operation are computed for static performance and analyzed. The outcome of the analysis shows that it is not correct to ignore ground impedance for unbalanced analysis, nor it is correct to assume neutral voltage as ‘0’ in star connected systems.

Index Terms——
- Neutral voltage influence in unbalanced system analysis
- Ground Impedance influence on neutral voltage
- Unbalanced system analysis

Notations:
- abc — denotes 3 phase a,b,c
- abcn — denotes 3 phase a,b,c with neutral n
- Zel, Ze2 hypothetical ground impedance at source and load end to an assumed 0 potential reference point in the ground
- Yabcn - 3Φ, 4 wire admittance matrix
- Zabcn - 3Φ, 4 wire impedance matrix
- ZabcnS - 3Φ, 4 wire source impedance matrix
- ZabcnL - 3Φ, 4 wire load impedance matrix
- ZabcnTL- 3Φ, 4 wire, transmission line impedance matrix

I. INTRODUCTION

The present day challenges in power system analysis with modern trends can be listed as follows

1. The present day systems are hybrid systems, with significant penetration of renewable energy which generate AC or DC power and integrates with the grid network at various voltage levels through power electronics converters for frequency matching and power control. These converters are likely to be both single and 3 phase converters.

2. Many renewable energy sources have their own local loads which have both AC and DC components and again may use converters suitably to cater to these loads.

3. The system as a whole is therefore hybrid system having AC and DC generation, network and loads.

4. Further it is common practice to intentionally ground the 3 phase 4 wire AC system using appropriate impedance to limit the earth fault current. Practical existing systems have both solidly grounded neutral and deliberately ungrounded systems.

5. Many continuous process industries are intentionally ungrounded to prevent stopping of the running process plant in the event of single phase to earth fault.

6. In addition, traditional power system analysis algorithms make typical assumption that ground potential is ‘0’ and neutral voltage of 3 phase 4 wire system is ‘0’ and simplify the modeling of the AC system.

IEEE standard 81 [2, 3] specifies (states) on earth resistivity, indicating earth resistivity can vary from $10^{-2} \Omega \cdot m$ to $10^6 \Omega \cdot m$ for different types of earth, soil conditions, and temperatures. It is evident from this, assuming uniform earth resistivity in the estimation of transmission line parameters may not be accurate assumption.

The following quote is reproduced from IEEE std 81, section 7.1 [2, 3].

“Earth resistivity varies not only with the type of soil but also with temperature, moisture, salt content, and compactness. The literature indicates that the values of earth resistivity vary from 0.01 to 1 $\Omega \cdot m$ for sea water and up to $10^6 \Omega \cdot m$ for sandstone. The resistivity of the earth increases slowly with decreasing temperatures from 25°C to 0°C. Below 0°C the resistivity increases rapidly. In frozen soil, as in the surface layer in winter the resistivity may be exceptionally high.”

Hence it is clear assumptions of ‘0’ resistance or constant resistance or ‘0’ potential of the earth is not valid. It makes sense to represent earth resistance and/or intentional ground impedance accurately in system mathematical models along with explicit neutral voltage variable modeling which is unlikely to be 0 under unbalanced operation conditions.

One of the standard assumptions made in 3 phase 4 wire model of transmission lines is that the neutral is solidly grounded and is at ‘0’ potential and hence by applying Kron’s reduction technique one may eliminate neutral voltage and current variable from the 4 variable equation of phases $a, b, c$ and $n$ to 3 phase variable equations of $a, b, c$. However, considering the IEEE standard 81 [2, 3], where earth resistivity is not ‘0’ and having varying ground potential for...
substation ground mat design [4, 5], it can be concurred, that assumption of ‘0’ voltage for the neutral is not correct. Further, many systems are deliberately grounded through impedance or ungrounded, in which case neutral voltage ‘0’ condition is not satisfied and Kron’s reduction from 4 variable a,b,c,n to 3 variable a,b,c equation model is not applicable.

