A Comparative Study of Control Theories for Realizing APFs in Distribution Power Systems

Pradeep Kumar
Department of Electrical Engineering
Indian Institute of Technology Madras
Email: ee14m039@ee.iitm.ac.in

Mahesh K. Mishra
Department of Electrical Engineering
Indian Institute of Technology Madras
Email: mahesh@ee.iitm.ac.in

Abstract—Concerns of power quality problems, especially for sensitive loads are well known. The integration of microgrid and renewable energy sources with power converters is propounding this problem. This challenges us to know more about their control techniques so as to optimize their performance, both in terms of time and memory. This paper is concerned to analyze the formulation of three control theories (ISC, IRP, SRF) in terms of computational time and memory, used in control of Active Power Filters to mitigate these problems. This assists in optimizing a given DSP algorithm for APFs and also reducing the overall operational time. A comparative analysis is presented with their Big-O polynomial to comment upon their efficiency in time and memory space.

I. INTRODUCTION

It would be prudent to say that the problem of power quality (PQ) being faced is a reflection of the industrial growth achieved so far. Moreover, electrical networks with poor PQ result in financial loss, environmental impacts and/ or safety concerns. These losses are cascaded back to the utility, power plants and results in increased CO2 emissions. Therefore, mitigation of PQ problems in power systems more so on dynamic and real time basis is a need of an hour in todays industrial world.

As power system suffers from different PQ problems, numerous methods/researches are proposed to indentify and mitigate such events [1], [2]. Since the advent of Active Power Filters (APF), numerous designs based on different configurations, control strategies, economic and technical considerations and their applications in power systems were evolved [3]. In order to implement an APF, selection of control strategy plays an important role. Therefore, control of an APF includes overall system control, extraction of reference signals, controlling d-link capacitor voltage or inductor current and gating signal generation. Accordingly, researchers made use of various control strategies both in time and frequency domain to implement the custom power devices to achieve load compensation and voltage regulation. The commonly used power control theories used in implementing APFs are Instantaneous Symmetrical Component (ISC) Theory, Instantaneous Reactive Power Theory (IRP or pq) and Synchronous Frame Theory (SRF or dq). The generalization of IRP theory was an interesting proposition in unifying both the ISC and IRP theory to achieve shunt compensation for a balanced and unbalanced distribution system [4]. So far, the control theories have been characterized and compared based on their applicability in meeting the compensation goals and to achieve desired THD % (quantified parameter), as specified in IEEE-519. Though, the symmetrical component theory is superior to other theories, in terms of its computational efficiency and simplicity, however, a very little information is available with the power quality engineers to comment upon the very basic nature of these theories, specifically their performance in terms of computational time and memory.

The paper aims to bring out a generic comparison in terms of time and space (memory) involved by deriving a polynomial relation (Big-O), amongst the theories. The scope of the comparison involves, analyzing the formulation of ISC theory, IRP (pq) and SRF (dq) in terms of their execution time and space occupied, under different source voltage conditions. A case study is also performed by analyzing the performance of the control theories on TMS320F28377S processor using CCS software. The paper also incorporates simulation studies to compare transient performance of the given theories.

II. MOTIVATION

Knowing the growing challenges of designing a grid interactive filter to provide real time mitigation/ reduction of the power quality problem, the implementation of APFs requires meticulous designing at each stage [5], [6]. The advent of microgrids and their unpredictable operation also impinges sudden variations in the source conditions for implementing an APF. As over a period, the use of digital control systems has taken over the analog control everywhere, the implementation of more complex digital signal processing algorithms has also been made possible. The advent of microgrids and their unpredictable operation also impinges sudden variations in the source conditions for implementing an APF. As over a period, the use of digital control systems has taken over the analog control everywhere, the implementation of more complex digital signal processing algorithms has also been made possible. In order to implement such APFs in digital domain, aspects starting from currents and voltage measurements, signal galvanic isolation, choice of sampling rate, choice of number of bits, choice of sequential versus simultaneous sampling, synchronization with line voltage, signal filtration and separation etc. needs to be deliberated. Therefore, in order
to design an efficient APF the vetting of a given control theory assumes utmost importance. Fig. ?? explains the methodology proposed in the paper for analyzing the control theories and their comparative study.

III. CONTROL THEORIES

The three widely used control theories in Power Quality analysis: ISC, IRF, and SRF have been used for realizing the APFs by generating reference currents to compensate the load unbalance and harmonics. The equations involved in reference current generation in three theories have been discussed here.

A. Instantaneous Symmetrical Component Theory (ISC)

The source voltage \((v_{sa}, v_{sb}, v_{sc})\) and load currents \((i_{ta}, i_{tb}, i_{tc})\) must be known to implement ISC using following equations [?], [?].

