An Efficient Hybrid Technique for Correlated Probabilistic Load Flow Study with Photovoltaic Generations

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***Abstract*—This paper proposes an efficient hybrid technique for probabilistic load flow study. A mixture of correlated Gaussian and non-Gaussian as well as discrete distributions is considered for input random variables. Distributions of desired random variables pertaining to the input random variables are found to be multimodal. Analysis using Gaussian mixture approximation is promising in this context, but computational burden increases significantly with the increase in number of discrete random variables. In contrast, the proposed method precisely obtains distribution of desired random variables in considerably less time without compromising accuracy. Multiple input correlations are effectively incorporated. Accuracy of the proposed method is examined in IEEE 14-bus and 57-bus test systems. Results are compared with combined cumulant-Gaussian mixture approximation method and Monte-Carlo simulation.**

***Index Terms*—Cumulant method; photovoltaic generation; probabilistic load flow; product moment correlation coefficient; sequence operation theory.**

I. Introduction

EVER since its inception by Borkowska in 1974, probabilistic load flow (PLF) has been a topic of rigorous research for more than four decades [1]. PLF considers uncertainties associated with input bus powers as random variables and obtains the distributions of desired random variables (bus voltages and line power flows). Accurate estimation of distributions of desired random variables in reduced computational effort is a challenging task.

Monte Carlo simulation is considered as an accurate method for PLF but is computationally burdensome. It is considered as a reference method for validation of other PLF methods. Cumulant method (CM) is an analytical method which obtains the cumulants of the desired random variables with less computational effort. Series expansion methods such as Gram Charlier Method (GCM), Cornish Fisher Method and Edgeworth method, can apt the shape of distribution from the cumulants [2]. The presence of discrete inputs yields distributions of desired random variables with multiple modes. These expansion methods cannot accurately approximate the multimodal distributions [3].

Under such cases, Gaussian mixture approximation (GMA) technique can accurately approximate the distributions of the desired random variables but, computational burden increases significantly with the increased number of discrete random variables comprising of multiple impulses [4]. Moreover, the Gaussian mixture reduction technique [5] is not applicable in this context. Sequence operation theory (SOT) can deal with discrete random variables aptly [6].

Correlated PLF using analytical methods are discussed in [2], [7]-[10] but, there is no remarkable work done using discrete random variables in the literature. PLF considering discrete inputs lead to multimodality in the distribution of desired outcomes. Under such cases existing expansion methods fail to approximate the distribution accurately. However, if continuous and discrete input random variables are dealt separately with two different methods that are efficient in their respective domains, then the distribution of the desired random variables can be established accurately with proper hybridization. In this regard, a hybrid method (HM) combining CM and SOT is proposed.

PLF model is elaborated in section II. Various input uncertainties are described in section III. Section IV discusses the proposed HM. Computational procedure for the implementation of proposed HM is provided in section V. In section VI MCS, GMA and proposed HM are applied for PLF and results are compared in two IEEE test systems. Finally, the conclusion is given in section VII.

II. Probabilistic Load Flow Model

In a power system comprising of ‘ n ’ number of buses (‘ m ’ $P|V|$ buses, $m \le n$) and ‘ l ’ number of branches, the linearized PLF model [4] is given as,

$$x_i = \sum_{j=1}^{2n-m-2} a_{ij}\left(y_j^{D} + y_j^{C}\right) + x_{i0} \tag{1}$$

$$z_i = \sum_{j=1}^{2n-m-2} b_{ij}\left(y_j^{D} + y_j^{C}\right) + z_{i0} \tag{2}$$

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where, $x_{i0} = x_i^0 - \sum_{j=1}^{2n-m-2} a_{ij} y_j^0$, $z_{i0} = z_i^0 - \sum_{j=1}^{2n-m-2} b_{ij} y_j^0$.

In (1) and (2), $\bar{x}$, $\bar{y}$ and $\bar{z}$ are the vectors of bus voltages, bus power injections and line power flows respectively. x^0, y^0 and z^0 are expected values of x, y and z respectively. a_{ij} and b_{ij} are sensitivity coefficients. Scalars x_{i0} and z_{i0} can be obtained from deterministic load flow [4].

