Incorporation of Asynchronous Generators as PQ Model in Load Flow Analysis for Power Systems with Wind Generation

James Ranjith Kumar. R, *Member, IEEE,* Amit Jain, *Member, IEEE,*
Power Systems Division,
Central Power Research Institute, India.
jamesrk@cpri.in, amitjain@cpri.in

***Abstract*—Asynchronous Generators are commonly used in wind based generating units as electrical power generated by them will be the same as the frequency of the grid irrespective of their shaft speed. In this paper, a new methodology for carrying out the load flow calculations in a power system having asynchronous generators has been presented. The model, which is proposed in this paper, utilizes the solution of the quadratic equation and hence, it will not deteriorate the convergence characteristics of the base load flow program. The proposed model has been incorporated in Newton's method based load flow technique and tested on 30 bus and 118 bus systems which have wind generation. The simulation results are benchmarked with the exact impedance model of the asynchronous generator available in literature in order to demonstrate the usefulness of the proposed model.**

***Keywords*—asynchronous generator, induction generator, load flow analysis, PQ model, quadratic solution, wind generating system**

I. Introduction

Wind energy has gained much popularity among different renewable based power generation technologies as it requires a fair capital investment and very less running costs. Due to this more and more wind based generating units are being integrated in the power grid at both transmission and distribution levels. Thus there is a need for proper modelling of these units in the traditionally used computation techniques for power system planning, operation and control. Each wind generating unit consists of two major well known components. The first part is the wind turbine which converts the kinetic energy of the wind flow into rotary mechanical energy. For almost every wind turbine manufacturer will provide the data sheet for their wind turbine which gives the relation between the velocity of the wind and the mechanical power generated by the turbine. So the shaft power can be quantified in the power system studies for the given situation.

The second major component is the electrical generator which generates electrical power from its shaft mechanical power. These wind generating units can be classified into three categories namely fixed speed, limited variable speed and variable speed [1]. In all these categories, the most common type of generator used in a wind generating unit is the asynchronous generator. The main reason for preferring asynchronous generators over synchronous machines is that their generated power will have the same frequency of the grid to which they are connected irrespective to their shaft speed. Apart from this, asynchronous machines possess other inherent advantages like these are robust, these do not need any synchronising and DC excitation equipment. Even though it has various advantages, the modelling of induction generators, which are basically asynchronous machines, is different from that of a synchronous generators. As asynchronous machines operate on the principle of mutual induction, it shares the model similar to a transformer. So the shunt component of this model is extremely important as it contains the core loss and magnetising components of the machine. Moreover, in order to operate this induction generator in self excited mode, the reactive power for the magnetising component need to be locally supplied by capacitor banks. For designing a proper reactive power compensation and for planning the network strengthening activities in power grid, Load Flow is the primary and important computational tool.

Load flow techniques for transmission systems have been studied and developed by researchers over the past many decades [2]. Among these techniques, the Newton's method [3] is the popular and well known technique and it has been utilized in major power system studies. The Newton's method outweighed other load flow techniques till date because of its quadratic convergence characteristics. It can handle complex power controlled buses (PQ), voltage magnitude and real power controlled buses (PV) and reference bus where voltage magnitude and angle are specified. But this method cannot directly handle Asynchronous Generators since the complex power injected by the machine in its terminals is a variable depending upon the bus voltage. So it needs to be modelled such that it can be incorporated in the load flow program.

Different Load Flow models for Asynchronous Generators have been proposed in the past which are available in the literature. A comprehensive survey was made in [4] on the popular induction machine models. These models can be differentiated under two categories. In the first category, which is PQ type model, the equivalent complex power injection at the machine terminals will be computed. Under the second category or RX type model, the parameters of equivalent impedance network of the asynchronous machine will be found that is seen from the machine terminals. One of the earliest models, which is a RX type model, was developed by Murthy et al. [5] in which the slip of the machine is first computed and its equivalent thevenin's circuit is formed. The application of this model in Radial Load flow solution was presented in [6].

