# Efficient Holomorphic Based Approach for Unit Commitment Problem

*Anup Shukla, Saurabh Kesherwani, Student Member IEEE , S.N.Singh, Senior Member IEEE*
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Kanpur, UP, India, 208016
anups@iitk.ac.in, skwani@iitk.ac.in, snsingh@iitk.ac.in

***Abstract*—Due to increasing energy demand and introduction of competition in the electricity sectors, the Unit Commitment (UC) has become a challenging task in the power system. In the operational planning of the modern power system UC plays an important role and saves significant amount of the cost per year. In this paper, optimal loading of the generating units is obtained using particle swarm optimization along with Holomorphic Embedded Load Flow (HELF) technique, under consideration of equality and inequality constraints of different units and power flow. IEEE 30-bus system is considered to test the effectiveness of the proposed approach. The simulation results thus obtained have been compared with the results obtained from NRLF load flow.**

***Index Terms*—Unit Commitment, Holomorphic Embedding, Hybrid Approach, Security.**

## I. Nomenclature

| | |
|---|---|
| $a_n$,$b_n$,$c_n$ | Cost coefficients. |
| $C_{shn}$ | Cold start hour of $n^{th}$ thermal unit. |
| $C_{scn}/H_{scn}$ | Cold/Hot start cost of $n^{th}$ thermal unit. |
| $L_{Dt}$ | Power demand at hour $t$. |
| $R_{Dn}/R_{Un}$ | Ramp down/up limit of $n^{th}$ thermal unit. |
| $FC_n$ | Fuel cost of $n^{th}$ thermal units. |
| $G_{ij}+jB_{ij}$ | $ij^{th}$ element of Y-Bus. |
| $I_{nt}$ | Schedule state of $n^{th}$ thermal unit for hour $t$. |
| $N$ | Number of thermal units. |
| $N_b$ | Number of Buses. |
| $N_g$ | Number of generator buses. |
| $N_{gq}$ | Number of PV buses and synchronous condensers. |
| $P_{gi}/Q_{gi}$ | Generation of real/reactive power at $i^{th}$ bus. |
| $P_i/Q_i$ | Demand of real/reactive power at $i^{th}$ bus. |
| $P_{losst}$ | Losses at hour $t$. |
| $P_{nt}$ | Generation of $n^{th}$ thermal unit at hour $t$. |
| $P_{nmax}/P_{nmin}$ | Max/Min generation of $n^{th}$ thermal unit. |
| $P_{wt}$ | Total actual wind generation at hour $t$. |
| $Pwn$ | Rated wind power output. |
| $PF_{ij}$ | Power flow from $i^{th}$ to $j^{th}$ bus. |
| $SR_t$ | Spinning reserve requirements at hour $t$. |
| $SU_{nt}/SD_{nt}$ | Startup/down cost of $n^{th}$ thermal unit at hour $t$. |
| $T$ | Number of time interval (hours). |
| $t_{n,on}/t_{n,off}$ | Continuously online/offline time of $n^{th}$ thermal unit. |
| $T_C$ | Total operating cost over the given time interval. |
| $T_n^{up}/T_n^{down}$ | Minimum up/down time of $n^{th}$ thermal unit. |
| $TL_{ij}^{max}$ | Transmission line rating. |
| $V_i$ | Voltage at $i^{th}$ bus. |
| $v_{wt}$ | Wind speed at hour $t$. |
| $v_1$ | Cut-in wind turbine speed. |
| $v_2$ | Rated wind turbine speed. |
| $v_3$ | Cut-out wind turbine speed. |
| $\phi_i$ | Voltage angle at $i^{th}$ bus. |

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## II. Introduction

The competition in power sector forcing the system operators to run the system ensuring users demand to be met with maximum benefits to both users (consumer) and power industries (supplier). Moreover, due to the volatile nature of demand and electricity prices, it has become a challenging task for the power utilities to perform a proper scheduling of the thermal units to meet the power demand with minimum operating cost. The study and operation of power system regarding these issues involve many different objective functions. The Unit Commitment is one of them. Unit Commitment (UC) generally refers to ON/OFF status and optimal loading of thermal units over a given scheduling horizon, with the objective of minimizing total operating cost while satisfying different equality and inequality constraints[1], [2].