Simplification of (1) and (2) yields,

$$x_i = \left(\sum_{j=1}^{2n-m-2} a_{ij} y_j^{\mathrm{D}} \right) + \left(\sum_{j=1}^{2n-m-2} a_{ij} y_j^{\mathrm{C}} \right) + x_{i0} \tag{3}$$

$$z_i = \left(\sum_{j=1}^{2n-m-2} b_{ij} y_j^{\mathrm{D}} \right) + \left(\sum_{j=1}^{2n-m-2} b_{ij} y_j^{\mathrm{C}} \right) + z_{i0} \tag{4}$$

Formulations in (3) and (4) can be represented by the sum of equivalent random variables given as,

$$x_i = x_i^{\mathrm{eD}} + x_i^{\mathrm{eC}} \ , \ z_i = z_i^{\mathrm{eD}} + z_i^{\mathrm{eC}} \tag{5}$$

where, $x_i^{\mathrm{eD}} = \sum_{j=1}^{2n-m-2} a_{ij} y_j^{\mathrm{D}}$, $x_i^{\mathrm{eC}} = \sum_{j=1}^{2n-m-2} a_{ij} y_j^{\mathrm{C}} + x_{i0}$,

$z_i^{\mathrm{eD}} = \sum_{j=1}^{2n-m-2} b_{ij} y_j^{\mathrm{D}}$, $z_i^{\mathrm{eC}} = \sum_{j=1}^{2n-m-2} b_{ij} y_j^{\mathrm{C}} + z_{i0}$.

Superscripts 'D' and 'C' stands for discrete and continuous respectively. x_i^{eD} and x_i^{eC} are equivalent discrete and continuous parts of x_i respectively whereas z_i^{eD} and z_i^{eC} are equivalent discrete and continuous parts of z_i respectively. The proposed method can easily be implemented on (5). It separately deals with discrete and continuous parts with the help of SOT and CM respectively.

III. Description of Input Uncertainty

A. Conventional Generation

The uncertainty in real power generation of a conventional generator is described by Bernoulli distribution [10]. For a power plant consisting of several generating units (with different forced outage rates (FORs)) operating independently, the power output of the plant is obtained by the convolution of several independent Bernoulli distributions.

B. Photovoltaic Generation

In PV systems, the real power generation P_{PV} strongly depends on solar irradiance which is unpredictable due to climatic conditions (cloud and fog).

According to [2], P_{PV} is given as,

$$P_{\mathrm{PV}} = \eta_{\mathrm{g}} r A \left(1 - K_{\mathrm{T}} \Delta T_{\mathrm{F}}\right) \tag{6}$$

where, r is the irradiance, ΔT_{F} is the forecasting error of PV cell temperature and K_{T} is the temperature coefficient.

Introducing random variables R and T as functions of r and ΔT respectively, (6) can be revised as,

$$P_{\mathrm{PV}} = RT \tag{7}$$

where, $R = \eta_{\mathrm{g}} r A$, $T = 1 - K_{\mathrm{T}} \Delta T$.

The linearization of (7) is given as,

$$P_{\mathrm{PV}} = \mu_{\mathrm{R}} T' + R \tag{8}$$

where, μ_{R} is the mean value of R , $T' = T - 1$.

The linearization of (8) can be inferred from [4].

For a solar park consisting of 'nr' number of PV units, aggregate generation is given by,

$$P_{\mathrm{PVA}} = P_{\mathrm{PV}_1} + P_{\mathrm{PV}_2} + \cdots + P_{\mathrm{PV}_{\mathrm{nr}}} \tag{9}$$

C. Probabilistic Modeling of Load Power

Probability distribution of aggregate bus load power is decided depending on the information available regarding the load power. Gaussian distribution is a predominant assumption to represent forecasting error (small variance). The distribution parameters i.e. mean and standard deviation are obtained from the historical data. In the absence of data, mean value is chosen as the base load power and standard deviation between $\pm 5\%$ to $\pm 10\%$. On the other hand if the data available does not fit any functional form of distribution, but simply in terms of probability values corresponding to different range of load values then a discrete distribution is chosen for its statistical characterization. In case of load power being forecasted precisely, one-point distribution is selected.