In [7], two models have been developed for asynchronous generators, one PQ type and another RX type. These models were developed based on the assumption that the core loss component in the shunt branch of the asynchronous machine is negligible. Chen et al. [8] further simplified the first model proposed in [7] by neglecting the real components of stator and rotor impedances. A further improvement in [7] was made by Eminoglu [4] where the active power injection of the asynchronous machine will be updated in every iteration according to the value of machine terminal voltage.

Models which can handle fixed speed, limited variable speed and variable speed wind generating units have been proposed in [9]. Feijóo [10] developed a simple iterative technique for computing the power injections at the machine terminals using the mechanical shaft power. So this method takes sub-iterations in every main iteration when it is incorporated in the load flow programs. An Impedance model for asynchronous machine was proposed in [11] which compute the slip of the machine using shaft mechanical power and the previous estimate of rotor side voltage. This slip value is used to transform the shaft power into equivalent impedance and this simplified linear system is solved to further update the complex power injection at the machine terminals. Haque [12] has proposed that the induction generators can be handled by creating internal buses at each node of the machine so that any load flow program can compute the solution at these buses. An induction generator model is presented in [13] where the previous estimate of the rotor side voltage is used to update the rotor side voltage for the next iteration by using the solution of the quadratic equation. Feijóo and Villanueva [14] developed an updating rule for updating the values of real and reactive power injections of the asynchronous generator. This model does not provide a direct solution as it requires the estimate of stator side and rotor side voltages at the beginning of every iteration.

The present paper is focused on developing a model for an asynchronous generator using the solution of the quadratic equation. Standard star to delta transformation is used to simplify the machine model without losing any accuracy. The proposed model can directly compute the rotor side voltage and from that the complex power supplied at the machine terminals is calculated. The proposed model has been incorporated in the Load Flow program based on Newton's method and its performance is compared with the impedance model presented in [11]. The proposed model is tested on 30 bus and 118 bus transmission systems having 3 and 12 wind farms respectively and the simulation results from these two test cases has been presented and compared with the model presented in [11] to establish the effectiveness of the proposed model.

II. Load Flow Model For Asynchronous Generator

The methodology of incorporating the proposed PQ Model for Asynchronous Generator has been explained in this section. The first part of this section explains the Load Flow Technique which was adopted in this paper. The second part will detail the mathematical formulations developed for the Asynchronous Generator model. Finally the consolidated load flow algorithm has been provided for handling the proposed PQ Model for Asynchronous Generator.