Several optimization methods have been reported in literature to solve the UC problems, which are divided into two different classes. First class belongs to classical or numerical optimization techniques. But, the numerical convergence and solution quality problems are the major concerns for most of these approaches [3], [4], [5]. The second class is the stochastic search or evolutionary techniques. These approaches can successfully handle the complex nonlinear constraints and provide high-quality solutions, but curse of dimensionality is a major concern [6], [7], [8], [9], [10]. Later, efforts have been made to develop hybrid techniques, for better and faster optimal results, to overcome these problems. Hybrid approaches are the combination of above two classes or combination of approaches from the same class [11], [12], [13], [14].

The UC with load flow constraints increases the computational complexity of the problem. But, load flow is important for the improvement as well as future enhancement of the existing power system. While, there are several techniques available in literature to solve the load flow problem such as Gauss-Seidel (GS) [15], Newton-Raphson (NR) [16], Fast Decoupled Load Flow (FDLF) [17], etc. In all the conventional methods discussed above, either storage or computational burden is a limiting constraint for fast and accurate evaluation. Therefore, a new non- iterative Holomorphic Embedded Load Flow (HELF) technique is proposed in [18]. The HELF method overcomes the convergence and computational burden problem of conventional power flow methods. This paper proposed the application of HELF technique for fast computation of UC problem with less computational burden.

## III. Problem Formulation

The UCP has commonly been formulated as a nonlinear mixed-integer optimization problem with the objective of minimizing the total operating cost while satisfying different equality and inequality constraints. To solve this problem, the scheduling period is divided into T time in intervals (24-hours). The mathematical model is formulated as follows:

$$\text{Minimize } T_C = \sum_{t=1}^{T}\sum_{n=1}^{N} FC_n(P_{nt}) + SU_{nt} + SD_{nt} \tag{1}$$

The fuel cost function is assumed as quadratic function of real power output as follows:

$$FC_n(P_{nt}) = a_n * (P_{nt})^2 + b_n * P_{nt} + c_n \tag{2}$$

Subjected to the following constraints:

### A. Thermal Constraints

- Power balance constraint:

$$\sum_{n=1}^{N} I_{nt} * P_{nt} = L_{Dt} + P_{losst} \tag{3}$$

  The total power generation of thermal units should exactly satisfy the power demand and network losses for that hour. Assumed power network losses: $P_{losst} = 0$.
- Reserve constraint:

$$\sum_{n=1}^{N} I_{nt} * P_{nt} \geq L_{Dt} + P_{losst} + SR_t \tag{4}$$

  To maintain system reliability, adequate spinning reserves are required.
- Power limit:

$$P_{nt}^{min} \leq P_{nt} \leq P_{nt}^{max} \tag{5}$$

  Power output of each thermal unit should be limited in a specified range.
- Ramp rate limit: Output of the thermal units is limited by

$$\begin{aligned} P_{nt}^{min} &= min(P_n^{max}, P_{n(t-1)} + \tau R_{Un}) \\ P_{nt}^{max} &= max(P_n^{min}, P_{n(t-1)} - \tau R_{Dn}) \end{aligned} \tag{6}$$

- Minimum up and down time constraints:

$$\begin{aligned} t_{on,n} &\geq T_n^{up} \\ t_{off,n} &\geq T_n^{down} \end{aligned} \tag{7}$$

  Owing to operational limits, once a unit is committed, it should not be decommitted immediately and vice versa.
- Startup cost: A simplified time-dependent start-up cost is given as follows

$$STC_{it} = \begin{cases} HSC_i & T_i^{down} \leq t_{i,off} \leq T_i^{down} + CSH_i \\ CSC_i & t_{i,off} \leq T_i^{down} + CSH_i \end{cases} \tag{8}$$

where, $\tau$ in Equation (6) is a UC time step which is equal to 60 min.