1) Computation of zero sequence source voltage, \(v_{s0}\) and instantaneous active power, \(p(t)\):

\[
v_{s0} = \frac{1}{\sqrt{3}}(v_{sa} + v_{sb} + v_{sc})
\]

\[
p(t) = [v_{sa} \quad v_{sb} \quad v_{sc}] \begin{bmatrix} i_{ta} \\ i_{tb} \\ i_{tc} \end{bmatrix}
\]

2) Computation of \(P_{avg}\) using Moving Average Filter (MAF):

\[
P_{avg} = \frac{1}{T} \int_{t_{1}-T}^{t_{1}} (v_{sa}i_{ta} + v_{sb}i_{tb} + v_{sc}i_{tc})dt
\]

for implementation purposes (??) in discrete form is stated as:

\[
P_{avg} = \frac{1}{k} \sum_{i=(j-kt_{d})}^{j} (v_{sa}(i)i_{ta}(i) + v_{sb}(i)i_{tb}(i) + v_{sc}(i)i_{tc}(i))
\]

where, \(T = kt_{d}\), \(t_{d}\) is the discretization time (sec), \(k\) is the number of points in a cycle.

3) Computation of reference currents- \(i_{fa}^{*}, i_{fb}^{*}, i_{fc}^{*}\):

\[
i_{fa}^{*} = i_{ta} - i_{sa} = \frac{v_{sa} + \gamma(v_{sb} - v_{sc})}{\Delta} P_{avg}
\]

\[
i_{fb}^{*} = i_{tb} - i_{sb} = \frac{v_{sb} + \gamma(v_{sc} - v_{sa})}{\Delta} P_{avg}
\]

\[
i_{fc}^{*} = i_{tc} - i_{sc} = \frac{v_{sc} + \gamma(v_{sa} - v_{sb})}{\Delta} P_{avg}
\]

where, \(\Delta = \sum_{j=a,b,c} v_{ji}^{2} - 3v_{sa}^{2}\), \(\gamma = \tan(\phi^{+})\) and \(\phi^{+}\) is the power factor between the positive sequence voltage \((v_{sa}^{+})\) an current \((i_{sa}^{+})\).

However, in case of unbalanced and/ or distorted source voltage condition the reference current generation will be decided by using the Fundamental positive sequence component of unbalanced/distorted source voltages \((v_{sa}^{+}, v_{sb}^{+}, v_{sc}^{+})\) - as described in [?].

B. Instantaneous Reactive Power Theory (IRP or pq)

The formulation of p-q theory to derive the reference currents, involves following equations [?].

1) Transformation of source voltage and load current to \(\alpha-\beta-0\) domain: The famous Clark’s transformation is used to perform this step.

\[
\begin{bmatrix} v_{0} \\ v_{a} \\ v_{b} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix}
\]

\[
\begin{bmatrix} i_{0} \\ i_{a} \\ i_{b} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{ta} \\ i_{tb} \\ i_{tc} \end{bmatrix}
\]

2) Computation of instantaneous real, \(p_{\alpha\beta}\) and reactive components, \(q_{\alpha\beta}\) of power in \(\alpha-\beta\) domain: The instantaneous real, reactive and zero sequence powers, denoted by \(p, q,\) and \(p_{0}\), respectively are expressed in \(\alpha\beta\) plane [?].

\[
\begin{bmatrix} p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} v_{a} & v_{b} \\ -v_{b} & v_{a} \end{bmatrix} \begin{bmatrix} i_{a} \\ i_{b} \end{bmatrix}
\]

\[
p_{0} = v_{0} \times i_{0}
\]

3) Computation of mean real power \((P_{\alpha\beta})\) using MAF: The mean active power in \(\alpha\beta\) plane, \(p_{\alpha\beta}\) is calculated using (??)

\[
P_{\alpha\beta} = \frac{1}{T} \int_{t_{1}-T}^{t_{1}} (v_{a}i_{a} + v_{b}i_{b})
\]

4) Generation of instantaneous reference filter currents in \(\alpha\beta\) domain \(i_{fa}^{*}, i_{fb}^{*}\):
5) Transformation of reference currents in abc domain:

\[
\begin{bmatrix}
i_{fa}^r \\
i_{fb}^r \\
i_{fc}^r
\end{bmatrix} = \sqrt{2/3} \begin{bmatrix}
\sqrt{2}/2 & 1/2 & \sqrt{3}/2 \\
\sqrt{2}/2 & -1/2 & \sqrt{3}/2 \\
\sqrt{2}/2 & -1/2 & -\sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
i_{fa} \\
i_{fb} \\
i_{fc}
\end{bmatrix}
\]

However, in case of unbalanced and/or distorted source voltage condition, the process of fundamental positive sequence extraction to be followed in addition to above mentioned steps [1].

