Optimal PMU Placement Using Binary SOS Considering Measurement Redundancy and Channel Limitations

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Abstract— Reliable and precise monitoring of large power network can be assured with Phasor Measurement Units (PMUs). Optimal placement of PMU entrusts judicial investment in costly infrastructural support of synchrophasor measurement. This paper proposes a new method for solving optimal PMU placement (OPP) problem with maximum measurement redundancy and channel limitation constraint. The proposed method uses Binary Symbiotic Organism Search (BSOS) which has not been cited in literature before. The work is further extended to capital cost investigation for maximum measurement redundancy and channel limitation cases. Four standard IEEE test systems have been considered to underpin the proposed scheme. For worthy validation of the proposed OPP scheme, a comparative assessment has been drawn with some of the other existing methods from the literature. The proposed methodology can give efficient cost concerned OPP solution for the small as well as large-scale power system network with and without considering channel limitation.

Keywords— Binary Symbiotic Organism Search, Optimal PMU placement, Phasor Measurement Unit, wide area measurement systems

I. INTRODUCTION

After the invention of synchrophasor measurement technique, the wide area measurement, protection, and control (WAMPAC) has become an essential part of the modern power grid. Robust WAMPAC functionalities are supposed to immune the power system to small or large disturbances. A very basic demand of WAMPAC technology is complete observability of the network with PMUs which also improves situational awareness of system operators. One of the best options to get extremely reliable observability is to install the PMUs to each bus of the network. However, this scheme not only escalates the chances of high capital investment but also forces to handle large amount data. Moreover, the installation sites may also be limited to communication facilities and other physical constraints. If the full observability of the network can be guaranteed with a lowest number of PMUs then there will be a lot of saving in capital investment, digital space, and valuable control room space. This particular problem in literature has been specified as optimal PMU placement (OPP) problem and many approaches have been proposed in the literature to address this problem. Some of them related to this paper are discussed below.

The methods available for observability analysis can be broadly classified into topological and numerical observability analysis. Topological observability analysis [1-4] mainly focuses on building up of full rank spanning tree with the available measurements from the network. Baldwin et al. [5] have used a dual search algorithm with bisecting search and simulated annealing to fix minimum phasor measurement unit placement for complete system observability. Due to the complexity and inefficiency of building up of spanning tree, topological observability approach is not encouraged for large network systems. A novel numerical observability approach has been proposed by authors in [6-7] with orthogonal transformation and Hachtel’s augmented matrix method for state estimations. Korres et al. [8] have solved the observability problem with echelon matrix to avoid the zero-pivot error of the Jacobian matrix. Authors used factorization-based numerical observability analysis in [9]. The reduced Jacobian matrix based observability analysis has been used for energy management system application in some papers [10-11]. Roy et al. have used a heuristic approach in three stages to decide the installation locations of PMU [12]. The numerical approaches discussed above suffer from two main drawbacks, viz. increased computational burden due to a repetitive rank calculation of the Jacobian matrix and reduced accuracy because of the dominance of cumulative errors. Some optimization algorithms are also worth to mention for solving the OPP from the literature. The optimization algorithms can be categorized as mathematical approaches and evolutionary algorithms. The mathematical approach uses integer linear programming methods (ILP) for solving linear or linearized equations. Authors have used ILP extensively for solving OPP during last decades [13-15], but PMU placement using ILP can’t guarantee maximum measurement redundancy. The evolutionary algorithms are nature inspired and being popular for solving dynamic problems, capable of giving definite solutions, can be combined with other methods, and having large scale applications. Many evolutionary algorithms have been already applied to solve OPP problems considering various placement constraints. Among them, Tabu search [16-17], particle swarm optimization [18-19], genetic algorithm [20-21], binary cuckoo search [22] are worth to mention.
The prime objectives of the OPP are considered as minimization of the number of PMUs required for complete observability and maximization of measurement redundancy. Most of the papers do not consider maximization of measurement redundancy while solving OPP and thus limited to utilize the full potential of PMUs. The channel limitation of PMU is another important issue to be taken care of. The complete observability of the network can also be assured utilizing minimum measuring channels of PMU which will incur less expense for PMU hardware cost [23]. In this paper, binary symbiotic organism search (BSOS) has been used for the first time to find a suitable solution for OPP considering maximum measurement redundancy and measuring channel limitation. Cost analysis is shown further for both the cases and comparative cost analysis is drawn considering with and without measuring channel limitation of PMUs.