### B. Power output limits on wind energy system:

The power output function with respect to the wind speed is given by [19].

$$P_{wt} = \begin{cases} 0 & : v_{wt} \leq v_1 \quad or \quad v_{wt} \geq v_3 \\ \psi(v_{wt}) & : v_1 \leq v_{wt} \leq v_2 \qquad t \in \mathcal{T} \\ P_{wn} & : v_2 \leq v_{wt} \leq v_3 \end{cases} \tag{9}$$

### C. Power Flow Constraints

- Power balance in the network:

$$\begin{aligned} P_{gi} - P_i - \mid V_i \mid \sum_{j=1}^{N_b} \mid V_j \mid \Big(G_{ij}\cos\phi_{ij} + B_{ij}\sin\phi_{ij}\Big) = 0 \\ Q_{gi} - Q_i - \mid V_i \mid \sum_{j=1}^{N_b} \mid V_j \mid \Big(G_{ij}\sin\phi_{ij} - B_{ij}\cos\phi_{ij}\Big) = 0 \end{aligned} \tag{10}$$

- Generation limit:

$$\begin{aligned} P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \qquad & i = 1, ..., N \\ Q_{gi}^{min} \leq Q_{gi} \leq Q_{gi}^{max} \qquad & i = 1, ..., N_{gq} \end{aligned} \tag{11}$$

- Voltage limit:

$$\begin{aligned} & \mid V_{gi}^{min} \mid \leq \mid V_{gi} \mid \leq \mid V_{gi}^{max} \mid \\ & \phi_{gi}^{min} \leq \phi_{gi} \leq phi_{gi}^{max} \qquad i = 1, ..., N_b \end{aligned} \tag{12}$$

- Line flow limit:

$$TL_{ij}^{min} \leq TL_{ij} \leq TL_{ij}^{max} \tag{13}$$

- Transformer tap limit:

$$T_p^{min} \leq T_p \leq T_p^{max} \tag{14}$$

## IV. HELF Method

Due to exponentially increasing load demand, power system is generally operated near to its limits. Therefore, traditional Power Flow (PF) will not guarantees to give solution [20]. PF problem is a highly non-linear problem which has multiple set of solution depending on the initial guess point so traditional methods do not guarantee to converge at desired set of solution. Some methods have been proposed in [21], [22] to enhance the algorithms for existing methods.

Holomorphic Embedding (HE) is a novel non-iterative method of solving PF. This method guaranteed to find the High-Voltage (HV) solution if solution exists and gives unambiguous signal when solution does not exists [23]. In this method, original non-holomorphic (non-analytic) Power

Balance Equations (PBEs) are embedded such that resultant problem is analytic.

In HE method, the voltage function is embedded using a complex parameter, $s$, such that the resultant system of equations is holomorphic. In HE method, voltage function is expressed as a Taylor series that is a function of injected real & reactive powers and the coefficients of this power series are obtained by recurrence relation established from the embedded PBEs. Thus, calculation of Jacobian matrix is eliminated in this solution procedure. The methodology demonstrated in [23], [24] for solving Load Flow (LF) problem using HE is as shown below. Consider the PBE of a load bus whose complex power injection $S_i$ is known. If $m$ is the set of PQ buses, the PBE in analytic form for PQ bus $i$ will be expressed as:

$$\sum_{k=1}^{N} Y_{ik\ trans} V_k(s) = \frac{sS_i^*}{V_i^*(s^*)} - sY_{i\ shunt}V_i(s), \quad i \in m \tag{15}$$

where, $Y_{ik}$ is the $(i,k)$ element of the bus admittance matrix and $V_i$ is the voltage at bus $i$. The shunt elements are moved to the RHS in (15) and the equation is embedded with a complex parameter $s$. $Y_{ik\ trans}$ corresponds to the series branch part of the admittance matrix and $Y_{i\ shunt}$ model the shunt admittance components of the transmission line, off-nominal transformer tap model and/or shunt capacitors/ reactors in the network.

