Study of Consumer Benefit Functions for Demand Response Algorithm

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***Abstract*—Demand response (DR) is an effective tool for energy management at the consumer end. The effectiveness of DR model depends not only on cost minimization but also on consumer satisfaction and comfort level. The satisfaction of the consumer is measured by benefit functions. In this paper, three benefit functions viz. Quadratic, logarithmic, and exponential have been modeled to study the DR algorithm for effective energy management. And also suggested a method to calculate approximated benefit function. The simulation results are presented for different scenarios.**

***Index Terms*—Benefit function, consumer load scheduling, demand response, demand side management, energy management.**

NOMENCLATURE

$\mathcal{A}$	Appliance vector.
$\mathcal{A}_{EL}$	Appliances belongs to elastic loads.
$\mathcal{A}_{PEL}$	Appliances belongs to perfectly elastic loads.
$\mathcal{A}_{PIL}$	Appliances belongs to perfectly inelastic loads.
E_a	Energy consumption of a appliance a to complete a task in a day.
$M(.)$	Marginal utility.
p_a^t	Power consumption of appliance a at the time t.
P_{sch}^{max}	Maximum power limit on power consumption.
t_α	Starting time of a task.
t_β	Ending time of a task.
$U(.)$	Benefit function.
$\pi(t)$	Electricity price.

I. INTRODUCTION

The main objective of demand side management (DSM) or load side management is to alter or control the power consumption patterns of consumer end at peak load. It helps both the load serving entity (LSE) and consumers. The consumer alters the load consumption pattern such that it supports the system security(Grid). If LSE controls the load, then it is called as direct load control, where in LSE has control over the appliances presented in the consumer household premises. It is based on the contract or agreement between the utility company and the consumer. It will enhance the system security in case of emergencies. But the disadvantage of this technique is a privacy concern. However, consumers controls their load when it is indirect load control. In this technique, consumers voluntarily schedules or alter their load. It is based on different pricing schemes like time of use pricing, real time pricing, and critical peak pricing. Otherwise utility companies provide incentives to customers for participation in demand response programs [1]-[4].

The objective of energy management via DR is to minimize the energy payment towards LSE without compromising the consumer comfort and benefit [3]-[4]. In [5], appliance scheduling was done by using concept of consumer efficiency, it is defined as ratio of benefit and cost. It is inherently same as maximizing the benefit and minimizing the cost.

To implement energy management system, the user should know the type and characterises of the appliances. The different types of home appliances classified into mainly two types (from the scheduling point of view) one is non-shiftable loads and other are shiftable loads. These two loads are classified into two sub categories ON/OFF loads and controllable loads. ON/OFF loads consumes rated power when the appliance is ON, otherwise it consumes zero power. Controllable loads can control the power from minimum power to maximum power. The controllable loads can further classify into continuous power consumption loads (CPC) and discrete power consumption loads (DPC). CPC loads like batteries, plug in electric vehicles. These appliances will consume any amount of power (within limits) and at any rate. The DPC loads like, air conditioner, fan etc., for these types of appliances whenever we are changing one state to other state the power consumption level will change by steps. The customer responds based on spot prices and needs. If the price of electricity is high, consumer may be shifting the load, control the load, curtail the load or based on urgency consumer can utilize the electricity at that price level.

This paper presents home energy management by consumer welfare maximization. Construction of different types of benefit functions is also studied and suggested a method to calculate the co-efficient of benefit function, and presented the simulation results with different scenarios.

In this paper, some assumptions are taken those are: 1) user has smart meter and smart appliances, and those can be communicable. 2) Smart meter should have functionality of energy scheduling. 3) Forecasted price signal is anticipated either by LSE or customer. In this paper, we are supposing that the day ahead price is given by the LSE.

The rest of the paper organized as follows. The system model and concept of benefit functions theory and types of

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benefit functions discussed in Section II. Problem formulation is presented in Section III. Results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model and Benefit Functions

Fig. 1 describes the brief overview of home energy management system. As shown in the figure, smart meter is the key component and it has functionality of energy scheduling and it can communicate with appliances and LSE. The smart meter schedules appliances based on the user's specification and forecasted price signal from the LSE.