In view of the currently available computing facilities, computer memory, data storage facilities, computational speeds compared to those available few decades back, it now seems that many simplifying assumptions used in earlier models may not be necessary any more. In this regard, it can be recalled sparse matrix and sparse vector methods were developed in view of limitations on computer memory and computer speed in earlier days. This may be the time to start using single or multiphase system models with explicit representation of the earth resistivity and grounding impedance and retaining the neutral current and neutral voltage in the model.

II. KRON’S NETWORK REDUCTION
The network impedance model relates the network node/bus voltages to nodal injection currents with the simple relation

\[
[Z][I] = [V] \tag{1}
\]

Suppose that there are a subset of nodes where the node voltages are exactly ‘0’, equation (1) may be partitioned with ‘0’ and non-0 voltages and the equation (1) may be rewritten as

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
=
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \tag{2}
\]

Where, subscript ‘1’ denotes nodes with non-zero values of voltages and subscript ‘2’ denotes node voltages with ‘0’ values.

Expand the equation (2) to get

\[
Z_{11}I_1 + Z_{12}I_2 = V_1
\]

\[
Z_{21}I_1 + Z_{22}I_2 = V_2 \tag{3}
\]

Substituting \(V_2 = 0\), in the second equation to get

\[
I_2 = Z_{22}^{-1}[0 - Z_{21}I_1] = -Z_{22}^{-1}Z_{21}I_1 \tag{4}
\]

Substituting (5) in (3) to get the

\[
[Z_{11} - Z_{12}Z_{22}^{-1}Z_{21}]I_1 = V_1 \tag{6}
\]

In equation (6) the variable \(V_2 = 0\) is eliminated and it gives the equivalent equation with both \(V_2\) and \(I_2\) eliminated. In the context of this paper, \(V_2\) and \(I_2\) refer to the neutral voltage and the neutral current of 3 phase 4 wire system.

In most 3 phase modeling such variable eliminations are used to eliminate the earth wires, with the assumption the ground potential is ‘0’, which in the light of IEEE std 81[2,3] may not be true. Also references [2, 3], states widely varying resistivity of the earth. In addition deliberate grounding through impedances is also practiced and ungrounded systems are also used in continuous process industries. The intention of this paper is to eliminate these approximations of omitting neutral variables and study the impact of representing the ground impedances explicitly with non zero neutral voltage under unbalanced operation.

III. 3 Φ EQUIVALENT / 3 Φ - 4 WIRE LINE MODEL
Most distribution analysis algorithms uses 3 phase equivalent model where the fourth equation related to the neutral voltage and neutral current are eliminated with the assumption that neutral voltage will be 0 in solidly grounded systems. However in view of the IEEE standard 81[2,3], and varying degree of ground resistivities, ground potential variation, ungrounded systems in continuous process industries, Deliberate grounding through impedances, there are no guarantee that neutral voltage will be zero under unbalanced conditions. In this section the two models where neutral voltage is eliminated and neutral voltage explicitly considered are listed for comparison and further analysis in subsequent section.

The 3Φ model discussed in this section is taken from the reference [1] showing the 3 phase conductors a, b, c and earth wire n.

Diagram -1 4-wire grounded neutral system
Diagram 1 shows a four-wire grounded neutral line segment. The neutral to ground impedance in this diagram is shown as 0 [solidly grounded] and the ground impedance between the sending and receiving end is ignored. It is proposed that the ground impedance can be modeled as neutral to ground impedance in such a way the ground potential variation can be taken into account; this is an additional refinement in the model as compared to the reference [1]. The mathematical model of the line shown is as follows

\[
\begin{bmatrix}
V_{ag} \\
V_{bg} \\
V_{cg} \\
V_{ng}
\end{bmatrix}
=
\begin{bmatrix}
\hat{Z}_{aa} & \hat{Z}_{ab} & \hat{Z}_{ac} & \hat{Z}_{an} \\
\hat{Z}_{ba} & \hat{Z}_{bb} & \hat{Z}_{bc} & \hat{Z}_{bn} \\
\hat{Z}_{ca} & \hat{Z}_{cb} & \hat{Z}_{cc} & \hat{Z}_{cn} \\
\hat{Z}_{na} & \hat{Z}_{nb} & \hat{Z}_{nc} & \hat{Z}_{nn}
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c \\
I_n
\end{bmatrix} \tag{7}
\]