IV. Proposed HM for PLF

In the proposed HM, discrete and continuous inputs are dealt separately by SOT and CM respectively. A brief introduction of both the methods is described as under.

A. Sequence Operation Theory

In SOT, the following sequences are vital.

- *Discrete sequence:* It is defined as a series of numerical values at the instants $i = 0, 1, \cdots, N$ where, N represents the length of the sequence. A discrete sequence $a(i)$ of length N_a must satisfy $a(i) = 0$ for $i > N_a$ and $a(i) \neq 0$ for $i = N_a$.
- *Probability sequence:* A discrete sequence $a(i)$ of length N_a is said to be a probability sequence if it satisfies the condition $\sum_{i=0}^{N_a} a(i) = 1$.

In SOT based framework, arithmetic operations are carried out using sequence operations [11]. The sequence operations are addition type convolution (ATC), subtraction type convolution (STC) and sequence multiplication operation (SMO). For probability sequences $a(i_a)$ and $b(i_b)$ of lengths N_a and N_b respectively, the above three operations are given as,

$$x(i) = \sum_{i_a + i_b = i} a(i_a) b(i_b), \ i = 0, 1, \cdots, \ N_a + N_b \tag{10}$$

$$y(i) = \begin{cases} \sum_{i_a \le i_b} a(i_a) b(i_b), & i = 0 \\ \sum_{i_a - i_b = i} a(i_a) b(i_b), & 1 \le i \le N_a \end{cases} \tag{11}$$

$$s(i) = \sum_{i_a i_b = i} a(i_a) b(i_b) \quad , \; i = 0, 1, \cdots, N_a N_b \tag{12}$$

It is noteworthy that, (10) and (11) are strictly applicable to sequences having same sequence intervals. On the other hand, (12) is applicable to sequences with different sequence intervals and the resultant is the product of individual sequence intervals. The inputs associated with first summands of (5) are discrete and the equivalent probability sequences of the first summands can be obtained with the help of above formulations.

B. Cumulant Method

For two linearly correlated random variables X_1 and X_2, the cumulants of Y in (13) are given by (14) or (15) using (16) to (18) [12].

$$Y = X_1 \pm X_2 \tag{13}$$

$$C_{Y,k} = A_1(k) C_{X_1,k} + A_2(k) C_{X_2,k} \, , \; \sigma_{X_2} \geq \sigma_{X_1} \tag{14}$$

$$C_{Y,k} = C_{X_1,k} + A_1(k) A_2(k) C_{X_2,k} \; , \; \sigma_{X_1} \geq \sigma_{X_2} \tag{15}$$

where, $C_{X_1,k}$, $C_{X_2,k}$ and $C_{Y,k}$ are k^{th} order cumulants of X_1, X_2 and Y respectively.

$$A_1(k) = (1+r)^k - r^k \, , \; A_2(k) = (\pm 1)^k \tag{16}$$

$$r = \pm \rho_{X_1 X_2} \left(\frac{\sigma_{X_2}}{\sigma_{X_1}} \right), \; \sigma_{X_2} \geq \sigma_{X_1} \tag{17}$$

$$r = \pm \rho_{X_1 X_2} \left(\frac{\sigma_{X_1}}{\sigma_{X_2}} \right), \; \sigma_{X_1} \geq \sigma_{X_2} \tag{18}$$

In (17) and (18), $\rho_{X_1 X_2}$ is Pearson product moment correlation coefficient (PMCC) between X_1 and X_2, r relates (16) with that of (17) and (18).

Extending (13) for 'nr' correlated random variables, cumulants of Y in (19) are obtained using (20) to (22).

$$Y = X_1 \pm X_2 \pm \cdots \pm X_{\text{nr}} \tag{19}$$

A compact form of (19) is given by $Y = W_{\text{nr}-2} \pm X_{\text{nr}}$. For any new random variable W_i given by (20), σ_{W_i} and $\rho_{W_i X_{i+2}}$ are obtained using (21) and (22) respectively. Note $W_0 = X_1$.