Let assume a home, it has n appliances, appliance vector denoted by $\mathcal{A} = [a_1, a_2, ..., a_n]$. The classification of the appliances is presented in subsection D. Power consumption profile of a appliance over a day is given by

$$\mathbf{p_a^t} = [p_a^1, p_a^2, ..., p_a^T], \forall a \in \mathcal{A}, t \in \mathbf{T} \tag{1}$$

Here, $\mathbf{T} = [1, 2, 3, ..., T]$. In this paper, T taken as 24 and $p_a^t \geq 0$.

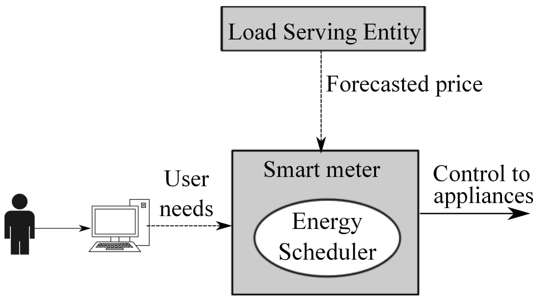

Fig. 1. Schematic diagram of energy scheduler.

The benefit function denotes the level of satisfaction of the consumer. The effective benefit function depends not only on the power consumption, but it depends on other parameters also those are amount paid for the consumed energy, time period during which appliance completed its task, and preference of appliance. If the price is high, consumer either controls or shifts the load based on the type of appliance. Some times user may neglect the some tasks to upgrade the individual benefit. Each appliance has adequate time limits, if the appliance accomplished its task during the specified period user will get more benefit. If a task is delayed, then the benefit is lowered. The properties of benefit functions [6]-[9] are presented in the following section.

A. Properties of Benefit Functions

- *Property 1*: Benefit functions are continuous, non-decreasing, and concave.

$$\frac{\partial U(P)}{\partial P} \geq 0 \tag{2}$$

- *Property 2*: Marginal benefit function is non-increasing.

$$\frac{\partial^2 U(P)}{\partial P^2} \leq 0 \tag{3}$$

- *Property 3*: Benefit at zero consumption of power is zero.

$$U(0) = 0 \tag{4}$$

B. Types of Benefit Functions

The construction of benefit functions based on properties discussed in previous section. The popular benefit functions are quadratic, logarithmic, and exponential [11].

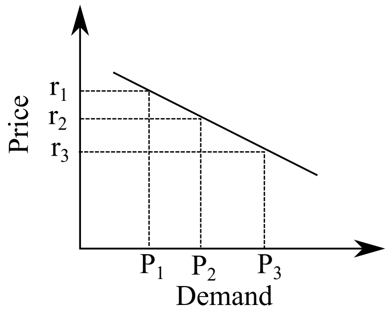

Fig. 2. Linear marginal benefit curve.

If the consumer response is linear and decreasing slope for increasing demand as shown in Fig. 2. It represented as

$$M(P) = a_1 - b_1 P \tag{5}$$

where b_1 is slope of the curve, P is power demand. The benefit function is integration of marginal benefit

$$U(P) = a_1 P - \frac{b_1 P^2}{2}. \tag{6}$$

In other case, consumer response is exponential decaying as shown in Fig. 3. It represented as

$$M(P) = a_2 - b_2 e^{kP} \tag{7}$$

here also, to get benefit function integrate the $M(P)$

$$U(P) = a_2 P - \frac{b_2 e^{kP}}{k}. \tag{8}$$

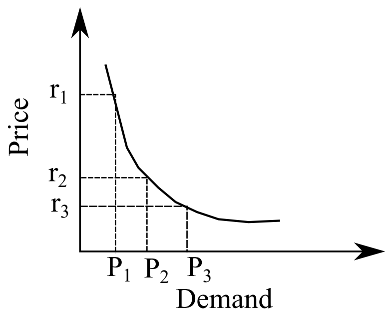

Fig. 3. Exponential marginal benefit curve.

There are many ways to represent benefit function. It is dependent on consumer's choice. Other types of benefit functions shown below [10].

$$\begin{aligned} U(P) &= k_1 log(1+P) && (9)\\ &= k_2 e^{k_3 P} && (10)\\ &= k_4 P^{\alpha} && (11)\\ &= a_3 - b_3 e^{-c_3 P} - d_4 P. && (12) \end{aligned}$$

Here, a, b, c, and k are general co-efficients of the benefit functions.