Let, $p$ denotes the set of PV buses for $N$-bus system. In case of PV bus, for handling reactive power in PBEs, the reactive power term is made a free variable, and represented as a function of the complex, $s$ parameter. According to this approach, the expression representing the PV bus with voltage magnitude constraint, in HE, is as follows:

$$\begin{gathered}\sum_{k=1}^{N} Y_{ik\ trans} V_k(s) = \frac{sP_i - jQ_i(s)}{V_i^*(s^*)} - sY_{i\ shunt}V_i(s) \\ V_i(s) * V_i^*(s^*) = 1 + s\left(|V_i^{sp}|^2 - 1\right)\end{gathered} \tag{16}$$

For (16), $i \in p$. Similarly, model for the slack bus is as follows:

$$V_i(s) = 1 + (V_i^{sp} - 1)\, s, \quad i \in slack \tag{17}$$

$$where, \delta_{ni} = \begin{cases} 1, & \text{if } n = i \\ 0, & \text{otherwise} \end{cases}$$

The voltage function $V(s)$, in (15), (16) and (17), and $V^*(s^*)$, in (15) and (16), are expressed, respectively, as:

$$V(s) = \sum_{n=0}^{\infty} V[n]\,(s)^n \tag{18}$$

$$V^*(s^*) = \sum_{n=0}^{\infty} V^*[n]\,(s)^n \tag{19}$$

Where, the coefficients $V[n]$ are complex numbers. Now, by substituting (18) and (19) in (15), and comparing the constant term and coefficients of $s$, $s^2$, $s^3$, $\cdots$, the general expression for calculating the power series coefficients $V_i[n]$, $i \in m$, for $n > 0$ is:

$$\sum_{k=1}^{N} Y_{ik\ trans} V_k[n] = S_i^* W_i^*[n-1] - Y_{i\ shunt} V_i[n-1] \tag{20}$$

where, $W(s)$ is the inverse of the $V(s)$. Equation (20) is used to evaluate the coefficients of the voltage series for load bus, $i \in m$. By expressing reactive power as a full power series, similar to voltage power series as expressed by (18), and substituting in (16) along with (18) to (19), the general expression for calculating the power series coefficients $V_i[n]$, $i \in p$, for $n > 0$:

$$\sum_{k=1}^{N} Y_{ik\ trans} V_k[n] = Rhs\_Known_i[n-1] - jQ_i[n] \tag{21}$$

$$\begin{aligned} Rhs\_Known_i[n-1] &= P_i W_i^*[n-1] \\ &- j\left(\sum_{k=1}^{n-1} Q_i[k]\, W_i^*[n-k]\right) - Y_{i\ shunt} V_i[n-1] \end{aligned} \tag{22}$$

$$V_{i\ re}[n] = \delta_{n0} + \delta_{n1}\frac{{V_i^{sp}}^2 - 1}{2} - \frac{1}{2}\left(\sum_{k=1}^{n-1} V_i[k]\, V_i^*[n-k]\right) \tag{23}$$