$$W_i = W_{i-1} \pm X_{i+1} \; , \; 1 \leq i \leq \text{nr} - 2 \tag{20}$$

$$\sigma_{W_i} = \sqrt{\sigma^2_{W_{i-1}} \pm 2 \rho_{W_{i-1} X_{i+1}} \sigma_{W_{i-1}} \sigma_{X_{i+1}} + \sigma^2_{X_{i+1}}} \tag{21}$$

$$\rho_{W_i X_{i+2}} = \frac{\rho_{W_{i-1} X_{i+2}} \sigma_{W_{i-1}} \pm \rho_{X_{i+1} X_{i+2}} \sigma_{X_{i+1}}}{\sigma_{W_i}} \tag{22}$$

The inputs associated with second summands of (5) are correlated continuous random variables. The first six cumulants of the equivalent random variable can be obtained with the help of the above formulations and the cumulative probability values can be approximated from the cumulants by using GCM. The process of obtaining probability sequence from the approximated cumulative probability at the desired sequence interval ΔS is discussed under:

According to [2], cumulative probability values of a normalized desired random variable X^* can be approximated as,

$$F(x^*) = \sum_{i=0}^{\text{n}_0} \frac{c_i}{i!} (-1)^i H_i(x^*) \varphi(x^*) \tag{23}$$

where, 'n_0' is the order of expansion, c is the constant coefficient of GCM, $H(x^*)$ is the Hermite polynomial, $\varphi(x^*)$ is the cumulative probability distribution of standardized X.

In (23), $F(x^*)$ is calculated for a range of values of X^*. The difference between any two consecutive values is set to $\Delta S / \sigma_X$, where, σ_X is the standard deviation of X. Value of n_0 is set to six. Now the cumulative probability values and possible values of X can be obtained as, $F(x) = F(x^*)$ and $x = \mu_X + (x^*) \sigma_X$.

V. Computational Procedures

A systematic procedure for PLF evaluation based on the proposed HM comprises the following steps:

1) List out the input random variables in the system under study and define their distributions.
2) Develop input PMCC matrix.
3) Obtain x_{i0} and z_{i0} from the converged deterministic load flow using Newton-Raphson iterative method.
4) Decide the value of ΔS.
5) Obtain equivalent probability sequence of first summands of (5) using the formulations as described in IV (A).
6) Calculate first six cumulants of the continuous input random variables from their distribution parameters.
7) Obtain first six cumulants of equivalent second summands of (5) using the formulations described in IV (B).
8) Approximate the cumulative probability of equivalent second summands of (5) for all the desired random variables using (23).
9) Obtain equivalent probability sequence of step-(8) with sequence interval same as that of equivalent probability sequence of step-(5).
10) Use ATC/STC operation (depending on the sign of the sensitivity coefficients) among the probability sequences of step-(5) and step-(9) to obtain the equivalent probability sequences of desired random variables in (5).
11) Obtain cumulative probability plot of step-(10).

VI. Case Studies and Comparison of Results

The proposed HM is tested on modified IEEE 14 and 57 bus systems. Bus and line data for both the systems are taken from [13]. The modification in the test systems is done by installing solar parks at certain buses where, load demand is high except for those where a conventional generator or synchronous condenser is connected. The particulars of the solar parks are given in Table I. The details of discrete generations and load powers for both the systems are specified in Table II and III respectively. The expected values of discrete generations and loads are same as the specified deterministic data. The penetration level of PV generation in both the systems is decided from local bus load demand. The following convention is considered to represent a desired random variable in rest of the paper: $|V_i|$ and δ_i indicates bus voltage

magnitude and bus voltage angle at i^{th} bus respectively. $P_{\text{L}i-j}$ and $Q_{\text{L}i-j}$ indicates real and reactive power flows in the line $i-j$ respectively. $P_{\text{D}i}$ and $Q_{\text{D}i}$ are real and reactive components of load power at i^{th} bus respectively. The programing codes are developed in MATLAB 7.10 and simulated on a computer with 3.4 GHz, i_7 processor and RAM size of 8 GB.

