Stochastic Unit Commitment using Two Point Estimate Method with Proliferation of Wind Power

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Abstract—The inherent flexibility of the power system is becoming weak with the proliferation of wind generators into the traditional power system. The intermittent and uncertain wind power adds to the problem of balancing load and generation. The proposal is made in two steps: in first step, a stochastic unit commitment model is presented for scheduling of the generating units while taking note of uncertainties in terms of load and wind generation by using two point estimate method. In the second step, an analytical study is presented such that the participation of different generating units for providing generation and flexibility. With this, a comparison between the proposed approach and the deterministic approach is also presented. The proposed approach is validated on modified IEEE RTS-96 system (including 96 conventional generating units and 19 wind farms).

I. INTRODUCTION

The never lasting increase in electricity demand and the technical modernization of supply has increased stress on the power system. Today, the power sector world is adopting clean, cheap and sustainable sources of power and there is no doubt that the renewable sources have emerged as great options. Wind energy source is quite popular for its sustainability and cleanliness at very low cost and is being integrated with power system. The load keeps changing with time and can be even estimated, but it shows significant fluctuation in real time. Hence, unit commitment is done for secured and reliable power delivery.

Unit commitment (UC) is an optimization problem used to determine the operation schedule of the generating units at every hour interval with varying loads under different constraints and environments. In other words, it can be stated as a problem of finding the order and schedule in which the units are to be started up and shut down over a period of time, say 24 hour, so the total operating cost is minimum subjected to the specified constraints. UC schedules the hourly on/off status and the MW outputs of each generating unit. The complexity of the UC problem proportionally increases with the number of units and considered constraints. For a large generating system, UC problem becomes a nonlinear, large scaled, mixed integer problem usually with large number of continuous and binary variables. Many different ways have been used to model a unit commitment problem depending on the selection of the constraints and optimization methods. In many research work, UC problem is modeled with classical optimization method like linear programming [1], lagrangian relaxation [2], dynamic programming [3], [4], mixed integer programming [5], branch and bound method [6], interior point method [7], benders decomposition [8]. Metaheuristic optimization methods like genetic algorithm [9], tabu search method [11], [12], neural network [11], particle swarm optimization technique [10], fuzzy algorithms [13] and binary fish swarm algorithm [14] have been used for modeling the complicated UC problem in many research work.

With integration of wind energy generators to the traditional power system, change in generating unit schedule and output is seen. The generation from costlier conventional generating unit reduces with increase in the level of wind penetration. Hence, the inherent flexibility of the power system gets weakened with integration of wind energy sources. Moreover, the random nature of wind results to intermittent power generation from the wind farm which consequently increases randomness in net demand. It raises problem in balancing the load and generation. The probabilistic methods such as monte carlo simulation method, Fast fourier transformation,point estimate method can be used for assessing the uncertainty in the net load of the power system. Monte Carlo Simulation (MCS) is the most commonly used simulation method due to its high accuracy [4]. Its execution can be extremely time-consuming. In this paper, two point estimate method (2PEM) is used to get rid of the time-consuming MCS method. The 2PEM requires a small level of data such as some of the statistical moments (i.e., mean, variance, skewness, and kurtosis) for approximating the probability functions of the uncertain variables [15].

Section II of this paper describes the two point estimate method which is used to provide generation schedule for real time considering uncertainties of wind generation and load. The problem formulation for both deterministic and stochastic UC models are discussed in Section III of the paper. Section IV demonstrates results and discussion and Section V presents the concluding remarks.

II. TWO POINT ESTIMATE METHOD

Two point estimate method is basically a variation of the original point estimate method (PEM). The problem is decomposed into several subproblems. The deterministic OPF is then run twice for each uncertain variable, once for the value
below mean, and once for the value above the mean, with other variables kept at their means. The steps are as follows:

