An Iterative Distributed Approach for Optimal Power Dispatch in a Smart Grid Environment

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Abstract—An iterative distributed algorithm is proposed for existing power system optimization problems. Here the centralized optimal dispatch problem is explained in a decentralised manner. The line flow constraints along with generation limits are distributed into different nodes of the given network. In order to achieve a globally optimal solution, the incremental cost and the penalty factor for congestion are varying iteratively. The distributed algorithms provide flexibility, robustness, and scalability. The algorithm is tested for optimal power dispatch in an IEEE 39-bus system and welfare maximization problem in a 3-bus system. The results are validated with the standard centralised approach.

I. INTRODUCTION

One of the most significant operational problems in smart grid is the economic dispatch problem (EDP). The main purpose of this problem is to achieve minimum cost operating condition. The traditional operational methods solve the problem in a centralised manner where a centralised control centre is calculating the output power reference for each generator. The Lambda-iteration method, gradient methods [1] are the basic centralised approaches. The fast converging methods like Newton Raphson approach and dynamic programming are also available. The economic dispatch problem is an optimization problem, hence heuristic optimization methods like Neural Network approach, particle swarm optimization and genetic algorithm are also applicable. The economic operation can be achieved with transmission losses and congestion management.

In recent years, due to the advancements in power system, more and more renewable or distributed generations have been installed in conventional power systems. Thus the power system became more decentralised than before. So the traditional centralised control algorithms may not be accurate. One of the main challenges of the central controller is that it requires a high level of connectivity. Another challenge is the unknown smart grid topology. It may be because of different communication topologies or may be because of “Plug-and-Play” mechanism. So a robust algorithm is required for efficient decentralized operation in the presence of unreliable, limited communication. Hence a distributed control strategy is more acceptable for this problem.

There are different approaches are available for distributed optimization problem such as dual decomposition, primal decomposition, benders decomposition etc. Dual decomposition together with subgradient approach is being used to solve a cost minimization problem. The dual algorithm is converging faster than the primal decomposition algorithm with less communication, can be used for analysis and synthesis of distributed feedback controllers.

Consensus protocol is one of the efficient methods for distributed control. Distributed control approaches for distributed energy storage systems based on consensus algorithm are being discussed [2], [3]. A cooperative control strategy for coordinating Energy Storage Systems (ESSs) is proposed in [2]. In this approach, each agent can utilise its own information and exchange information with its neighbours. A leader-follower distributed control algorithm under cyber-attacked conditions is used in [3]. Distributed economic dispatch problem is being solved using consensus protocol-based approaches [4]–[8]. An incremental cost consensus (ICC) algorithm is being proposed to solve EDP [4]. Incremental cost is used as consensus variable and leader-follower consensus approach is used to represent the communication network. In [5], average power mismatch is used as consensus variable. A modified λ consensus algorithm is explained in [6]. During each iteration, a consensus + innovation term are updated, where consensus term ensures that the incremental cost is same at all nodes and the innovation term guarantees that the generation-demand equality constraints were met. In [7], a distributed approach for EDP based on consensus protocol is proposed. It includes two parallel running consensus algorithms, the first algorithm takes care of the local power mismatch and the second algorithm works on consensus on the most up-to-date information strategy. The consensus protocol is used to solve a non-convex economic dispatch problem [8]. The minimum cost operation is obtained by an auction mechanism. The bids generated by each generating unit is shared among the network agents using consensus protocol.

The economic dispatch problem gives an optimum economic operation without considering the line-flow limits. If the EDP results in congestion in any one of the transmission lines, then the next optimum solution has to be considered. The major problem in considering line-flow limits along with EDP is the decomposition of constraints. The optimal dispatch of Virtual Power Plant (VPP) is obtained by a distributed primal-dual subgradient algorithm [9]. This algorithm maximises the profit of the VPP.

A combination of dual decomposition and consensus al-
algorithm is proposed in this paper. The dual decomposition decomposes the centralised optimal dispatch problem and the global optimum solution is reached by using the proposed iterative distributed algorithm. The paper is composed as follows. The basic graph theory concepts and consensus protocols, which are useful in later sections, are explained in section 2. Section 3 formulates the problem and also discusses the centralized approach for the problem. Section 4 introduces the proposed distributed optimal dispatch strategy. The results of numerical simulation are explained in section 5. Finally, Conclusions are made in section 6.