The general term for voltage magnitude constraint is calculated by (23). Where, $V_{i\ re}[n]$ represents the real part of the n-th coefficient of voltage power series from the voltage magnitude constraint. Now by separating real and imaginary part of (20) and (21) and rearranging the known terms together,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ G_{l1} & -B_{l1} & 0 & -B_{l2} & \dots & G_{lN} & -B_{lN} \\ B_{l1} & G_{l1} & 1 & G_{l2} & \dots & B_{lN} & G_{lN} \\ G_{j1} & -B_{j1} & 0 & -B_{j2} & \dots & G_{jN} & -B_{jN} \\ B_{j1} & G_{j1} & 0 & G_{j2} & \dots & B_{jN} & G_{jN} \end{bmatrix} \begin{bmatrix} V_{slack\ re}[n] \\ V_{slack\ im}[n] \\ Q_l[n] \\ V_{l\ im}[n] \\ V_{j\ re}[n] \\ V_{j\ im}[n] \end{bmatrix} = \begin{bmatrix} \delta_{n0} + \delta_{n1}(V_{slack} - 1) \\ 0 \\ Q_l[n] \\ V_{l\ im}[n] \\ V_{j\ re}[n] \\ V_{j\ im}[n] \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ G_{l2} \\ B_{l2} \\ G_{j2} \\ B_{j2} \end{bmatrix} V_{i\ re}[n] \tag{24}$$

where, $l \in p$ and $j \in m$. The matrix on the LHS of (24), that is constructed by breaking the $Y_{ik\ trans}$ matrix into real and imaginary components and correcting the coefficients corresponding to slack bus and PV buses; will be referred as iteration matrix [23]. Equation (24) gives the voltage power series of PQ buses and (23) & (24) are used for computation of voltages power series and reactive power series of PV buses.

Analytic continuation technique is employed to find the converged value of this voltage function and reactive power function (i.e. the voltage and reactive power solution). To find the maximal analytic continuation, Padé approximants is used [24]. After calculating complex power for PQ buses and active power for PV buses mismatch is calculated. The process is repeated until convergence. Reactive power limit handling is given in [24], [23].

## V. Method For UC Calculation

To solve the unit commitment problem in a given scheduling time interval hybrid approach is utilized [25]. Hybrid approach combines dynamic programming with the particle swarm optimization to solve the problem. The approach is divided into two different dependent sub problems: a) Outer (discrete) sub-problem named as Particle Search Space (PSS) that consists of unit status (binary states 1 or 0). b) Inner (continues) sub problem named as search space which consist of optimal loading of generating units along with load flow. The objective of the approach is to select the best scheduling path so that total operating cost over given scheduling period can be minimized while satisfying different unit and load flow constraints. Steps involved in solving the SCUC problem are described below:

- **Step 1:** Total hourly load demand is obtained by summing up the load at all buses.
- **Step 2:** Obtain PSS, representing a group of generating units.
- **Step 3:** Rough idea of losses is obtained by performing ED with all the units to be available for 24-hr of scheduling horizon.
- **Step 4:** Fitness function is evaluated by calling PSO and load flow subroutines using HE method to obtained optimum unit status subjected to equality and inequality constraints listed in Section III.
- **Step 5:** Repeat Step 4 for every hour with minimum transition cost reference to the previous hour state, while satisfying minimum up/down time constraint.
- **Step 6:** Repeat Step 4 and Step 5 until the particles are able to find the best optimal commitment while satisfying different constraints for 24-hr of scheduling interval.

The method for UC evaluation using HELF has the following features

1. There are no repeated matrix (analogous to Jacobian in conventional Newton's method).
2. This matrix is a function of the system admittance and thus it is constant for a given system.
3. It provide extremely fast evaluation of UC as compared to NRLF methods.

## VI. Numerical Results

The proposed approach has been implemented entirely in MATLAB. Simulation are run on an Intel Core of 3.40 GHz computer with 4 Gb of RAM. Effectiveness of approach is tested on IEEE 30-bus system, unit data, load flow data and line MVA limits are taken from [8], [25]. For all analysis on this system, $V_i^{min}$,$V_i^{max}$, $\phi_i^{min}$,and $\phi_i^{max}$ for bus $i$ are considered to be 0.9 p.u, 1.1 p.u, -45 degree and 45 degree, respectively.