C. Co-efficient calculation for Benefit Function

The procedure to calculate co-efficients of a benefit function

- First, schedule the house load i.e., P_{sch}.
- Based on scheduled data and forecasted prise calculate the benefit at each time t.

$$U_{act}(P(t)) = P_{sch}(t)\pi(t) \tag{13}$$

  Here, U_{act} is actual benefit function.
- The objective function formulation to calculate unknown functions as follows

$$\textbf{minimize} \quad \sum_{t=1}^{T}(U_{act}(P(t)) - U(P(t)))^2 \tag{14}$$

- Solve the above optimization problem to estimate unknown co-efficients.

D. Classification of Loads

Home appliances are classified based on the concept elasticity. Those are namely,

- Perfectly inelastic loads
- Elastic loads
- Perfectly elastic loads.

1) Perfectly Inelastic Loads (PIL): Perfectly inelastic loads are must run loads. Power consumption of these appliance is independent of the price. These loads are non-controllable. $\mathcal{A}_{PIL} \in \mathcal{A}$.

$$\begin{aligned} p_a^t &= E_a, \forall t \in T_a, a \in \mathcal{A}_{PIL} & (15)\\ &= 0, \text{otherwise.} & (16)\end{aligned}$$

2) Elastic Loads (EL): Elastic loads are controllable loads. Power consumption of these appliances will vary based upon the elasticity (the slope of the demand curve). $\mathcal{A}_{EL} \in \mathcal{A}$.

$$\begin{aligned} p_a^t &= E_a, p_a^{min} \leq p_a^t \leq p_a^{max}, \forall t\, a \in \mathcal{A}_{EL} & (17)\\ &= 0, \text{otherwise.} & (18)\end{aligned}$$

3) Perfectly Elastic Loads (PEL): Perfectly elastic loads are completely shiftable loads from one time period to another time period. $\mathcal{A}_{PEL} \in \mathcal{A}$.

$$\begin{aligned} p_a^t &= E_a, 0 \leq p_a^t \leq p_a^{max}, \forall t, a \in \mathcal{A}_{PEL} & (19)\\ &= 0, \text{otherwise.} & (20)\end{aligned}$$

The preference order of the appliances as follows

$$\mathcal{A}_{PIL} > \mathcal{A}_{EL} > \mathcal{A}_{PEL}. \tag{21}$$

In optimal scheduling mainly depends on EL and PEL loads only. PIL loads are ineffective, because they will consume fixed amount of power.

III. Problem Formulation

In this section, objective function is formulated to maximize the consumer welfare ($\mathcal{W}$). The consumer welfare function defined as difference between benefit of consumer for power consumption and pay off [11].

$$\mathcal{W}(P) = U(P) - P(t)\pi(t) \tag{22}$$

$$\underset{P_t}{\textbf{maximize}} \quad \sum_{t=1}^{T} U(P_t) - \sum_{t=1}^{T}\big(\sum_{a\in\mathcal{A}} p_a^t\big)\pi(t) \tag{23}$$

subjected to

$$\sum_{t=1}^{T} p_a^t \times t = E_a, \forall\, a \in \mathcal{A} \tag{24}$$

$$\begin{aligned} X_a &= \sum_{t=t_\alpha}^{t=t_\beta} p_a^t \times t = E_a, p_a^{min} \leq p_a^t \leq p_a^{max} & (25)\\ &= 0\ \forall\, t \notin (t_\alpha, t_\beta), \forall \mathcal{A} & (26)\end{aligned}$$

$$\sum_{t=1}^{T}\Big(\sum_{a\in\mathcal{A}} p_a^t\Big) \leq P_{sch}^{max}, \forall t. \tag{27}$$

In the objective function, first term is the benefit function and the second term is cost payment towards LSE for the power consumption. Cost payment is the product of sum of power consumption of all the appliances and price at each hour. The benefit functions are described in section II (B). Constraint (24) is energy consumption of a appliance to complete a specified task over a day. It is provided for the user. Constraint (25)-(26) is, user can specify time bounds to complete a particular task. During that specified time that appliance must run, irrespective of the electricity price. The time interval (t_α^a, t_β^a) is not need be continuous. It can be multiple time periods also like ($t_{\alpha1}^a$,... $t_{\beta1}^a$,...., $t_{\alpha n}^a$...$t_{\beta n}^a$). The appliance consumes the power during the specified time period only, other than specified time period power consumption of the appliance is zero.(X_a is feasibility vector). Constraint (27) is maximum power consumption limit on total power consumption of all the appliances at time t. This constraint is specified by LSE. In day ahead scheduling or real time scheduling the consumer may go beyond specified power limit. That is named as day ahead power deviation or real time power deviation. For the deviated power consumption, user charged depending upon wholesale electricity prices. Those charges are typically high.