II. PRELIMINARY

A. Basic Definitions of Graph Theory

In general, the topology of any communication network of the given power system network is modeled as a graph \( \mathcal{G} = (V, E) \), where \( V \) and \( E \) are the set of nodes (or vertices) and the set of edges, respectively. Each of \( e \in E \) is a set of two vertices. An undirected, unweighted graph without any multiple edges and loops is called a simple graph. The term graph is usually used to represent a simple graph. If there is a path from \( x \) to \( y \) in \( \mathcal{G} \) for every pair of distinct vertices \( x, y \in V(\mathcal{G}) \), then the graph is called connected graph. A simple graph may be either connected or disconnected. When the directions are given in the edges of a graph, then it is called directed graph or digraph.

Let \( y \) be a node in a graph \( \mathcal{G} \). The neighborhood of \( y \) in \( \mathcal{G} \) is \( N_\mathcal{G}(y) = \{ x \mid xy \in E(\mathcal{G}) \} \). The matrices \( L_\mathcal{G}, D_\mathcal{G} \) and \( A_\mathcal{G} \) for a graph are defined as the Laplacian matrix, Degree matrix and the Adjacency matrix. The adjacency matrix \( A_\mathcal{G} \) of a graph has all the same information that contained in the graph. The graph and the adjacency matrix are the two different representation of the same data. The adjacency matrix \( A_\mathcal{G} = [a_{xy}] \) is an \( n_\mathcal{G} \times n_\mathcal{G} \) matrix where \( n_\mathcal{G} \) is the number of nodes. The element \( a_{xy} \) has a non-zero entry when there exists an edge from vertex \( x \) to vertex \( y \). The adjacency matrix of a simple graph is a symmetric (0,1)-matrix with diagonal elements as zeros. The degree matrix \( D_\mathcal{G} = [\deg(x)] \) is a \( n_\mathcal{G} \times n_\mathcal{G} \) diagonal matrix, which carries the information about the degree of each vertex. The cardinality of the neighbourhood set \( N_x \) of vertex \( x \) is the degree \( \deg(x) \). The dynamic characteristics of the graph can be represented as a matrix called Laplacian matrix, \( L_\mathcal{G} = [\ell_{xy}] \) and is defined as

\[
\ell_{xy} = \begin{cases} 
  a_{xy}, & \text{for } x = y; \\
  -a_{xy}, & \text{for } x \neq y.
\end{cases}
\]

For an undirected graph, the laplacian matrix is symmetric positive semi-definite. The laplacian matrix can be calculated as

\[
L_\mathcal{G} = D_\mathcal{G} - A_\mathcal{G}.
\]

The column sum or row sum of laplacian matrix is a zero vector. The eigenvalues of laplacian matrix have a lot of information about the network dynamics. The algebraic connectivity of the graph is related to the second smallest eigenvalue of laplacian matrix. The convergence speed of the consensus algorithm depends on the algebraic connectivity of the network graph [11].

B. First Order Discrete Consensus Protocol

Let \( x_k \in \mathbb{R} \), is the state variable of each node \( k \), can be any one of the physical quantities like power mismatch, incremental cost, output power etc. The system reaches consensus when all the nodes attain the same state i.e. \( x_k = x_1 \) for all \( k,l \). Considering all the nodes have first-order dynamics, a standard linear consensus protocol [11] is given by,

\[
x_k(t) = \sum_{l} a_{kl} (x_l(t) - x_k(t)).
\]

If a group of agents is following the protocol in Eq.(3), then the collective dynamics can be written as

\[
x_k(t) = -L_{\mathcal{G}} x_k(t).
\]

where, \( L_{\mathcal{G}} \) denotes the Laplacian matrix of the graph. A discrete time linear consensus protocol [11] having first order dynamics can be represented as,

\[
x_k(p+1) = x_k(p) + u_k(p) \Rightarrow x_k(p+1) - x_k(p) = u_k(p).
\]

where, \( u_k(p) \) depends on the information state of the neighbours of \( k^{th} \) node and is given by,

\[
u_k(p) = \sum_{l} a_{kl} (x_l(p) - x_k(p)).
\]

From (5) and (6), the discrete time consensus algorithm can be written as,

\[
x_k(p+1) = \sum_{l=1}^{n_\mathcal{G}} s_{kl} x_l(p),
\]

where, \( p \) denotes the discrete time index; \( s_{kl} \) is the \( (k,l) \)th element of the row-stochastic matrix \( \mathcal{S} \), which can be defined as,

\[
s_{kl} = \frac{\ell_{kl}}{\sum_{l=1}^{n_\mathcal{G}} |\ell_{kl}|}.
\]

From equation (7), it is clear that the current state of each state variable depends on the previous state of the other state variables (state variables of the neighbouring nodes).