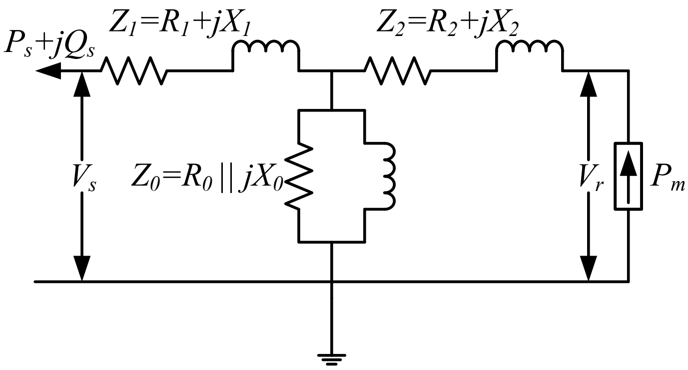

Fig. 1. Equivalent circuit of Asynchronous Generator for Steady State Analysis

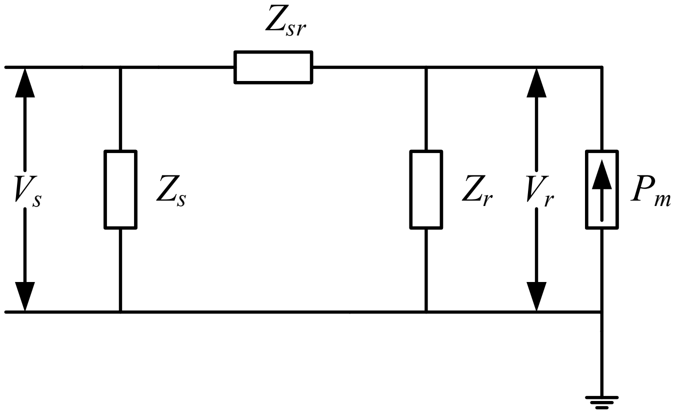

Fig. 2. Steady State π model of Asynchronous Generator

A. *Load Flow Method*

It is well known that, Load flow is the computational method for calculating the node voltages of the electric network by solving the power balance equations. In this paper, the complex node voltages are considered in rectangular form, i.e. $V_k = e_k + jf_k$. The power balance equations at bus k is as follows.

$$P_k = e_k \sum_{m \in k} (G_{km} e_m - B_{km} f_m) + f_k \sum_{m \in k} (G_{km} f_m + B_{km} e_m) \tag{1}$$

$$Q_k = f_k \sum_{m \in k} (G_{km} e_m - B_{km} f_m) - e_k \sum_{m \in k} (G_{km} f_m + B_{km} e_m) \tag{2}$$

In these equations, G_{km} and B_{km} are the real and imaginary parts of the element of Bus Admittance Matrix corresponding to buses k and m. For buses where the voltage magnitude are controlled, the following equation is utilised.

$$V_k^2 = e_k^2 + f_k^2 \tag{3}$$

Though the model proposed in this paper will work with any load flow technique, this paper has adopted the Newton's Method [3] for computing the node voltages computed in rectangular form [15] which is given in (4).

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V^2 \end{bmatrix} = \begin{bmatrix} S & T \\ U & W \\ EE & FF \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \tag{4}$$

The elements of the Jacobian matrix in (4) are defined as given in [15]. The mismatch vector can be calculated from the power balance equations and the voltage magnitude expressions.

B. Asynchronous Generator model

The equivalent circuit of a typical asynchronous generator is shown in Figure 1 which is given in single phase per unit (pu) form.

R_1 and X_1 are defined as stator resistance and stator leakage reactance from which the stator impedance can be found as $Z_1 = R_1 + jX_1$. In a similar way, impedances are defined for rotor, $Z_2 = R_2 + jX_2$ and for shunt branch, $Z_0 = \frac{jR_0X_0}{R_0+jX_0}$. The shaft power at the mechanical side and the output power at the electrical side are P_m and $(P_s + jQ_s)$ respectively.

It can be observed that model give in Figure 1 has T structure and it can be transformed into an equivalent π model using standard star to delta transformation technique.

$$Z_{sr} = \frac{Z_1Z_0 + Z_2Z_0 + Z_1Z_2}{Z_0} \tag{5}$$

$$Z_s = \frac{Z_1Z_0 + Z_2Z_0 + Z_1Z_2}{Z_2} \tag{6}$$

$$Z_r = \frac{Z_1Z_0 + Z_2Z_0 + Z_1Z_2}{Z_1} \tag{7}$$

By Kirchoff's Current Law, the power balance equation of the asynchronous generator model shown in Figure 2 can be written as

$$\frac{P_m}{V_r^*} = \frac{V_r - V_s}{Z_{sr}} + \frac{V_r}{Z_r} \tag{8}$$

which can be algebically rewritten as

$$V_rV_r^* = \frac{Z_p}{Z_{sr}}V_sV_r^* + Z_pP_m \tag{9}$$

where

$$Z_p = \frac{Z_{sr}Z_r}{Z_{sr} + Z_r} \tag{10}$$

By taking complex conjugate, (9) can be rewritten as

$$V_r^*V_r = \frac{Z_p^*}{Z_{sr}^*}V_s^*V_r + Z_p^*P_m \tag{11}$$

After subtracting (9) with (11)

$$\frac{Z_p^*}{Z_{sr}^*}V_s^*V_r - \frac{Z_p}{Z_{sr}}V_sV_r^* = \left(Z_p - Z_p^*\right)P_m \tag{12}$$