TABLE I
DETAILS OF SOLAR PARKS

Parameters	14-Bus System		57-Bus System	
	Solar Park 1	Solar Park 2	Solar Park 1	Solar Park 2
Bus No.	13	14	16	17
PV Units	2	2	2	2
PV Penetration	20 % (1st Unit), 25 % (2nd Unit)			

TABLE II
DETAILS OF DISCRETE POWER GENERATIONS

Test System	14-Bus System		57-Bus System					
Bus No.	2		3		12			
Unit No.	1	2	1	2	1	2	3	4
Capacity (p.u.)	0.22	0.22	0.22	0.22	0.84	0.84	0.84	0.84
FOR (p.u.)	0.10	0.08	0.10	0.08	0.09	0.09	0.07	0.07

A. The IEEE 14-Bus Test System

The system consists of 2 generating units, 3 synchronous condensers, 17 transmission lines and 3 transformer branches. The real and reactive load powers at buses 5, 13 and 14 are assumed to follow Gaussian distribution. Loads at remaining buses except 9 are supposed to follow one point distributions. The sets (PV_1,PV_2) and (PV_3,PV_4) represent real power generation of units pertaining to solar parks 1 and 2 respectively.

PMCC matrix for this system is decided as per Table IV. This PMCC matrix is considered as the base case correlation matrix. MCS, CCGMA and proposed HM are applied for PLF. Let, N_r be the total number of Gaussian components required to approximate a desired random variable. Value of N_r is 1600 in case of CCGMA. It is found that mean values of desired random variables using above three methods are almost identical. However, compared to MCS differences are mainly observed in higher order moments. Absolute percentage errors in calculating standard deviation σ for any random variable X is given as,

$$e_X = \left| \frac{P_{X,\text{MCS}} - P_{X,\text{COM}}}{P_{X,\text{MCS}}} \right| \times 100$$

where, $P_{X,\text{MCS}}$ and $P_{X,\text{COM}}$ are parameters of X obtained using MCS and the comparing method respectively.

In Fig. 1, cumulative probability plots for $|V_9|$, δ_2, $P_{\text{L}1-2}$ and $Q_{\text{L}5-6}$ using the proposed HM are compared with that of CCGMA and MCS.

TABLE III
DETAILS OF DISCRETE POWER DEMANDS

Test System	Bus Number	Real Power		Reactive Power	
		Capacity (p.u.)	Probability	Capacity (p.u.)	Probability
14-Bus System	9	0.134	0.10	0.075	0.10
		0.196	0.15	0.110	0.15
		0.302	0.30	0.170	0.30
		0.348	0.25	0.196	0.25
		0.373	0.20	0.210	0.20
57-Bus System	47	0.134	0.10	0.070	0.10
		0.196	0.15	0.090	0.15
		0.308	0.30	0.120	0.30
		0.349	0.25	0.150	0.25
		0.373	0.20	0.110	0.20

TABLE IV
BASE CASE PMCC MATRIX

Random variable	R_1	T_1'	R_2	T_2'	R_3	T_3'	R_4	T_4'	$P_{\text{D}5}$	$P_{\text{D}13}$	$P_{\text{D}14}$	$Q_{\text{D}5}$	$Q_{\text{D}13}$	$Q_{\text{D}14}$
R_1	1	0.3	0.7	0	0	0	0	0	0	0.3	0	0	0	0
T_1'	0.3	1	0	0.2	0	0	0	0	0	0	0	0	0	0
R_2	0.7	0	1	0.3	0	0	0	0	0	0.3	0	0	0	0
T_2'	0	0.2	0.3	1	0	0	0	0	0	0	0	0	0	0
R_3	0	0	0	0	1	0.3	0.7	0	0	0	0.3	0	0	0
T_3'	0	0	0	0	0.3	1	0	0.2	0	0	0	0	0	0
R_4	0	0	0	0	0.7	0	1	0.3	0	0	0.3	0	0	0
T_4'	0	0	0	0	0	0.2	0.3	1	0	0	0	0	0	0
$P_{\text{D}5}$	0	0	0	0	0	0	0	0	1	0.2	0.2	0.5	0	0
$P_{\text{D}13}$	0.3	0	0.3	0	0	0	0	0	0.2	1	0.5	0	0.5	0.2
$P_{\text{D}14}$	0	0	0	0	0.3	0	0.3	0	0.2	0.5	1	0	0.2	0.5
$Q_{\text{D}5}$	0	0	0	0	0	0	0	0	0.5	0	0	1	0.2	0.2
$Q_{\text{D}13}$	0	0	0	0	0	0	0	0	0	0.5	0.2	0.2	1	0.5
$Q_{\text{D}14}$	0	0	0	0	0	0	0	0	0	0.2	0.5	0.2	0.5	1