Diagram 1 is modified along with equation (7) in Diagram 2, where the ground impedance and any intentional neutral wire
impedance grounding are included. The impedances $Z_{e1}$ (sending end neutral to ground impedance) and $Z_{e2}$ (receiving end neutral to ground impedance) and all voltages are referred to a ground reference point which is at ‘0’ potential [‘0’ potential is an assumption and denotes the reference voltage for other voltage measurements]. The idea is, as per IEEE standard 80, 81 [2, 3, 4, and 5], the ground potential is not uniform, but varies along the ground resulting in step and touch potential. Consequently it is essential to refer all voltages with respect to a reference point, which is chosen as ‘0’ potential point or a reference voltage point.

Where the shunt elements indicated in the diagonal of the Y bus of the equation (11) denotes the admittance to ground of the shunt elements, inclusive of loads, Norton source admittances, intentional grounding impedances, ground impedances.

From equation (11) it is now clear that admittance matrix formulation can be extended to any number of buses of 3 phase 4 wire systems. It may be noticed from the structure of equation and $[Z_{abcn}]$, that by simply specifying appropriate impedance values of the 4x4 matrix, any unbalanced, short circuited or open circuited conditions can be defined. Typically short circuit conditions are specified with numerically low values of the impedance and open circuit conditions may be specified with numerically very high values of impedances permitted in the computer. All other values may be specified within these two limits to denote either balanced or unbalanced operation. Further the ground impedance values may also be specified as shunt elements to be absorbed in the Y bus diagonal elements as part of either $Y_{shunt}$ or $Y_{shunt}'$.

At this stage only problem is to define impedances of the ground from different network points to the chosen reference point of ‘0’ voltage [a common ground point for all nodes], which is the scope for future research work.

IV. THE WORK OF CARSON ON OVERHEAD LINE PARAMETERS 

John Carson [1, 7] developed primitive impedance matrix calculations where the electric circuit current return path is ground. In his paper, Carson assumes the earth is an infinite, uniform solid with a flat uniform upper surface and a constant resistivity. Any end effects introduced at the neutral grounding points are not large at power frequencies, and are therefore neglected. The assumption of the constant resistivity and other specifications in this paper are possibly not valid considering the IEEE standards [2, 3]. The assumption the neutral voltage is ‘0’ is also not used in the example of this paper.

V. CASE STUDIES 

The example considers a small system data for examining the ground impedance influence on the neutral voltage and current, by modeling the neutral and ground impedances explicitly. The system considered has balanced source impedance, balanced transmission line impedance and unbalanced load impedance. All parameters considered are in ohms, volts, amperes and per unit calculations was not considered. The transmission line impedance parameters are taken from the example given in the reference [1]. The primitive line impedance model is given as

$$Z_{abcn} = \begin{bmatrix}
0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 & 0.0953 + j0.7524 \\
0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7802 & 0.0953 + j0.7865 \\
0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 & 0.0953 + j0.7674 \\
0.0953 + j0.7524 & 0.0953 + j0.7865 & 0.0953 + j0.7674 & 0.4013 + j1.4133
\end{bmatrix}$$
The source impedance is considered balanced and grounded through a 125 \( \Omega \) resistance as in conventional analysis and is given by the following matrix.

\[
Z_{abcS} = \begin{bmatrix}
0.0 + j0.1 & 0.0 + j0.0 & 0.0 + j0.0 \\
0.0 + j0.0 & 0.0 + j0.1 & 0.0 + j0.0 \\
0.0 + j0.0 & 0.0 + j0.0 & 0.0 + j0.0 \\
\end{bmatrix}
\]  \( (13) \)

The load impedance is considered with about 25% variation between the impedances of phase a, phase c, with respect to the phase b as follows. This degree of unbalance [25\%] is considered normal range of unbalance in loads in distribution network.