III. PROBLEM FORMULATION

A. Optimal Dispatch Problem

The main goal of optimal economic dispatch problem [1] is to minimize power generation cost and is given by,

\[
F(P_G) = \sum_{i=1}^{n_G} F_i(P_{Gi})
\]

where, \( F_i(P_{Gi}) \) and \( P_{Gi} \) are the cost characteristics and the active power output of generator \( i \) and \( n_G \) is the number
of generating units. The generation cost function can be approximated as a quadratic function and expressed as,

\[ F_i(P_{Gi}) = c_i P_{Gi}^2 + b_i P_{Gi} + a_i \]  (10)

where, \( c_i, b_i \) and \( a_i \) represents the cost coefficients of the generator unit \( i \). In optimal dispatch problem, the most economic operating schedule of the generators is obtained by considering several operational constraints. These constraints may include both equality and inequality constraints. The first operational constraint is the total power generation should be equal to the total load in the system. This is the generation-demand equality constraint and is given by

\[ \sum_{j=1}^{n} P_{Dj} = \sum_{i=1}^{n_G} P_{Gi}. \]  (11)

where, \( n \) denotes the total number of nodes and \( P_{Dj} \) denotes the load at each node. The inequality constraints are the generation limits and line flow limits.

\[ P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, i = 1, 2, \ldots, n_G \]  (12)

\[ |P_{\text{line}}| \leq P_{\text{line}}^{\max}. \]  (13)

where, \( P_{Gi}^{\min} \) and \( P_{Gi}^{\max} \) are the upper and lower active output power limits of generator \( i \), respectively. \( P_{\text{line}} \) is the active power flow through lines. \( P_{\text{line}}^{\max} \) is the power flow limits of lines. The optimal dispatch problem is a basic optimization problem. It will give an optimum operating schedule for generators without any congestion on transmission lines.

### B. Centralized Approach

There are different kinds of solution methods are available for optimal dispatch problem. One of the centralized approaches is Lambda-iteration. The fundamental idea behind the centralized solutions is the incremental cost \( \lambda \). It is the additional cost for adding or subtracting one unit of power. By neglecting the generation limits and the line flow limits, the Lagrange function of the problem is given by

\[ \mathcal{L}(P_{Gi}, \lambda) = \sum_{i=1}^{n_G} F_i(P_{Gi}) + \lambda(P_D - \sum_{i=1}^{n_G} P_{Gi}). \]  (14)

where, \( P_D \) denotes the total load in the system. At optimum condition, the partial derivative of Lagrange function is zero.

\[ \frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{dF_i(P_{Gi})}{dP_{Gi}} - \lambda = 0 \implies \lambda = \frac{dF_i(P_{Gi})}{dP_{Gi}}. \]  (15)

If the generator constraints and the line flow limits are added to the problem, then the Lagrange function becomes

\[ \mathcal{L}(P_{Gi}, \lambda) = \sum_{i=1}^{n_G} F_i(P_{Gi}) + \lambda(P_D - \sum_{i=1}^{n_G} P_{Gi}) + \sum_{i=1}^{n_g} \mu_i(P_{Gi}^{\min} - P_{Gi}) + \sum_{i=1}^{n_G} \mu_i(P_{Gi}^{\max} - P_{Gi}) + \sum_{j=1}^{n_{\text{line}}} \mu_j^{\text{line}}(|P_{\text{line}}| - P_{\text{line}}^{\max}) \]  (17)

where, \( n_{\text{line}} \) is the number of lines in the system and \( \mu, \bar{\mu} \) and \( \mu^{\text{line}} \) are the penalty factors for minimum generation, maximum generation and line flow limits violations, respectively. The essential condition for the existence of a global minimum operating condition is that all incremental costs must have the same value. The optimal incremental cost, \( \lambda^* \) is given by

\[ \lambda^* = \frac{dF_i(P_{Gi})}{dP_{Gi}} - \mu_i + \bar{\mu}_i + \sum_{j} H_{\mu_j^{\text{line}}}. \]  (18)

where, \( H \) denotes the Power Transfer Distribution Factor (PTDF) matrix. At optimum condition, \( \lambda^* \) will be same at all the generators but the locational marginal price at each generator will vary depending upon the penalty factors. The generated power \( P_{Gi}^* \) is given by

\[ P_{Gi}^*(p) = \begin{cases} \frac{\lambda_i(p) - b_i}{2c_i}, & \text{if } P_{Gi}^{\min} < P_{Gi}^*(p) < P_{Gi}^{\max} \\ P_{Gi}^{\min}, & \text{if } P_{Gi}^*(p) \leq P_{Gi}^{\min} \\ P_{Gi}^{\max}, & \text{if } P_{Gi}^*(p) \geq P_{Gi}^{\max} \end{cases} \]  (19)

where, \( \lambda_i(p) = \lambda^* + \mu_i - \sum_j H_{\mu_j^{\text{line}}} \). This indicates that the generators when operating on limits \( P_{Gi}^{\min} \) or \( P_{Gi}^{\max} \) have incremental cost \( b_i + 2c_i P_{Gi}^{\min} \) or \( b_i + 2c_i P_{Gi}^{\max} \), respectively while the others have the same incremental cost.