The effectiveness of the proposed approach is tested on IEEE 30-bus system with two different cases.

**Case (a)** Base Case: Without considering wind power.
**Case (b)** UC with forecasted wind power.

### A. Case (a)

The system consists of six generating units and 41 transmission lines. Data are taken from [8], [25]. The spinning reserve is assumed to be 10% of the load demand. Simulation results, unit schedule and total operational cost at each hour obtained using Dynamic Programming (DP), Branch and Bound (BB), Ant Colony System (ACS) and hybrid approach using NRLF and HELF for Case (a) are given in Table I. There are six generators and digit 1 represents the ON status and 0 represents the OFF status of the generating units. From Table I, it is found that the approach using HELF provide same cost as compared to NRLF. But, the application of HELF technique for UC problem provide fast computation with less computational burden as compared to NRLF method.

### B. Case (b)

The system consists of six generating units and Wind Power Generation (WPG). The forecasted wind data is taken from [19]. In this study, the impact of WPG with conventional thermal unit is taken into account for solving the UC schedule. Wind generator of 50 MW is integrated in the system at the $21^{st}$ bus to test the performance of UC. The result of the generation scheduling plan for Case (b) in the 24-hr time interval is shown in Table II. As it is well known that the WPG operational cost is zero, so, it should be run at its maximum value to minimize overall cost. But, when WPG is integrated with UCP, it should run with the system while satisfying different constraints, i.e., it can be run below its maximum value (curtailment of wind power) so that different constraints of the system are satisfied. For example, at the $24^{th}$ hour of the scheduling interval, WPG is generating 45.19 MW as shown in Table II, but its maximum generation at this hour is 50 MW i.e 4.81 MW of wind power is curtailed. From simulation results for Case (a) and Case (b) as given in Table I and II, it is found that after incorporating WPG in the system, total operating cost and losses are reduces, respectively. Losses for Case (a) in 24 hour of scheduling interval is found to be 141.80 MW and losses for Case (b) are given in Table II.

## VII. Conclusion

In this paper, hybrid approach is used to solve unit commitment problem. Optimal load sharing of generating unit is performed using particle swarm optimization. Newton-Raphson and non-iterative Holomorphic Embedding method is utilizes for solving the ac power-flow problem with two different cases. The HELF method overcomes the convergence and computational burden problem of conventional power flow methods. The effectiveness of proposed approach is tested on IEEE 30-bus system. The simulation results obtained from two different cases revels that

• Since the proposed method with HELF is taking much lesser

TABLE I: Unit status and total operating cost for Case (a).