1) Determine the number of uncertain variables (κ)
2) Set \( \xi(y) = 0 \) and \( \xi(y^2) = 0 \)
3) Set variable, \( v = 1 \)
4) Calculate the locations of concentrations, \( \eta_{v,1} \) and \( \eta_{v,2} \)
   \[ \eta_{v,1} = \sqrt{\kappa} \]  
   \[ \eta_{v,2} = -\sqrt{\kappa} \]  
5) Calculate the probability of both concentrations, \( P_{v,1} \) and \( P_{v,2} \)
   \[ P_{v,1} = P_{v,2} = \frac{1}{2\kappa} \]  
6) Finding out the two concentrations, \( x_{v,1} \) and \( x_{v,2} \)
   \[ x_{v,1} = \mu_{X,v} + \eta_{v,1}\sigma_{X,v} \]  
   \[ x_{v,2} = \mu_{X,v} + \eta_{v,2}\sigma_{X,v} \]  
7) Run OPF for both concentrations, \( x_{v,1} \) and \( x_{v,2} \) using \( X = [\mu_{X,1}, \mu_{X,2}, ..., \mu_{v,1}, ..., \mu_{v,k}, ..., \mu_{X,k}] \) and \( X = [\mu_{X,v}, \mu_{X,2}, ..., \mu_{v,1}, ..., \mu_{X,k}] \) respectively.
8) Update the \( \xi(y) \) and \( \xi(y^2) \)
   \[ \xi(y) = \sum_{v} \sum_{i} (P_{v,i} h(\mu_{X,1}, ..., \mu_{v,i}, ..., \mu_{X,k})) \]  
   \[ \xi(y^2) = \sum_{v} \sum_{i} (P_{v,i} h(\mu_{X,1}, ..., \mu_{v,i}, ..., \mu_{X,k}))^2 \]  
9) Evaluate the mean and standard deviation
   \[ \mu_y = \xi(y) \]  
   \[ \sigma_y = \sqrt{\xi(y^2) - \mu_y^2} \]  
10) Repeat steps 4 to 9 for all uncertain variables

### III. Problem Formulation

UC plays vital role in secured and reliable operation of the power system. It schedules on/off pattern and generation output of all generating units to meet the load at minimum operating cost for a given time interval. The problem formulation is done for lossless dc models for deterministic and stochastic UC problems.

A. Deterministic unit commitment with wind penetration

The objective of deterministic unit commitment is to minimize the total generation cost subjected to power balance, generating unit minimum and maximum outputs, ramp limits, minimum up and down times and transmission line constraints. With the assumption that system operator owns the wind generators, no cost is associated with the wind farms and it act as negative loading.

a) Objective function: The objective is minimization of generating cost i.e. sum of fixed, production and start-up cost as explained in (10)

\[
\text{Min } \sum_{i=1}^{T} \sum_{t=1}^{T} \left( A_{i} u_{i}(t) + \sum_{s=1}^{S} k_{s} g_{s}(t) + \text{SU}^{\text{cost}}_{i}(t) \right)
\]

where \( t \) denotes hours (1 to T), \( i \) denotes generating units (1 to I), \( A_{i} \) is fixed production cost for generating unit \( i \), \( u_{i}(t) \) is binary variable which is equal to 1 if generating unit \( i \) produces at time \( t \) otherwise equals to 0, \( s \) denotes cost curve segments for generating units (1 to S), \( k_{s} \) denotes slope for the cost curve segment of generating unit \( i \), \( g_{s}(t) \) denotes output of generating unit \( i \) on segment \( s \) at time \( t \), \( \text{SU}^{\text{cost}}_{i}(t) \) is start-up cost of generating unit \( i \) at time \( t \).

b) Constraints: Constraints reveal the limitations of the generating units for secure and reliable operation of power system.

- **Power balance constraints**
  \[
  \sum_{i=1}^{I} G_{i}(t) + \sum_{w=1}^{W} W_{G_{w}}(t) - \sum_{\{m,n\}} \delta_{m}(t) - \delta_{n}(t) X_{mn} \leq 0 \]
  \[
  + \sum_{\{m,n\}} \delta_{n}(t) - \delta_{m}(t) X_{mn} = d_{m}(t) \forall t \leq T, m \leq B
  \]

  where \( G_{i}(t) \) is output of generating unit \( i \) at time \( t \)

\[
G_{i}(t) = \sum_{s=1}^{S} G_{i,s}(t)
\]

and \( W_{G_{w}}(t) \) is wind generation by wind generating unit \( w \) at time \( t \), \( w \) is index for wind generating unit, 1 to W, \( B \) indicates buses(1 to B), \( l \) shows lines(1 to L), \( \delta_{m}(t) \) and \( \delta_{n}(t) \) denote voltage angle at bus \( m \) and bus \( n \) in radian at time \( t \), respectively, \( X_{mn} \) shows reactance of line connecting nodes \( m-n \) and \( d_{m}(t) \) is demand at bus \( m \).