The \( \lambda \) iteration method can be used to solve this problem. All the generating units have to sent their active power output and cost functions to the centralized controller. The centralized controller determines the optimized solution for all the generating units.

### IV. DISTRIBUTED APPROACH FOR OPTIMAL DISPATCH

An iterative distributed optimal dispatch strategy is used to solve the optimal dispatch problem. The proposed method is based on dual decomposition and consensus protocol. The centralized optimization problem is decomposed into several sub-problems by dual decomposition algorithm. The sub-problems converge to a global optimum solution by the iterative distributed approach.

In this proposed method, the Lagrange dual function given in (17) is decomposed into sub-problems. The entire system is divided into different nodes. The buses having generating units are assumed as nodes. Each node will be updating their output
power based on the sub-problem solution. By neglecting the generation limits, the eq. (17) becomes,

$$ L(P_{Gi}, \lambda) = F_i(P_{Gi}) + \lambda(P_D - \sum_{i=1}^{n_G} P_{Gi}) + \sum_{j=1}^{n_{line}} \mu_j^{\text{line}} \left( |P_{line}| - P_{\text{line}}^{\max} \right). $$

The objective function of sub-problem is to minimize the Lagrange function given in (20) subjected to the generation constraints

$$ P_{\text{min}}^{\text{Gi}} \leq P_{Gi} \leq P_{\text{max}}^{\text{Gi}}. $$

The optimization problem given in eq.(20) and (21), are solved in each of the generation nodes.

In the proposed approach, leader-follower algorithm is used to converge to a globally optimal solution. In optimal dispatch problem, the incremental cost $\lambda$ is equal for all generators except for bounded generators. In this algorithm, the leader generator receives the output powers from all the follower generators. The leader checks the generation-equality constraint and calculates the transmission line flows using PTDF matrix. The leader fixes the values of $\lambda$ and $\mu_{\text{line}}$ based on the power mismatch and line overloading. The new $\lambda$ and $\mu_{\text{line}}$ is sent to generators using first-order discrete consensus algorithm [11]. Here, incremental cost and the penalty factor matrix are the consensus variables. The incremental cost [12] can be calculated as

$$ \lambda_i = \sum_{j=1}^{n_G} s_{ij} \lambda + \epsilon \Delta P_i. $$

where, $s_{ij}$ is the $(i, j)^{th}$ element of row stochastic matrix, $S$ and $\epsilon$ is a constant. In a row stochastic matrix, the sum of elements of each row is one. The penalty factor is calculated as

$$ \mu_{\text{line}}(\Delta P) = \sum_{j=1}^{n_{line}} s_{ij} \mu_{\text{line}} + \epsilon \Delta F_i. $$

The output power of each generator is given by,

$$ P_{Gi}^*(p) = \begin{cases} \lambda_i^{\text{Gi}}(p) - b_i \frac{2c_i}{P_{\text{Gi}}^{\max}}, & \text{if } P_{\text{Gi}}^{\min} < P_{Gi}^* (p) < P_{\text{Gi}}^{\max} \\ P_{\text{Gi}}^{\min}, & \text{if } P_{Gi}^* (p) \leq P_{\text{Gi}}^{\min} \\ P_{\text{Gi}}^{\max}, & \text{if } P_{Gi}^* (p) \geq P_{\text{Gi}}^{\max} \end{cases} $$

where, $\lambda_i^{\text{Gi}}(p) = \lambda_i - \mu_{\text{line}} \sum H(:, i)$, and $H$ is the PTDF matrix. The proposed distributed algorithm is depicted in Fig.1.
V. SIMULATION RESULTS

The effectiveness of the algorithm is tested for two different problems in standard test systems. The optimal power flow problem is tested in IEEE 39-bus system. The system contains 10 generators and 46 lines. The characteristics of generators are given in the archive provided by MATPOWER [13]. The branch data and line limits are also taken from MATPOWER. The simulations are done in MATLAB and the results are shown in Fig. 2.