This expression is reoriented so that the value of V_r^* can be obtained in terms of V_r

$$V_r^* = \frac{\frac{Z_p^*}{Z_{sr}^*}V_s^*V_r}{\frac{Z_p}{Z_{sr}}V_s} + \frac{\left(Z_p^* - Z_p\right)P_m}{\frac{Z_p}{Z_{sr}}V_s} \tag{13}$$

The value of V_r^* given in (13) is substituted in (11)

$$\frac{\frac{Z_p^*}{Z_{sr}^*}V_s^*}{\frac{Z_p}{Z_{sr}}V_s}V_r^2 + \left(\frac{\left(Z_p^* - Z_p\right)P_m}{\frac{Z_p}{Z_{sr}}V_s} - \frac{Z_p^*}{Z_{sr}^*}V_s^*\right)V_r - Z_p^*P_m = 0 \tag{14}$$

As it is in standard form of a quadratic equation with coefficients and rewritten as

$$aV_r^2 + bV_r + c = 0 \tag{15}$$

where

$$a = \frac{\frac{Z_p^*}{Z_{sr}^*}V_s^*}{\frac{Z_p}{Z_{sr}}V_s} \tag{16}$$

$$b = \frac{\left(Z_p^* - Z_p\right)P_m}{\frac{Z_p}{Z_{sr}}V_s} - \frac{Z_p^*}{Z_{sr}^*}V_s^* \tag{17}$$

$$c = -Z_p^*P_m \tag{18}$$

This quadratic equation given in (15) can be directly solved using the well-known solution as given in (19).

$$V_r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{19}$$

It was observed that in the simulation, keeping positive sign gives a stable solution and hence the second solution with negative sign has been ignored. As the voltage at the rotor side has been calculated, the electric power injected by the asynchronous generator at the point of common coupling is given by (20).

$$P_s + jQ_s = P_m - \frac{|V_r - V_s|^2}{Z_{sr}^*} - \frac{|V_r|^2}{Z_r^*} - \frac{|V_s|^2}{Z_s^*} \qquad (20)$$

It is important to note that in this model, the impedance term Z_r is not lumped with the power injection P_m and V_r is directly updated by quadratic equation solution but it does not require previous value of V_r for this quadratic equation solution. So the proposed PQ model uses a simple quadratic equation based solution which gives accurate solution for the asynchronous generator by updating the real and reactive power injections by using V_r as given in (20) and where V_r is evaluated directly.

C. *Algorithm*

1) The network and bus data is read
2) The Bus Admittance Matrix is formed using the network data
3) The PQ injections of the asynchronous generator is calculated using the model given in (20).
4) Update these injections in the bus data corresponding to the nodes wher the asynchronous machies are connected
5) Calculate the mismatch vector using the calculated and given values of active and reactive power and squares of the voltage magnitudes at PV buses
6) Compute the elements of the Jacobian Matrix
7) The corrections required for the real and imaginary parts of voltages are solved using the Jacobian Matrix and the mismatch vector
8) These corrections are updated in respective bus voltages
9) Test for tolerance limit and repeat from step 3) of algorithm, if it is not satisfied

III. Simulation Results and Discussion

The proposed model is implemented in the Newton's technique for load flow solution based on rectangular coordinates [15]. This section presents the results which are obtained from this algorithm which has been programmed in MATLAB. This program has been tested on 30-bus and 118-bus systems which are modified by adding wind farms for which test system details are taken from [12]. These wind farms are composed of identical fixed speed wind turbines (Vestas V100, 1.8 MW) and Asynchronous Generators (1.8 MW, 575V, 0.9 pf). The wind speed for wind farm turbines has been taken as 12 m/s [16] in the present simulation study. The efficiency of the gear box is assumed to be 95%. The asynchronous generator parameters are R_1= 0.004843 pu, X_1= 0.1248 pu, R_2= 0.004377 pu, X_2= 0.1791 pu, X_0= 6.77 pu and the value of R_0 was ignored. Each of the wind farms are connected to the power system network through a step up transformer with a leakage reactance of 0.05 pu. New buses are formed on the secondary side of these transformers in both test cases. The results of proposed model are compared with the impedance model, given in [11].