- **Generation limit constraints**
  \[
  G_{i}^{\text{min}}(t) \leq G_{i}(t) \leq G_{i}^{\text{max}}(t)
  \]
  where \( G_{i}^{\text{min}}(t) \) and \( G_{i}^{\text{max}}(t) \) are minimum and maximum output of generating unit \( i \) at time \( t \), respectively.

- **Ramp limits constraints**
  Maximum ramp up rate
  \[
  \text{ramp}_{i}^{\text{up}} \geq G_{i}(t+1) - G_{i}(t)
  \]
  Maximum ramp down rate
  \[
  \text{ramp}_{i}^{\text{down}} \leq G_{i}(t+1) - G_{i}(t)
  \]
  where \( G_{i}(t+1) \) and \( G_{i}(t) \) indicate output of generating unit \( i \) at time hour \( t+1 \) and \( t \), respectively.

- **Minimum up and down times constraints**
  \[
  u_{i}(t) = \text{SU}^{\text{up},\text{off}}_{i}(t) \forall i \leq I, t \leq T \text{SU}_{i}^{\text{up},\text{min}} + T \text{SU}_{i}^{\text{down},\text{min}}
  \]
where $G_{i}^{on,off}(t)$ is on-off status of generating unit $i$ at time $t$, $T_{d}^{up,min}$ is duration of time for which generating unit $i$ has to be on at the beginning of the planning horizon, $T_{d}^{down,min}$ is duration of time for which generating unit $i$ has to be off at the beginning of the planning horizon.

$$\sum_{tt=t-G_{i}^{on,off}+1}^{t} u_{i}^{su}(tt) \leq u_{i}(t) \forall t \geq T_{d}^{up,min}$$

$$\sum_{tt=t-G_{i}^{on,off}+1}^{t} u_{i}^{sd}(tt) \leq 1 - u_{i}(t) \forall t \geq T_{d}^{down,min}$$

where $u_{i}^{su}(tt)$ and $u_{i}^{sd}(tt)$ are start-up and shutdown binary variables for generating unit $i$ at time $t$, respectively.

$$T_{d}^{up,min} = \max\{0, \min\{T, (G_{i}^{up} - G_{i}^{up,int}), G_{i}^{on,off}\}\}$$

$$T_{d}^{down,min} = \max\{0, \min\{T, (G_{i}^{down} - G_{i}^{down,int}) (1 - G_{i}^{on,off})\}\}$$

where $G_{i}^{up}$ and $G_{i}^{down}$ are minimum up and down time of generating unit $i$, respectively, $G_{i}^{up,int}$ and $G_{i}^{down,int}$ are the time for which generating unit $i$ has been up and down already before $t = 0$, respectively, $G_{i}^{on,off}$ is on-off status of generating unit $i$ at $t = 0$.

- Line flow constraints

$$l_{mn}^{\min} \leq \frac{\delta_{m}(t) - \delta_{n}(t)}{X_{mn}} \leq l_{mn}^{\max}$$

where $l_{mn}^{\min}$ and $l_{mn}^{\max}$ is minimum and maximum capacity of line between node $m$ and $n$.

- Voltage angle constraints

$$-\pi \leq \delta_{b}(t) \leq \pi \forall t \leq T, \forall b \leq B$$

voltage angle for all bus except the reference bus. For reference bus, the voltage angle is

$$\delta_{r}(t) = 0 \forall t \leq T$$

Wind energy penetration means the percentage of total annual demand supplied by wind energy. Assumed if the total wind energy generation is 300MWh for 24 hour and total demand for 24 hour is 900 MWh then wind energy penetration is 300/900 i.e 33.33%. It evaluates the wind contribution in total electrical energy consumption. However, the maximum allowable wind energy penetration is limited by the flexibility of the conventional generating unit.