The PMU placement model which has been followed in this paper has been given in Section II. The proposed BSOS algorithm has been described in Section III. The problem formulation for maximum measurement redundancy and measuring channel limitation cases with cost consideration has been discussed in Section IV. The Simulation results and output of the proposed scheme has been shown in Section V. Finally the paper has been concluded in Section VI.

II. MODELLING OF PMU PLACEMENT SCHEME

The basic OPP scheme for an N bus power network can be addressed as (1).

$$\text{Min } \sum_{i=1}^{N} C_i L_i; \text{ subject to } F(L) \geq b$$  \hspace{1cm} (1)

where $C_i$ is the cost of PMU at the $i^{th}$ bus, $C_i$ is unity if the cost of the PMU is considered to be the same at each bus. $L$ is a binary choice variable whose entries can be determined as below.

$$L_i = \begin{cases} 1, & \text{if PMU is installed at } i^{th} \text{ bus} \\ 0, & \text{otherwise} \end{cases}$$

for $i = 1, 2, 3, ..., N$.

The current phasors of all the incident lines and voltage phasors of the PMU installed buses are available from PMU. The adjacent buses' voltage phasors can be obtained using line parameters and Ohm’s law.

III. SOS AND PROPOSED BSOS ALGORITHM

A. Basic SOS algorithm

Symbiotic organism search has been proposed in 2014 by Cheng and Prayogo [24]. The word ‘symbiosis’ refers to the interactive behavior between organisms in nature. Organisms in nature do not live in separation and relay on each other for survival and growth. The symbiosis in nature can be seen as facultative (beneficial but non-essential relationship) and obligate (mutually beneficial and dependent relationship). The symbiotic relationships can be found in nature may be of three types viz. mutualism, commensalism, and parasitism. Mutualism refers to that relationship where both the organism gets benefited from each other. Commensalism denotes the relationship where one species get benefited from cohabitation and other remains neutral. In parasitism, one species get benefited but other gets harmed from the relationship. The SOS algorithm uses the above-said relationships to find out the fittest organism within an ecosystem (initial population). Like other evolutionary algorithms, some fitness value ($F_Y$) is calculated for each organism which refers to the degree of adaptation of the organism to the ecosystem. The example of each phase of SOS has been given below for better understanding of mutualism, commensalism, and parasitism in SOS.

A good example of ‘Mutualism’ relationship can be found among flowers and bees. Flowers need pollination for expansion of its species through pollen. Bees fly from flower to flower and gather nectar to convert it to honey. This activity of bees helps flower in pollination by distributing the pollen among flowers. To model this mutualistic relationship an organism randomly selected from the ecosystem as $X_j$ to interact with the $i^{th}$ organism $X_i$. Now, both $X_i$ and $X_j$ are involved in a mutualistic relationship with an objective to get benefited from the relationship and new organisms can be obtained from (2) and (3).

$$X_{i的新} = X_i + \text{rand} (0, 1) \ast (X_{\text{best}} - MV \ast BF_i)$$ \hspace{1cm} (2)

$$X_{j的新} = X_j + \text{rand} (0, 1) \ast (X_{\text{best}} - MV \ast BF_j)$$ \hspace{1cm} (3)

$$MV = \frac{x_i + x_j}{2}$$ \hspace{1cm} (4)

rand (0,1) generates a random number with lower bound as ‘0’ and upper bound as ‘1’. Values of $BF_i$ and $BF_j$ are 1 or 2 representing partial or complete benefit from the partner. $X_{\text{best}}$ is kept as a target point for the betterment of the organisms. The organisms are updated if the present fitness is better than the previous fitness.

Commensalism can be seen in between shark and remora fish. Remora fish stay attached with a shark and eats leftover foods. In this way, remora fish gets benefited from the shark but sharks do not get benefit from it. To model this phase in the same way as mutualism phase a random organism $X_j$ is selected from the ecosystem and interacted with $X_i$. Assume $X_j$ will try to have benefited from $X_j$ and $X_{i的新}$ can be modeled as (5).

$$X_{i的新} = X_i + \text{rand} (-1,1) \ast (X_{\text{best}} - X_j)$$ \hspace{1cm} (5)

A part of (5) i.e. $(X_{\text{best}} - X_j)$ reflects that $X_i$ is getting benefited from $X_j$.