A. *30-bus system*

The network and system data of the 30 bus system is taken from [17]. This system comprised of 5 PV buses, 4 Transformers with off nominal taps and 3 wind farms. The details of these wind farms, connected to the 30 bus system, are provided in Table I. The values of voltage magnitudes obtained at all buses after convergence from proposed model and impedance model given in [11] have been shown in Figure 3. The results obtained from both the models are in exact agreement to each other. In order to demonstrate the effectiveness of the proposed model, the maximum error on mismatch vector at the end of each iteration have been plotted in Figure 4 for both the models. It is easy to see that by incorporating the proposed model in the Newton's Method, the quadratic rate of convergence is not affected. But the exact impedance model tends to affect the convergence rate as it took 7 iterations for convergence whereas the propose model took 5 iterations for obtaining the solution at the prescribed tolerance limit.

TABLE I. Details of Wind Farms in 30-bus System

Connected Bus	Number of Wind Turbine & Asynchronous Generator Sets	Capacity in MW
14	5	9
26	10	18
30	15	27

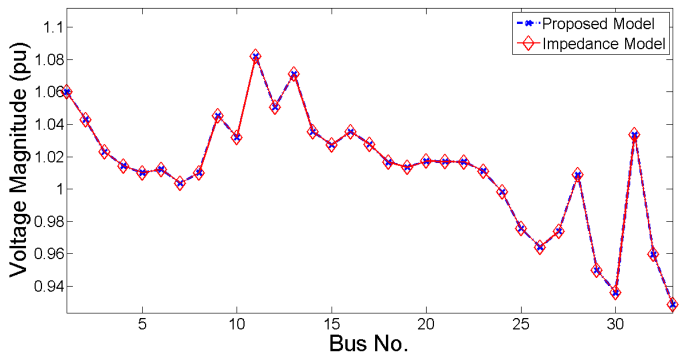

Fig. 3. Voltage Magnitude across all buses on the 30-bus system

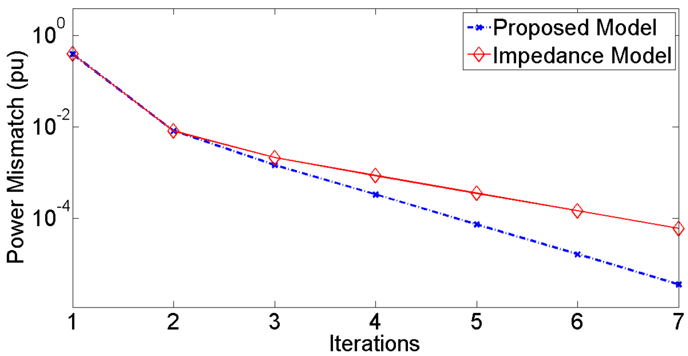

Fig. 4. Maximum error on mismatch vector of the 30-bus system

B. *118-bus system*

The second test case taken for evaluating the proposed model is the standard IEEE 118 bus system whose network and system are taken from [18]. This system has large number of PV buses. For the present simulation study, totally 12 wind farms are connected at different locations of this 118 bus system. Table II shows the details of these wind farms which are connected to this 118 bus system.

Figure 5 shows the profile of voltage magnitudes across all buses obtained after convergence from proposed model and impedance model given in [11]. Due to the presence of large number of PV buses, it can be observed that there are more variations on the voltage magnitudes between the adjacent buses. So the convergence is difficult as compared to the 30 bus test case. The solutions obtained at the end of convergence are exactly same for the proposed model and for the impedance model given in [11]. Though both the models are implemented in Newton's technique based load flow method and have the same tolerance limit, it took 5 iterations for convergence using the proposed model for asynchronous generator but on the other side, it took 7 iterations for the impedance model given in [11]. To demonstrate the convergence rate, the maximum value in the mismatch vector at each iteration from both models have been plotted in Figure 6. This plot shows that the proposed model does not deteriorate the quadratic convergence characteristics of the Newton's Method as the error value decreases at a rate of two decades/iteration.