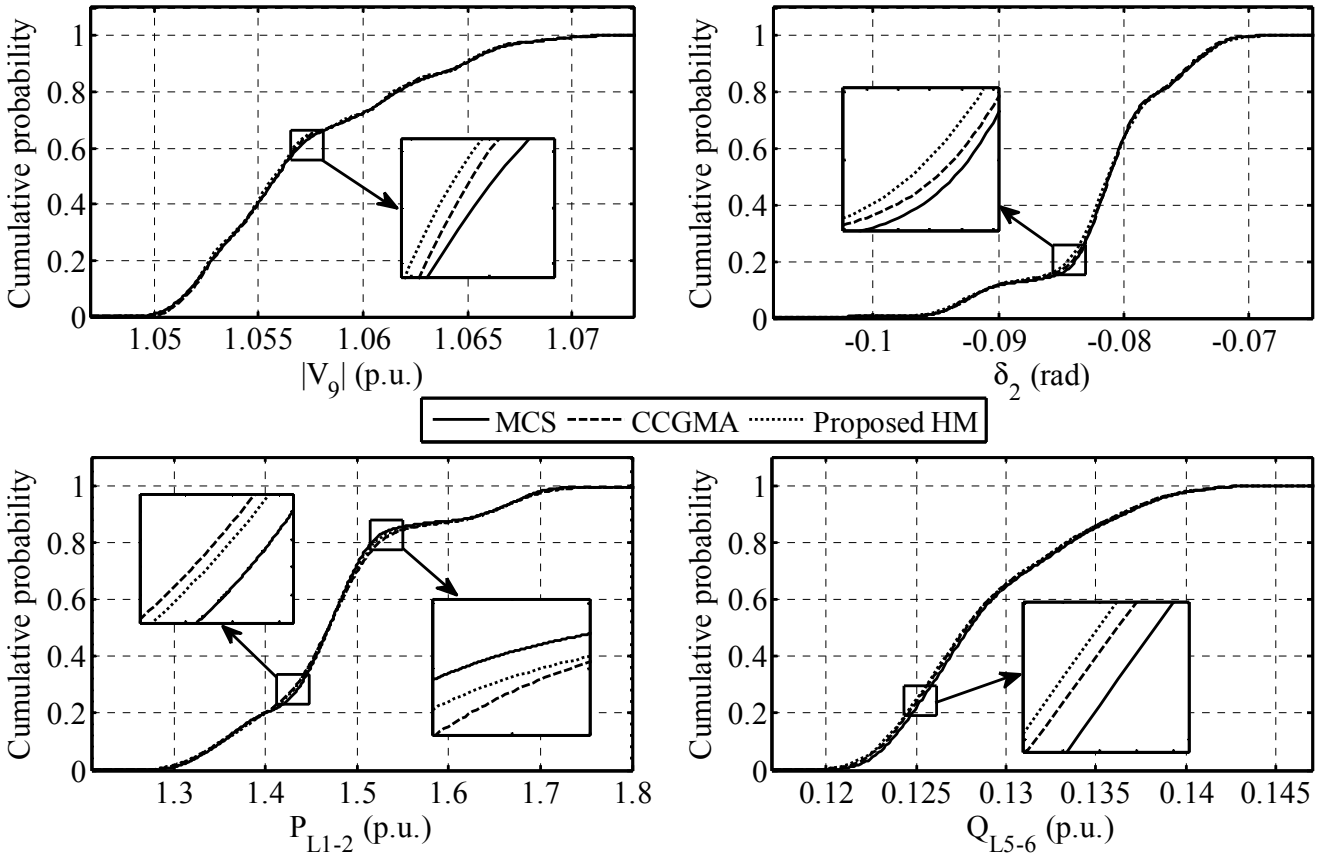

Fig. 1. Comparison of cumulative probability plots of $|V_9|$, δ_2, P_{L1-2} and Q_{L5-6}.