| Dynamic Programing (DP) | | Branch and Bound (BB) | | Ant Colony System (ACS) | | Hybrid Approach | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | NRLF | | HELF | |
| Unit Status | $F_T$ (\$) | Unit Status | $F_T$ (\$) | Unit Status | $F_T$ (\$) | Unit Status | $F_T$ (\$) | Unit Status | $F_T$ (\$) |
| 111101 | 483.31 | 111111 | 558.88 | 111111 | 558.87 | 111101 | 482.63 | 111101 | 482.63 |
| 111101 | 503.79 | 111111 | 505.27 | 111111 | 505.27 | 111101 | 502.88 | 111101 | 502.88 |
| 111101 | 611.52 | 111111 | 610.28 | 111111 | 610.28 | 111101 | 610.27 | 111101 | 610.27 |
| 111111 | 921.42 | 111111 | 741.42 | 111111 | 741.42 | 111101 | 743.55 | 111101 | 743.55 |
| 111111 | 800.6 | 111111 | 800.6 | 111111 | 800.6 | 111101 | 803.64 | 111101 | 803.64 |
| 111111 | 759.32 | 111111 | 759.32 | 111111 | 759.32 | 111101 | 761.71 | 111101 | 761.71 |
| 111111 | 667.77 | 111111 | 667.77 | 111110 | 699.09 | 111101 | 668.86 | 111101 | 668.86 |
| 111111 | 558.29 | 111111 | 558.29 | 111110 | 556.76 | 111100 | 556.29 | 111100 | 556.29 |
| 111111 | 493.12 | 111110 | 519.71 | 111110 | 489.71 | 111100 | 487.74 | 111100 | 487.74 |
| 111110 | 427 | 111110 | 397 | 111110 | 397 | 111100 | 392.86 | 111100 | 392.86 |
| 111110 | 357.33 | 111110 | 357.34 | 111110 | 357.34 | 111100 | 352.42 | 111100 | 352.42 |
| 111110 | 394.14 | 111110 | 394.14 | 111110 | 394.14 | 111100 | 389.92 | 111100 | 389.92 |
| 111110 | 423.16 | 111110 | 423.16 | 111110 | 423.16 | 111100 | 419.64 | 111100 | 419.64 |
| 111110 | 468.13 | 111110 | 468.13 | 111110 | 468.13 | 111100 | 465.66 | 111100 | 465.66 |
| 111110 | 540.48 | 111110 | 540.48 | 111110 | 540.48 | 111100 | 539.65 | 111100 | 539.65 |
| 111110 | 620.48 | 111110 | 620.47 | 111110 | 620.47 | 111100 | 621.27 | 111100 | 621.27 |
| 111110 | 669.09 | 111110 | 669.09 | 111110 | 669.09 | 111100 | 670.56 | 111100 | 670.56 |
| 111110 | 651.6 | 111110 | 651.6 | 111110 | 651.6 | 111100 | 652.83 | 111100 | 652.83 |
| 111110 | 634.25 | 111110 | 634.25 | 111110 | 634.25 | 111100 | 635.24 | 111100 | 635.24 |
| 111110 | 596.66 | 111110 | 596.66 | 111110 | 596.65 | 111100 | 597.04 | 111100 | 597.04 |
| 111110 | 527.59 | 111110 | 527.59 | 111110 | 527.59 | 111000 | 526.13 | 111000 | 526.13 |
| 111110 | 458.99 | 111100 | 509.28 | 111100 | 509.28 | 111000 | 453.98 | 111000 | 453.98 |
| 111110 | 397 | 111100 | 393.63 | 111100 | 393.63 | 111000 | 388.78 | 111000 | 388.60 |
| 111100 | 360.64 | 111100 | 308.64 | 111100 | 308.64 | 111000 | 301.64 | 111000 | 301.64 |
| Total cost (\$) | | | | | | | | | |
| 13,325.65 | | 13,213.01 | | 13,212.77 | | 13,025.19 | | 13,025.01 | |
| Execution Time (s) | | | | | | | | | |
| 27 | | 7,288 | | 68 | | 216 | | 55 | |

TABLE II: Total cost and scheduling for 24 hour for Case (b).