The relation between human host and plasmodium parasite is an example of parasitism. Plasmodium parasite which passes through anopheles mosquito enters in human body and reproduces itself, causing malaria in human. Thus plasmodium parasite is benefited and human host is actively harmed. The modeling of parasitism phase is quite different from mutualism and commensalism phase. A new organism is created here, called parasite vector ($PV$) by duplicating and modifying the dimension of $X_i$. After this, $PV$ places itself in the position of a randomly selected host $X_i$ from ecosystem. If the fitness of the $PV$ is better than the host it will replace the host from the ecosystem otherwise the host will remain in the ecosystem and the $PV$ will be discarded from the ecosystem.
B. Binary SOS for solving OPP:

The prime disparity between SOS and Binary SOS (BSOS) is in the initialization of population. BSOS starts with an initial population consists of only binary position vectors (0 or 1). The binary position ‘0’ defines a bus as a non-PMU bus and ‘1’ defines as PMU installed bus. So, each organism is an initial solution set for a specified OPP problem, same as other evolutionary algorithms. Each organism is having a \( F_i \) which refers to as the degree of adaptation of an organism in the ecosystem. The update of the organisms takes place if the \( F_i \) of the respective organism is better than the previous. The 3 phases of SOS help to explore the better solution for the problem posed.

IV. PROBLEM FORMULATION

A. Maximization of measurement redundancy with BSOS

The ratio of a total number of observable buses \( OB \) in the system by PMU to the number of buses in the network is termed as measurement redundancy \( MR \). The maximization of \( MR \) and minimization of PMUs for full observability of the network is a prime objective for robust PMU placement approach; though very fewer papers have addressed this issue. Better \( MR \) is always preferred as it assures reliable estimation of the power system states. In this paper, an objective function \( OF \) has been formulated in (6) to get maximum \( MR \). As (6) only assures maximum \( MR \) so it is combined with (1) and transformed into an overall minimization problem \( (F_i) \) to satisfy both the objectives stated above.

\[
\text{Max } \sum_{i=1}^{N} \frac{OBI_i}{\text{total number of lines}} \quad (6)
\]

\[
F_i = \text{Min } \sum_{i=1}^{N} L_i + \text{Min } \sum_{i=1}^{N} \frac{N}{OBI_i} \quad (7)
\]

B. Consideration of measuring channel limitation of PMU with BSOS

The complete observability of the network can also be assured considering measuring channel limitations of PMUs too. The literature confirms that most of the papers considered a abundant number of channels \([4-21]\) which does not reflect the practical situation while it comes about the economical placement of PMU. The basic objective of PMU placement is complete network observability and if it is done by using a lowest number of channels then a lot of capital investment can be saved. A new \( OF \) in (8) has been modeled to confirm the involvement of a minimum number of channels. Equation (8) has been combined with (1) to minimize the number of PMUs for full observability.

\[
\text{Min } \sum_{i=1}^{N} \frac{OBI_i}{\text{total number of lines}} \quad (8)
\]

\[
F_i = \text{Min } \sum_{i=1}^{N} L_i + \text{Min } \sum_{i=1}^{N} \frac{OBI_i}{\text{total number of lines}} \quad (9)
\]

C. Cost assessment with and without considering channel limitation of PMU

The total cost of PMU infrastructure depends on many factors e.g. substation disruption cost, communication infrastructure cost, CT-PT connections, the number of measuring terminals (channels) etc. However, the main reason of different PMU costs is the number of measuring channel as rest of the costs are same for all the PMU installations \([23]\).

The assumption of equal PMU cost in the network gives more realistic OPP solution and promotes economic planning for OPP. While maximum \( MR \) is considered, PMUs are assumed to have sufficient number of channels so that it can measure current phasors of all the incident lines and voltage phasors of all the adjacent buses, but it is seen that the large PMUs which can provide up to twelve phasors and frequency are much costlier than the small PMUs which can provide up to three phasors and frequency \([23]\). So, the maximum \( MR \) and high reliability or minimum \( MR \) and low-reliability placement will have exalted capital cost and hence should be investigated properly before deployment. In this paper, the investment cost has been assessed for maximum redundancy and considering channel limitation (minimum redundancy) cases.