The initial load demand is 5000MW. The load demand varied five times during the simulation. In optimal power dispatch problem, the optimization is distributed at generation nodes. One generator is treated as leader node, where $\lambda$ and $\mu_{line}$ are updated during each iteration, based on power mismatch and line overloading. The other nodes are treated as follower nodes. The generated power output $P_{G_i}$ at follower nodes is calculated based on the updated values of $\lambda$ and $\mu_{line}$ as given in eq.(24). The incremental cost and the penalty factors will get updated till a global optimum is reached. For a load demand of 6254.20MW, the system converges to global optimum with $\lambda = 20.40658$/MW. The total cost is minimised to 95578.2778 $. The proposed algorithm is validated with a standard centralized approach. The optimal power dispatch problem is solved in centralized approach in MATPOWER [13]. The comparison results are shown in Tables I - III. The algorithm comparison results for a 6-bus system having 3-generators and IEEE 39-bus system are given in Tables I and II, respectively. The proposed algorithm is verified in a larger system, and the results are compared with the centralized approach and are given in Table III.

The welfare maximization problem is solved in a 3-bus system having 2 generators connected at bus 1 and 3 respectively and the load is connected to bus 2. The generation and load characteristics of the system are given in table IV. In this problem, an additional term is added to the objective. The objective becomes the price - cost. The line-flow limits are also included along with other constraints such as generation-demand constraints and generation limits constraints. The optimization problem is being solved at each node and converges to common incremental cost. The system converges to an optimal solution after few iterations. The obtained generation and load values are $P_{G1} = 43.62$ MW, $P_{G3} = 52.06$ MW and $P_{D2} = 95.68$ MW. The incremental cost is obtained as 4.3 $/MW with welfare $ 407.69.

### Table I: Comparison for 6-bus system

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Centralized Approach</th>
<th>Proposed Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{G1}$</td>
<td>68.3310</td>
<td>68.3304</td>
</tr>
<tr>
<td>$P_{G2}$</td>
<td>116.1084</td>
<td>116.1081</td>
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<tr>
<td>$P_{G3}$</td>
<td>105.5606</td>
<td>105.5602</td>
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<tr>
<td>AS/MW</td>
<td>12.40</td>
<td>12.3974</td>
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<tr>
<td>Power (MW)</td>
<td>290</td>
<td>289.9986</td>
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<tr>
<td>Cost ($)</td>
<td>4021.04</td>
<td>4021.0279</td>
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### Table II: Comparison for 39-bus system

<table>
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<th>Parameters</th>
<th>Centralized Approach</th>
<th>Proposed Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{G1}$</td>
<td>239.834</td>
<td>239.8340</td>
</tr>
<tr>
<td>$P_{G2}$</td>
<td>506.473</td>
<td>506.4730</td>
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<tr>
<td>$P_{G3}$</td>
<td>644.8063</td>
<td>644.8064</td>
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<td>$P_{G4}$</td>
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<td>$P_{G5}$</td>
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<td>$P_{G6}$</td>
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<td>$P_{G8}$</td>
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<td>$P_{G9}$</td>
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<td>$P_{G10}$</td>
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<td>AS/MW</td>
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<td>Power (MW)</td>
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<tr>
<td>Cost ($)</td>
<td>95578.27</td>
<td>95578.2778</td>
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</table>

### Table III: Comparison for 118-bus system

<table>
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<th>Parameters</th>
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</tr>
</thead>
<tbody>
<tr>
<td>AS/MW</td>
<td>39.38</td>
<td>39.3814</td>
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<tr>
<td>Power (MW)</td>
<td>4242.0</td>
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<tr>
<td>Cost ($)</td>
<td>125947.88</td>
<td>125948.0387</td>
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![Simulation results for 3-bus system.](image-url)
TABLE IV
GENERATION AND LOAD CHARACTERISTICS OF 3-BUS SYSTEM

<table>
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<tr>
<th>Bus No.</th>
<th>o_i</th>
<th>b_i</th>
<th>c_i</th>
<th>P_{min}</th>
<th>P_{max}</th>
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<tbody>
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<td>1</td>
<td>0</td>
<td>4.03</td>
<td>0.0031</td>
<td>0</td>
<td>114</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3.53</td>
<td>0.0074</td>
<td>0</td>
<td>180</td>
</tr>
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VI. CONCLUSION

An iterative distributed approach for optimal dispatch is proposed. The algorithm is tested in standard test systems and compared with the centralized approach. Welfare maximization problem is also solved using the same distributed algorithm. The line flow constraints are effectively decomposed. This algorithm is scalable and feasible for larger system.

REFERENCES