Generating units ramp up and down, startup or shutdown to meet the net load at different time hour in different manner. This can be termed as generation cycle. The generation cycle is the percentage of difference between the generation output of a generating unit in two successive time hour to the maximum capacity of that generating unit and can be explained as (24).

$$Genracy_{i}(t) = \frac{G_{i}(t) - G_{i}(t-1)}{G_{i}^{max}} \times 100$$

B. Stochastic unit commitment with wind penetration

The stochastic unit commitment determines the operation schedule of the generating units at every hour interval with varying loads under different constraints and environments considering the uncertainties in the input parameters. Here, the loads and wind generation are taken as uncertain variables and two point estimate method (2PEM) is used for solving the stochastic problems. $\kappa$ is number of uncertain variable. $2\kappa$ are the number of scenarios which will get generated and deterministic unit commitment is run for all the scenarios. Different scenarios are generated by (1)-(5). The problem is formulated with objective function as (10) subjected to power balance constraint as (25) and other constraints as (12) - (23) for each scenarios and generation output is updated with (6)-(9).

$$\sum_{i=1}^{I} G_{i}(t) + \sum_{w=1}^{W} W G_{w}^{new}(t) - \sum_{\{m,n\} \in I | n \geq m} \frac{\delta_{m}(t) - \delta_{n}(t)}{X_{mn}} = d_{m}^{new}(t) \forall t \leq T, m \leq B$$

where $W G_{w}^{new}(t)$ is new uncertain wind generation by wind generating unit $w$ at time $t \kappa$ considering deviation in wind, $d_{m}^{new}(t)$ is new uncertain demand by bus $m$ at time $t \kappa$ considering deviation in load.

IV. Result and Discussion

A. Input Data

Unit commitment models are made in General algebraic modeling system (GAMS) 23.5.2 environment. GAMS solver CPLEX has been used to solve the mixed integer linear programming UC problem. These models are applied to modified IEEE RTS 96 system for 24 hour with hourly resolution. The cost functions, initial conditions and detailed data regarding the limits for generation, ramp and start up and shut down of the generating unit are taken from [16]. The data for system bus, load, transmission line and generating units at each bus is used from [17]. System load for performing UC is taken for Monday of the first week of the winter season considering 10% increase in the annual load. Hourly load is taken in percentage of daily peak load as per a weekday of the winter season. The total demand of the system for 24 hour is 150189.077 MWh. TABLE I shows position, capacity and generation of wind energy generating units for 24 hour time horizon.

B. Case Studies

Case 1. Deterministic unit commitment with wind penetration

The main aim of the deterministic UC is to schedule generating unit to satisfy the load at minimum generation
TABLE I  
WIND DATA

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</table>

- Objective cost
- Operation cost
- Startup cost

Fig. 2. Different types of cost associated in generation at different level of wind penetration (a) scenarios 1-4 without uncertainty in load and wind (b) scenarios 5-7 with uncertainty in load and wind.

![Net load of the system at different level of wind penetration](image)

Fig. 1. Net load of the system at different level of wind penetration

Case 1a. Deterministic unit commitment without wind penetration: No wind penetration is considered in this case and the conventional generating units of the system has to meet the demand (150189.077 MWh) for next 24 hour. Fig.1 depicts that maximum load is at $18^{th}$ and $19^{th}$ hour at 0% wind penetration. Fig.3 shows the contribution of different type of generating units in satisfying the demand in terms of percentage of total 24 hr demand. Scenario 1 of Fig.3 presents that maximum contribution in generation is of U400, U197, U101, U155, U76 and U350 (descending order). The generating unit U12, U20 and U50 are not scheduled for generation. However U12, U20 and U50 shut down and its participation is 3.12%, 2.0833% and 4.1667% of overall cycling by the generating units in 24 hour time horizon respectively and is depicted in scenario 1 of Fig.4. U155, U76 and U100 have maximum involvement in ramping to meet the demand while the generating unit U155 and U350 ramps up to meet the maximum demand at $18^{th}$ and $19^{th}$ hour.

Case 1b. Deterministic unit commitment with 5% wind penetration: For this case 5% wind penetration is allowed, which means conventional generating units of the system subject to a number of constraints. The unit commitment allows the cheaper generators to generate and meet demand firstly and then the costlier one are committed. Limits for ramp, transmission and minimum up and down time of generating units may schedule costlier generating unit before cheaper generating units to meet the demand. Wind energy penetration means the percentage of total day (24 hr) load shared by the wind. The increment in wind energy penetration leads to curtailment in the net load and hence, reduces the overall production cost from conventional sources. The costlier generator are avoided for production and changes is seen for the time at which maximum load occurs. This aggravate the condition in context to flexibility requirement. Higher flexible units shows more cycling to meet the demand. Fig.1 depicts netload to be served by conventional generating units at different levels of wind penetration. Fig.2 shows different type of cost associated in generation for different scenarios.
has to meet 142679.623 MWh demand for next 24 hour. Fig.1 depicts that maximum load is at 19th hour at 5% wind penetration and scenario 2 of Fig.3 shows maximum contribution in generation is of U400, U197 and U100. Generation of U76 increases while U155 decreases. The generating unit U12, U20 and U50 are not scheduled for generation. U155, U100 and U76 have maximum involvement in ramping to meet the demand as depicted in scenario 2 of Fig.4. The generating unit U155 and U350 ramps up to meet the maximum demand at 19th hour.