V. SIMULATION RESULTS AND ANALYSIS

A. BSOS for maximum measurement redundancy

The proposed algorithm has been applied to IEEE 14-bus, 30-bus, 57-bus, and 118-bus test systems \([25]\) for 30, 50, 500, and 1000 organisms and 50 iterations for each case respectively. A same number of organisms can be used for initialization also but in that case, there will be a repetitive initial solution for lower bus systems. The result obtained for this case has been given in Table I. The \( MR \) and \( FV \) as calculated from (7) have been given in Table I. For IEEE 14-bus system 4 PMUs are required for complete observability of the system. Table II shows the comparison of results in measurement redundancy and number of PMU perspective for the test systems under study. Table II shows, there is sufficient improvement in \( MR \) in the case of 30-bus, 57-bus, and 118-bus test systems with the proposed approach.

<table>
<thead>
<tr>
<th>Test cases</th>
<th>PMUs</th>
<th>PMU locations</th>
<th>MR</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-bus</td>
<td>4</td>
<td>2, 6, 7, 9</td>
<td>35.7</td>
<td>4.56</td>
</tr>
<tr>
<td>30-bus</td>
<td>10</td>
<td>2, 4, 6, 9, 10, 12, 15, 18, 25, 27</td>
<td>73.3</td>
<td>10.54</td>
</tr>
<tr>
<td>57-bus</td>
<td>17</td>
<td>1, 6, 15, 19, 22, 25, 27, 3, 36, 38, 39, 41, 46, 51, 52, 54</td>
<td>22.8</td>
<td>17.76</td>
</tr>
<tr>
<td>118-bus</td>
<td>32</td>
<td>1, 5, 10, 12, 15, 17, 20, 23, 29, 30, 35, 40, 43, 46, 49, 53, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 115</td>
<td>32.2</td>
<td>32.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test cases</th>
<th>PMUs</th>
<th>PMU locations</th>
<th>MR</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-bus</td>
<td>4</td>
<td>35.7</td>
<td>10</td>
<td>67.7</td>
</tr>
<tr>
<td>30-bus</td>
<td>4</td>
<td>NR</td>
<td>17</td>
<td>19.29</td>
</tr>
<tr>
<td>57-bus</td>
<td>4</td>
<td>10</td>
<td>66.7</td>
<td>NR</td>
</tr>
<tr>
<td>118-bus</td>
<td>4</td>
<td>NR</td>
<td>10</td>
<td>NR</td>
</tr>
<tr>
<td>Proposed</td>
<td>4</td>
<td>35.7</td>
<td>10</td>
<td>73.3</td>
</tr>
</tbody>
</table>

TABLE II. COMPARISON TABLE FOR MEASUREMENT REDUNDANCY

<table>
<thead>
<tr>
<th>Methods</th>
<th>14-bus</th>
<th>30-bus</th>
<th>57-bus</th>
<th>118-bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMU</td>
<td>MR</td>
<td>MR</td>
<td>MR</td>
<td>MR</td>
</tr>
<tr>
<td>Ref. [12]</td>
<td>4</td>
<td>35.7</td>
<td>10</td>
<td>67.7</td>
</tr>
<tr>
<td>Ref. [26]</td>
<td>4</td>
<td>35.7</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Ref. [27]</td>
<td>4</td>
<td>10</td>
<td>66.7</td>
<td>NR</td>
</tr>
<tr>
<td>Ref. [28]</td>
<td>4</td>
<td>NR</td>
<td>10</td>
<td>NR</td>
</tr>
<tr>
<td>Proposed</td>
<td>4</td>
<td>35.7</td>
<td>10</td>
<td>73.3</td>
</tr>
</tbody>
</table>

NR: not reported
B. BSOS for OPP considering channel limitation

In Section IV need of considering the channel limitation has been described. The OPP solution with higher measurement redundancy involves a maximum number of measuring channels where if channel limitation is considered the OPP solution obtained will give minimum redundancy. So, the benefits of higher MR are sacrificed for sake of installment cost. The simulation results obtained for the OF \((F_{2})\) has been given in Table III. For IEEE 14-bus case minimum measuring channels will be involved if PMUs are placed at bus number 2, 8, 10, and 13.