IV. Results and Discussion

In this section, we provide results for three different case studies. Appliance details, power consumption levels and run time of the appliances are given in Table I. In that, appliances are B and C are belonging to PIL, appliances A and D to G are belongs to EL, and appliances H and I are belonging to PEL. Price vector taken from Indian Energy Exchange (IEX)

[12], which is shown in Fig. 4. In IEX, price gave for every 15 minutes block, in this paper we taken the average value to calculate hourly price. The optimization model which is described in section III is solved using CONOPT solver under GAMS environment, on an Intel i7 3.4 GHz processor with 8 GB of RAM. The output of the demand response algorithm is hourly schedule of all the appliances. For nine appliances (Table I) the size of scheduling vector is 9×24.

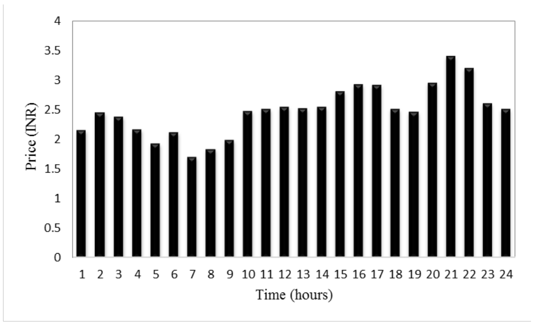

Fig. 4. Day ahead price signal.

TABLE I
TYPES OF APPLIANCE AND ENERGY CONSUMPTION

Appliance	p^a_{min} (W)	p^a_{max} (W)	Time	E_a (Wh)
A	50	100	1-6 and 17-24	1000
B	200	200	17-21	1000
C	200	200	7 and 17-21	1400
D	100	1200	21-24	1800
E	500	5000	1-6 and 19-24	6250
F	100	1000	1-4 and 12-24	2500
G	75	1500	2-23	2250
H	0	2000	1-24	2000
I	0	500	1-24	500

A- Light, B- Fan, C- Television, D- Dishwasher, E-PHEV, F- Air conditioner, G- Refrigerator, H- Battery-1, I-Battery-2

A. Case 1: Cost Minimization

In this case study, scheduling of appliances based on only cost minimization is presented (benefit maximization is not considered). And assume that maximum power scheduled by LSE (i.e., P^{max}_{sch}) is very high.

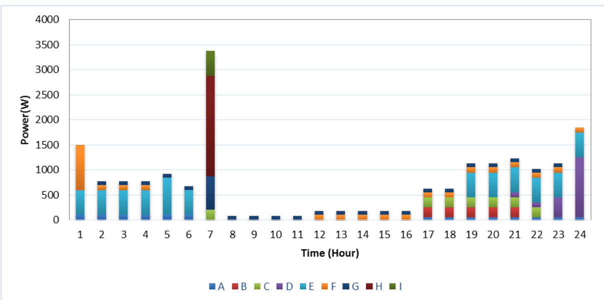

Fig. 5. Optimal power schedule for appliances.

The optimal schedule of all appliances is shown in the Fig. 5. All the appliances are scheduled according to the predefined constraints. Elastic loads and perfectly inelastic loads are scheduled when the price is lower. Elastic loads are consuming minimum power when the price is higher. In the case of perfectly inelastic appliances H and I are scheduled at only 7th hour. The reason is price at that time is minimal compared to all the time slots in a day. That is shown in Fig. 6. The cost comparison of cost and PAR shown in the Table II. The pay off is less in case of optimal consumption. In case of nominal consumption consumer will pay more. Here, nominal consumption means a user can consume the power or use appliances at any desired time of the day. The PAR in the optimal schedule is higher than nominal consumption. This is due to the objective function is only cost minimization. So, most of the appliances will schedule at that time only.