TABLE II. DETAILS OF WIND FARMS IN 118-BUS SYSTEM

Connected Bus	Number of Wind Turbine & Asynchronous Generator Sets	Capacity in MW
3	5	9
16	10	18
20	15	27
33	20	36
41	25	45
53	30	54
62	5	9
74	10	18
84	15	27
98	20	36
106	25	45
117	30	54

It is well known that the Newton's method have quadratic convergence characteristics [3]. This is because the fact that majority of the power balance equations are quadratic in nature and Taylor series for them do not have more than second order terms. As Newton's method ignores only the second order, in other words Hessian components, error value tends to decrease in a quadratic rate if the initial guess is closer to the final solution.

Though impedance model given in [11] compute the rotor side voltage and using it, the P and Q values injected by the asynchronous generator is updated in every iteration but it linearizes the shaft power which is a quadratic term into equivalent impedance. So in this case, the previous estimate of the rotor side voltage is required for calculating the new values of rotor side voltage. This leads to reducing the speed of convergence of the load flow program.

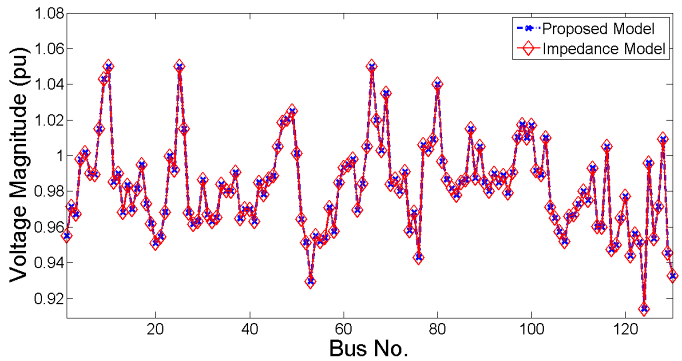

Fig. 5. Voltage Magnitude across all buses on the 118-bus system

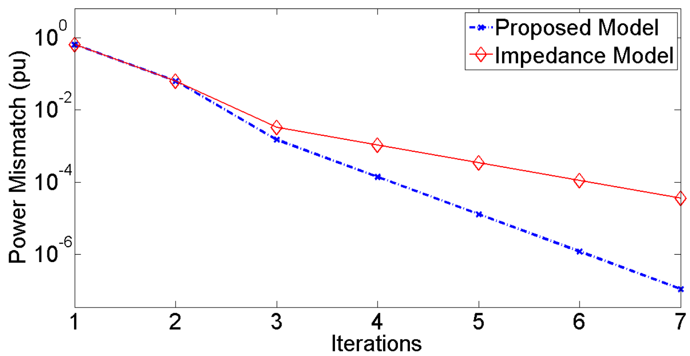

Fig. 6. Maximum error on mismatch vector of the 118-bus system

In the proposed model, the rotor voltages are calculated directly from the machine terminal voltage using the solution of quadratic equation and does not require any prior estimate of rotor voltage. So the values of P and Q for the asynchronous generator will be updated with the direct solution instead of an estimated value. Hence the convergence rate of the Newton's method will not be affected by using the proposed model.

IV. CONCLUSION

A load flow solution with asynchronous machine model has been developed in this paper which uses the shaft mechanical power and the terminal voltage to compute the active and reactive power supplied by the machine. This model first computes value of the rotor side voltage using the direct solution of the quadratic expression. Due to this reason, the convergence characteristics of the proposed load flow solution will not be affected. The effectiveness of the proposed model has been tested on 30 bus and 118 bus system using the Newton's method based load flow in rectangular coordinates. The results of the proposed model have been compared with the impedance model of the asynchronous machine given in [11]. It indicates that, by using the proposed model for asynchronous machine, the load flow program converges in a quadratic manner and better in performance. Also the high variation in the voltages between the adjacent buses in the 118 bus system and the large number of the wind farms did not have effect on the performance of the proposed model which shows the effectiveness of the proposed model. The authors are working to apply the proposed solution for power systems with doubly fed induction generator (DFIG), which will be presented in future work.

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