**Case 1c. Deterministic unit commitment with 10% wind penetration:** With 10% wind penetration, the net demand to be met by conventional generating units in next 24 hour is 135170.169 MWh. Fig.1 depicts that maximum load is at 10th hour at 10% wind penetration and scenario 3 of Fig.3 shows maximum contribution in generation is of U400, U197 and U100. Generation of U350 increases while U155 decreases. Again, in this case the units U12, U20 and U50 are not scheduled for generation. U155, U100 and U76 have maximum involvement in ramping to meet the demand shown in scenario 3 of Fig.4. The generating unit U155 and U350 ramps up to meet the maximum demand at 10th hour.

**Case 2. Stochastic unit commitment with wind penetration**

For stochastic unit commitment, both wind and load are taken as variables. 2PEM is used for generating different scenarios by considering deviation of 5% in wind generation and 10% in load demand. Curtailment in the total generation cost is seen with increase in wind penetration. Even, the total generation cost of stochastic UC is lesser in comparison to deterministic UC of the system. The overall generation cost reduces with increase in wind energy penetration where startup cost increases while operational cost decreases for lower penetration level and vice versa for higher wind energy penetration level. The cycling of generating unit increases in the stochastic scenarios.

**Case 2a. Stochastic unit commitment with 5% wind penetration:** The generating cost is lower than deterministic UC but startup cost increases as unit U50, U20 and U12 are made ON to meet the uncertain fluctuation in net demand. The pattern of contribution in generation by these units i.e. U400, U197, U100, U76 and U155 contributes in descending order is shown as scenario 5 of Fig.3. U155, U76 and U100 shows maximum participation in cycling and can be depicted as scenario 5 of Fig.5. Generating units shows more cycling on considering uncertainties. U20, U50, U155 and U350 ramp up to meet the maximum demand at 10th hour.

**Case 2b. Stochastic unit commitment with 10% wind penetration:** In this case, the generation cost reduces but startup cost increases in comparison to deterministic UC. The unit U350, U50, U20 and U12 are started up to meet significant change in net demand. U155, U100 and U76 have maximum involvement in ramping to meet the demand as shown in scenario 6 of Fig.5. Maximum contribution in generation is of U400, U197 and U100 and is depicted as Scenario 6 of Fig.3. The generating unit U12, U20, U50, U155 and U76 shows high fluctuation while U100, U350 and U400 shows little variance to meet the maximum demand at 10th hour.

**Case 2c. Stochastic unit commitment with 20% wind penetration:** In this case, both the cost for generation and startup decreases in comparison to deterministic UC. The unit U350, U50, U20 and U12 are started up to meet significant change in net demand. U155, U100 and U76 have maximum involvement in ramping to meet the demand shown in scenario 7 of Fig.5. Scenario 7 of Fig.3 depicts maximum contribution in generation is of U400, U197 and U100. The generating unit U12, U20, U50, U76 and U155 shows high variation to meet the maximum demand at 9th hour.
Fig. 5. Participation of different type of generating units in cycling on considering uncertainties at different level of wind penetration

Fig. 6 shows the participation of different types of generating units in meeting maximum demand for deterministic and stochastic schedule with different level of wind penetration at the forecasted peak hours. Significant variation in cycling of the generating units is depicted.

V. Conclusion

In this paper, a stochastic UC problem is modeled by applying 2PEM considering the uncertainties of wind and demand. The generation schedule varies dramatically on including uncertainty at different level of wind energy penetration. On analyzing the results of both deterministic and stochastic UC, it can be concluded that the contribution of generating unit U400 and U197 remains maximum while contribution of unit U155 and U350 in generation decreases in meeting the net demand with increase in wind penetration. Unit U100 and U76 participation in generation increases and the costlier generating units with high flexibility have high contribution in providing flexibility for the significant change in demand on considering uncertainties due to deviation of wind and load and increment in wind energy penetration.

REFERENCES