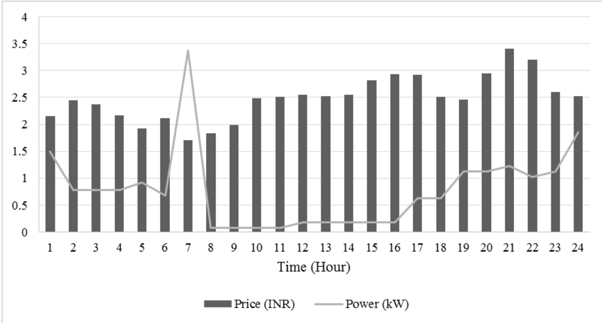

Fig. 6. Power consumption and price.

TABLE II
COMPARISON OF NOMINAL POWER CONSUMPTION AND OPTIMAL POWER CONSUMPTION

	Pay off (INR)	PAR
Nominal consumption	50.76	3.69
Optimal consumption	44.90	4.82

B. Case 2: Comparison of Benefit Functions

In this case study, comparison of three different benefit functions is presented. Forecasted price and appliance data taken as in Case I. The co-efficients of benefit function are calculated as presented in Section II. The co-efficient values are presented in Table III. The optimal schedules for the quadratic, logarithmic, and exponential benefit functions shown in Fig. 7 to Fig. 9. The pay off cost and PAR are shown in Table IV.

In this case study, pay off the cost is less for exponential benefit function. The reason is, the benefit function is approximately same as actual benefit function. In this case study, we did not focus on PAR minimization. The benefit function is intrinsically reducing the PAR.

TABLE III
CO-EFFICIENTS OF BENEFIT FUNCTION

Benefit function	Equation	Co-efficients
Quadratic	(6)	a= 14.817, b= 0.002
Exponential	(8)	a= 13.98, b= 60.68, c= 0.01 d=0
Logarithmic	(9)	k_1= 1984.28

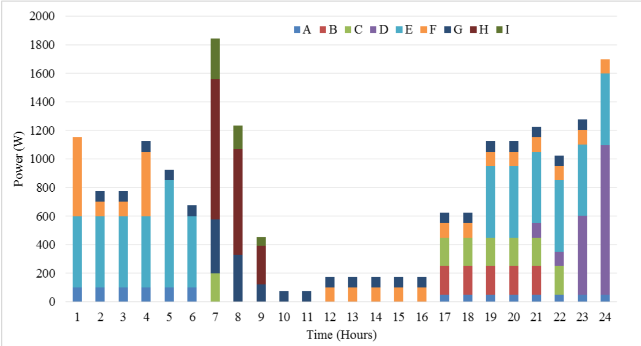

Fig. 7. Optimal schedule for quadratic benefit function.

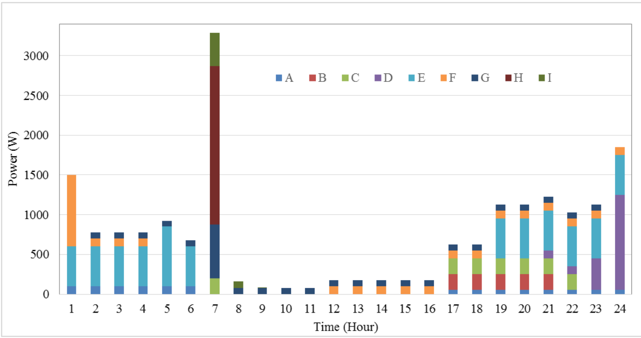

Fig. 8. Optimal schedule for exponential benefit function.

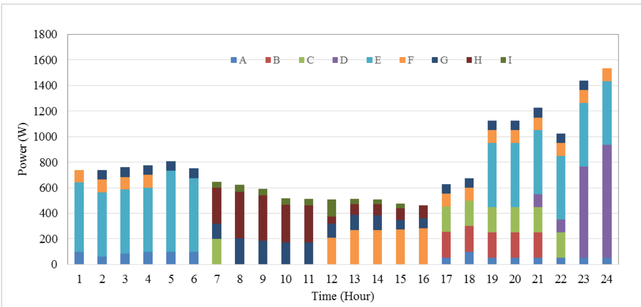

Fig. 9. Optimal schedule for logarithmic benefit function.