| Hour | Power of generating units (MW) | | | | | | | Generation (MW) | Load (MW) | Losses (MW) | Cost (\$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | U1 | U2 | U3 | U4 | U5 | U6 | WPG | | | | |
| 1 | 98.55 | 31.28 | 15.12 | 10.00 | 0.00 | 12.00 | 2.61 | 169.57 | 166.00 | 3.57 | 476.17 |
| 2 | 122.16 | 36.92 | 16.96 | 10.00 | 0.00 | 12.00 | 2.92 | 200.96 | 196.00 | 4.96 | 494.77 |
| 3 | 150.97 | 43.89 | 19.24 | 10.00 | 0.00 | 12.00 | 0.00 | 236.10 | 229.00 | 7.10 | 611.06 |
| 4 | 173.58 | 49.43 | 21.15 | 20.03 | 0.00 | 12.00 | 0.00 | 276.18 | 267.00 | 9.18 | 744.07 |
| 5 | 182.54 | 51.65 | 21.93 | 25.11 | 0.00 | 12.28 | 0.00 | 293.52 | 283.40 | 10.12 | 804.02 |
| 6 | 176.36 | 50.12 | 21.39 | 21.60 | 0.00 | 12.00 | 0.00 | 281.46 | 272.00 | 9.46 | 762.18 |
| 7 | 161.91 | 46.56 | 20.15 | 13.45 | 0.00 | 12.00 | 0.00 | 254.06 | 246.00 | 8.06 | 669.53 |
| 8 | 147.56 | 43.02 | 18.89 | 10.00 | 0.00 | 0.00 | 0.00 | 219.47 | 213.00 | 6.47 | 556.84 |
| 9 | 127.85 | 38.26 | 17.34 | 10.00 | 0.00 | 0.00 | 3.57 | 197.02 | 192.00 | 5.02 | 477.14 |
| 10 | 103.45 | 32.42 | 15.44 | 10.00 | 0.00 | 0.00 | 3.24 | 164.54 | 161.00 | 3.54 | 384.18 |
| 11 | 94.75 | 30.34 | 15.00 | 10.00 | 0.00 | 0.00 | 0.00 | 150.09 | 147.00 | 3.09 | 353.25 |
| 12 | 105.24 | 32.83 | 15.56 | 10.00 | 0.00 | 0.00 | 0.00 | 163.63 | 160.00 | 3.63 | 390.71 |
| 13 | 113.18 | 34.73 | 16.17 | 10.00 | 0.00 | 0.00 | 0.00 | 174.08 | 170.00 | 4.08 | 420.40 |
| 14 | 102.93 | 32.34 | 15.51 | 10.00 | 0.00 | 0.00 | 28.02 | 188.80 | 185.00 | 3.80 | 382.74 |
| 15 | 111.85 | 34.49 | 16.26 | 10.00 | 0.00 | 0.00 | 39.98 | 212.58 | 208.00 | 4.58 | 416.19 |
| 16 | 129.60 | 38.77 | 17.67 | 10.00 | 0.00 | 0.00 | 41.73 | 237.77 | 232.00 | 5.77 | 484.92 |
| 17 | 143.52 | 42.12 | 18.77 | 10.00 | 0.00 | 0.00 | 38.27 | 252.69 | 246.00 | 6.69 | 541.07 |
| 18 | 142.75 | 41.93 | 18.68 | 10.00 | 0.00 | 0.00 | 34.16 | 247.52 | 241.00 | 6.52 | 537.81 |
| 19 | 144.30 | 42.30 | 18.79 | 0.00 | 0.00 | 0.00 | 37.43 | 242.83 | 236.00 | 6.83 | 510.79 |
| 20 | 132.78 | 39.52 | 17.88 | 0.00 | 0.00 | 0.00 | 40.85 | 231.03 | 225.00 | 6.03 | 464.07 |
| 21 | 116.00 | 35.48 | 16.55 | 0.00 | 0.00 | 0.00 | 40.85 | 208.87 | 204.00 | 4.87 | 398.47 |
| 22 | 101.82 | 32.08 | 15.42 | 0.00 | 0.00 | 0.00 | 36.60 | 185.92 | 182.00 | 3.92 | 345.34 |
| 23 | 81.73 | 27.32 | 15.00 | 0.00 | 0.00 | 0.00 | 39.98 | 164.04 | 161.00 | 3.04 | 277.09 |
| 24 | 53.04 | 20.00 | 15.00 | 0.00 | 0.00 | 0.00 | 45.19 | 133.23 | 131.00 | 2.23 | 186.69 |
| | Total | | | | | | | | | 132.55 | 11,689.51 |

time to evaluate UC as compared to NRLF, so there is large amount of time saving for the large scale UC problem.

• By efficiently utilize wind energy, the total fuel cost of thermal units can be reduced significantly. For future studies, the authors will explore the potential of proposed approach in optimization involving uncertainties in renewable energy sources on large scale unit commitment problem.

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