\[
Z_{abcL} = \begin{bmatrix}
1.43050 + j0.705 & 0.0 + j0.0 & 0.0 + j0.0 & 0.0 + j0.0 \\
0.0 + j0.0 & 1.907 + j0.94 & 0.0 + j0.0 & 0.0 + j0.0 \\
0.0 + j0.0 & 0.0 + j0.0 & 2.3835 + j1.175 & 0.0 + j0.0 \\
0.0 + j0.0 & 0.0 + j0.0 & 0.0 + j0.0 & 0.0 + j0.0 \\
\end{bmatrix}
\]  \( (14) \)

Where, RLN is the load neutral to earth reference resistance value in ohms. The value of RLN is varied from \( 10^{-2} \) \( \Omega \) to \( 10^{7} \) \( \Omega \) denoting well grounded to ungrounded system, and the influence of earth resistance on neutral voltage and currents is computed for analysis.

The Thevenin’s source voltage at the sending end is considered at 1250 volts and the 3 phase source voltages are considered balanced with 120 electrical degrees apart and the source neutral voltage is at exact 0 voltage potential and is the reference for measuring all other node voltages.

The total impedance to the ground from the Thevenin’s voltage is given by

\[
Z_{abcnTotal} = Z_{abcnS} + Z_{abcnTL} + Z_{abcnL} \quad (15)
\]

Using this total impedance and the Thevenin’s voltage, the current through the source to load via the return ground point for unbalanced operation is computed. The influence of the assumed range of ground resistance denoted by RLN in the load impedance is estimated. The results are summarized in the Table 1.

The plots of neutral voltage and neutral current variation at the load ends are also provided in Figures 1 and 2. These plots consider log-log plots considering the wide range of resistances to ground from load neutral.

### VI. Observations and Conclusions

It is seen from the figure 1 and 2 that the load neutral voltage increases almost linearly with the increase in the earth resistivity and soon reaches a saturation point. Conversely the neutral current decreases with increased earth resistivity for the given unbalanced condition. The source and transmission line unbalances was not considered and also the load unbalance considered was conservative in nature. In actual systems, source, transmission line and higher load unbalance can easily exist. It can therefore be concluded the neutral voltage under unbalanced condition which was around 6\% of phase voltage cannot be ignored and Kron’s approximation with the assumption of 0 neutral voltage may not be valid from engineering point of view. Further in view of hybrid nature of the present day power systems having both AC and DC sources and loads, both single and 3 phase AC network, AC and DC loads, and numerous converter interconnections with the grid, it is essential to make more detailed representation of the system for most system analysis for accurate results.

#### Table 1: Summary of neutral resistance (Ohms), voltage (mV) and current (mA)

<table>
<thead>
<tr>
<th>Resistance to Ground in Ohms</th>
<th>Load Neutral Voltage in mV</th>
<th>Load Neutral Current in mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{2} )</td>
<td>5.15</td>
<td>515.28</td>
</tr>
<tr>
<td>( 10^{1} )</td>
<td>51.49</td>
<td>514.91</td>
</tr>
<tr>
<td>( 10^{0} )</td>
<td>511.26</td>
<td>511.26</td>
</tr>
<tr>
<td>( 10^{-1} )</td>
<td>4773.77</td>
<td>477.38</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
<td>28710.9</td>
<td>287.11</td>
</tr>
<tr>
<td>( 10^{-3} )</td>
<td>57585.82</td>
<td>57.59</td>
</tr>
<tr>
<td>( 10^{-4} )</td>
<td>64024.72</td>
<td>6.40</td>
</tr>
<tr>
<td>( 10^{-5} )</td>
<td>64748.69</td>
<td>0.65</td>
</tr>
<tr>
<td>( 10^{-6} )</td>
<td>64829.33</td>
<td>0.06</td>
</tr>
</tbody>
</table>

![Figure 1: Load Neutral Voltage vs Neutral Resistance Variation](image1)

![Figure 2: Load Neutral Current vs Neutral Resistance Variation](image2)
VII. REFERENCES


[10] Web link on renewable energy sources, en.wikipedia.org/wiki/Renewable_energy_in_India