TABLE IV
COMPARISON OF BENEFIT FUNCTIONS FOR CASE II

Benefit function	Pay off (INR)	PAR
Quadratic	45.42	2.36
Exponential	45.17	4.218
Logarithmic	47.27	1.969

C. Case 3: Consumer Welfare Maximization with Power limit Constraint

In this case study, consumer welfare maximization is studied, with maximum power limit constraint. The maximum power scheduled by the LSE is considered as 2 kW. The limit considered as same throughout the day. The optimal schedule of all the appliances for the three benefit functions is shown in Fig. 10 to Fig. 12. From the figures we can observe that, for all the benefit functions the maximum power scheduled at particular is not exceeded more than 2 kW.

For the maximum power limit constraint, performance of the three benefit functions presented in Table V. In the table C.E is consumer efficiency, which is defined as the ratio of benefit to cost payment towards LSE [5]. The quadratic benefit function is giving more consumer efficiency for the consumption of the same power level. Even though the power consumption is same, because of the pay off the consumer efficiency is vary from benefit function to benefit function. In case of, exponential benefit function the pay off is increased compared to the previous case study (i.e., Case II). The reason is in the previous case there is no limit on the hourly power consumption. So, appliances are scheduled lower price levels. But in this case, the excess power is distributed to other time slots where the price is high. Hence the payoff is increased. For quadratic and logarithmic benefit functions the pay-off is same compared to Case II. The reason is, the peak power consumption (in Case II) at a hour is less than the 2 kW. Hence, the maximum power limit constraint does not effect the pay-off.

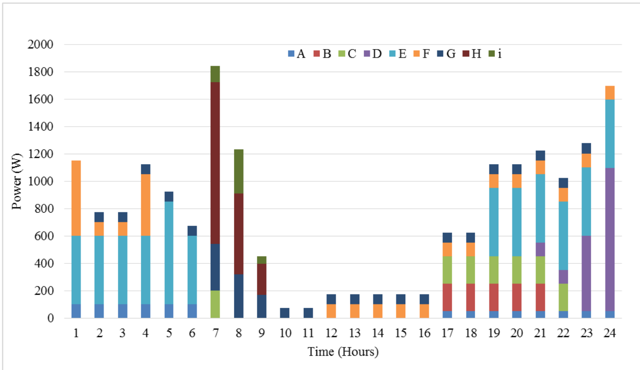

Fig. 10. Optimal schedule for quadratic benefit function with power limit constraint.

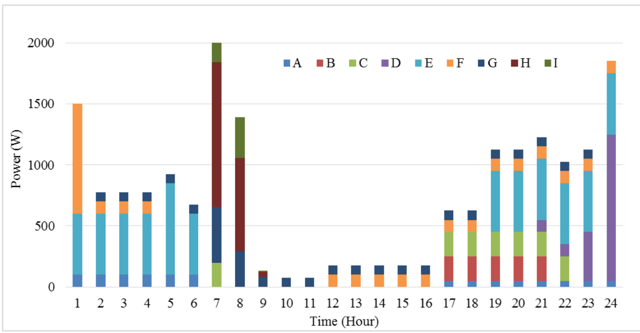

Fig. 11. Optimal schedule for exponential benefit function with power limit constraint.

V. CONCLUSION

In this paper, optimal schedule of appliances with cost minimization objective and consumer welfare maximization is examined for home energy management system. Construction

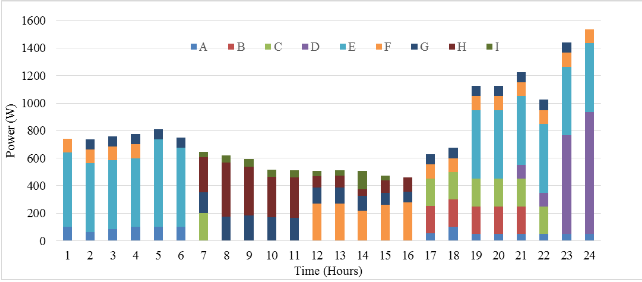

Fig. 12. Optimal schedule for logarithmic benefit function with power limit constraint.

TABLE V
COMPARISON OF BENEFIT FUNCTIONS FOR CASE III

	Pay off (INR)	Power (kW)	C.E
Logarithmic	47.27	18.7	2.88
Quadratic	45.42	18.7	5.64
Exponential	46.81	18.7	5.58

of various benefit functions is presented, and suggested a method to calculate the co-efficients of the benefit function. Studies and suggested methodology explained with a detailed case study. The selection of benefit function depends on the power consumption profile of consumer and its user dependent. In this case study, quadratic benefit function is given more consumer efficiency compared to other benefit functions.

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