![Fig.2. PMU observability considering channel limitation](image)

Table III. Solution of OPP considering minimum measuring channel

<table>
<thead>
<tr>
<th>Test cases</th>
<th>PMUs</th>
<th>PMU locations</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-bus</td>
<td>4</td>
<td>2, 8, 10, 13</td>
<td>4.70</td>
</tr>
<tr>
<td>30-bus</td>
<td>10</td>
<td>1, 5, 8, 10, 11, 12, 19, 23, 26, 29</td>
<td>10.85</td>
</tr>
<tr>
<td>57-bus</td>
<td>17</td>
<td>1,2,6,13,19,22,25,27,32,36,41,45,47,51,52,54,57</td>
<td>17.82</td>
</tr>
<tr>
<td>118-bus</td>
<td>32</td>
<td>1,5,10,12,13,17,21,25,29,34,37,41,45,49,52,56,62,63,68,72,73,75,77,80,85,87,90,94,102,105,110,114</td>
<td>32.84</td>
</tr>
</tbody>
</table>

C. Cost analysis with and without channel limitation:

The different PMUs may have varied measuring channels and depending on that PMUs may incur different costs. In this section, the setting up cost of PMU has been analyzed with and without considering channel limitation. The following assumption has been made for the investment assessment purpose. As the basic installation cost of a PMU is same for all types of PMU so, the cost of a single channel PMU has been assumed as 1 per unit (pu) [23]. The additional channel cost has been considered as 0.1 pu for each channel e.g. the cost of a 4 channel PMU will be 1.3 pu. The observability diagram of IEEE 14-bus system without (maximum redundancy/sufficient measuring channels) and with (minimum redundancy/limited measuring channel) considering channel limitation has been given in Fig. 1 and 2 respectively. Fig. 1 shows that if the channel limitation is not considered for OPP then it takes total 15 measuring channel (total connectivity of PMU buses in solid lines) for complete observability of the system but it takes only 10 channels (total connectivity of PMU buses in solid lines) if channel limitation is considered. So, there will definite capital saving in the scheme shown in Fig. 2. Table IV shows the per unit cost assessment for IEEE test systems under study for without (case I) and with (case II) channel limitation.

![Fig.1. PMU observability without channel limitation](image)

Table IV. Channel requirement and per unit cost assessment

<table>
<thead>
<tr>
<th>Test systems</th>
<th>No. of PMUs</th>
<th>Number of channels (case I)</th>
<th>Number of channels (case II)</th>
<th>Investment cost (case I)</th>
<th>Investment cost (case II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-bus</td>
<td>4</td>
<td>15</td>
<td>10</td>
<td>5.1</td>
<td>4.6</td>
</tr>
<tr>
<td>30-bus</td>
<td>10</td>
<td>42</td>
<td>24</td>
<td>13.2</td>
<td>11.4</td>
</tr>
<tr>
<td>57-bus</td>
<td>17</td>
<td>53</td>
<td>47</td>
<td>20.6</td>
<td>20</td>
</tr>
<tr>
<td>118-bus</td>
<td>32</td>
<td>158</td>
<td>122</td>
<td>41.3</td>
<td>40.8</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper, two crucial issues of OPP viz. measurement redundancy and measuring channel limitation have been addressed with BSOS. Two new objective functions have been suggested to maximize the redundancy and minimize the number of measuring channel for complete observability of the network. It has been seen that there is a significant improvement in redundancy with the proposed method. The consideration of channel limitation helps to reduce the number of measurement channels requirement for complete observability. The cost analysis with and without considering channel limitation gives a better idea of PMU investment costs in the network. It has been seen that a lot of capital investment can be saved if PMUs are placed considering channel limitation, though there may be a reduction in measurement redundancy. The solution of OPP for minimum measuring channels will help the utility at the planning stage to save huge capital investment. The proposed algorithm can give OPP solution for small as well as large scale power systems.

REFERENCES