B. The IEEE 57-Bus Test System

The system consists of 4 generating units, 3 synchronous condensers, 65 transmission lines and 15 transformer branches. The real and reactive components of load powers at buses 13 to 17 are assumed to follow Gaussian distributions. Loads at remaining buses except bus 47 are supposed to follow one point distributions. PMCC matrix is same as Table IV which is defined among the random variables R_1, T_1', R_2, T_2', R_3, T_3', R_4, T_4', P_{D13}, P_{D16}, P_{D17}, Q_{D13}, Q_{D16} and Q_{D17}. Sensitivity coefficients representing the coupling between ($|V_i|$, Q_{Li-j}) and injected real bus powers as well as (δ_i, P_{Li-j}) and injected reactive bus powers are less than 0.005 hence, are neglected. Value of N_r is 5120 in case of CCGMA. The cumulative probability plots of δ_{16} and P_{L12-16} using the three methods are compared in Fig. 2 and Fig. 3 respectively.

Average of e_X for standard deviation σ pertaining to all the desired random variables using CCGMA and the proposed HM for both the systems is indicated in Table V. For instance, the values 2.99 and 5.32 in Table V indicate the average of e_X in obtaining σ for all the desired random variables using the proposed HM for both the systems respectively.

Further, it is observed from Table V that simulation run time is much less using the proposed HM than that of CCGMA for both the test systems. The results obtained using both the test systems verify that the proposed HM incorporates correlated input random variables in PLF with increased computational efficiency as compared to CCGMA.

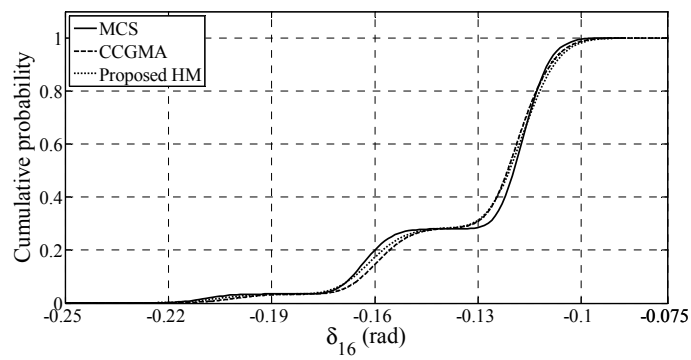

Fig. 2. Comparison of cumulative probability plots of δ_{16}.

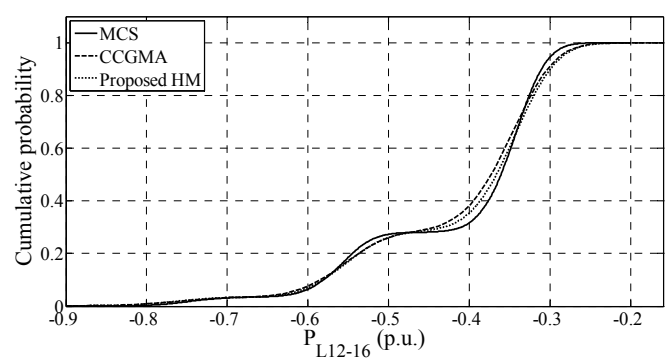

Fig. 3. Comparison of cumulative probability plots of P_{L12-16}.

TABLE V
SIMULATION RUN TIME AND ACCURACY COMPARISON

Test System		14-Bus	57-Bus
Average e_σ	CCGMA	02.58	04.45
	Proposed HM	02.99	05.32
Simulation Run Time (sec)	MCS	20.06	100.30
	CCGMA	08.15	62.37
	Proposed HM	03.17	16.64

VII. CONCLUSION

A hybrid method for probabilistic load flow is successfully implemented on IEEE 14 and 57 bus test systems with photovoltaic systems installed. The uncertainties associated with conventional generations, photovoltaic generations and aggregate bus load powers are modeled as input random variables. The proposed method separately deals with discrete and continuous input random variables by SOT and CM respectively. Correlation cases such as generation-generation, generation-load and load-load are effectively incorporated in the proposed method. Distributions of multi modal desired random variables are obtained in less time by the proposed HM compared to CCGMA and MCS.

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