C. Synchronous Reference Frame Theory (SRF or dq)

This control theory makes use of load currents (i_{la}, i_{lb}, i_{lc}) to implement shunt APF using following equations, thereby reducing the number of inputs to be sensed to three [2]. In order to implement this theory, the information related to 'θ' is assumed to be obtained using PLL, which can be made as an in-built feature in microprocessors, thereby reducing separate execution time (delay time) and memory in overall implementation.

1) Transformation of load current to dq0 reference frame:

\[
\begin{bmatrix}
i_{ld} \\
i_{dq} \\
i_{d0}
\end{bmatrix} = \sqrt{2/3} \begin{bmatrix}
cos\theta & cos(\theta + \frac{2\pi}{3}) & cos(\theta - \frac{2\pi}{3}) \\
sin\theta & sin(\theta + \frac{2\pi}{3}) & sin(\theta - \frac{2\pi}{3}) \\
\sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2
\end{bmatrix} \begin{bmatrix}
i_{fa} \\
i_{fb} \\
i_{fc}
\end{bmatrix}
\]

The execution of (13) would necessitate three Cosine calculations and three Sine calculations sequentially after 1 SUB and 1 ADD. Therefore, the implementation of Sin-Cos calculator is an additional computational step in this theory, which is implemented using a look-up table in the processor.

2) Computing average of d-component of load current (I_{ld}) using MAF:

\[
I_{ld} = \frac{1}{2T} \int_{t_{i} = T}^{t_{f} = T} i_{ld}(t)dt
\]

From implementation point of view, (14) takes the discrete form of (14).

\[
I_{ld} = \frac{1}{k} \sum_{i = (j - k t_{d})}^{j} i_{ld}(i)
\]

3) Computation of reference currents in abc domain: The matrix in (15) is the transpose matrix of equation (14). Therefore, while implementing (15) no new memory is allocated for this stage as memory addressing mode is used to optimize the memory space as there were no distinct values involved.

\[
\begin{bmatrix}
i_{fa}^r \\
i_{fb}^r \\
i_{fc}^r
\end{bmatrix} = \sqrt{2/3} \begin{bmatrix}
cos\theta & cos(\theta - \frac{2\pi}{3}) & cos(\theta + \frac{2\pi}{3}) \\
sin\theta & sin(\theta - \frac{2\pi}{3}) & sin(\theta + \frac{2\pi}{3}) \\
\sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2
\end{bmatrix} \begin{bmatrix}
i_{ld} - I_{ld} \\
i_{dq} \\
i_{d0}
\end{bmatrix}
\]

IV. COMPARATIVE ANALYSIS

The performance of these control theories are quantified in terms of memory occupied and execution time involved. The Big-O analysis, which is generally used in solving a computer science problem when there are usually more than one method/algorithm is available to solve a problem, is used here in MATLAB.

The definition of Big-O analysis is that it measures the efficiency of an algorithm based on the time it takes for the algorithm to run as a function of the input size (here t_{d}).

The performance of each control theory is represented in the form of a general polynomial, which can be analyzed and compared based on the available microprocessor used for implementing an APF. Following cases are considered to analyze and compare the performance of the control theories in time and memory domain

- Case I: Comparison of control theories under balanced source voltage condition
- Case II: Comparison of control theories under unbalanced source voltage condition

### TABLE I

<table>
<thead>
<tr>
<th>Mathematical Operations</th>
<th>ADD</th>
<th>SUB</th>
<th>MUL</th>
<th>DIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISC</td>
<td>pq</td>
<td>dq</td>
<td>ISC</td>
<td>pq</td>
</tr>
<tr>
<td>T/t</td>
<td>T/t</td>
<td>T/t</td>
<td>T/t</td>
<td>T/t</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>1</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>+3</td>
<td>+3</td>
<td>+3</td>
<td>+3</td>
<td>+3</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Mathematical Operations</th>
<th>ADD</th>
<th>SUB</th>
<th>MUL</th>
<th>DIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISC</td>
<td>pq</td>
<td>dq</td>
<td>ISC</td>
<td>pq</td>
</tr>
<tr>
<td>2T/t</td>
<td>2T/t</td>
<td>2T/t</td>
<td>2T/t</td>
<td>2T/t</td>
</tr>
<tr>
<td>+31</td>
<td>+31</td>
<td>+31</td>
<td>+31</td>
<td>+31</td>
</tr>
</tbody>
</table>

### TABLE III

<table>
<thead>
<tr>
<th>Memory Allocation (in floats)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cases</td>
</tr>
<tr>
<td>Case I</td>
</tr>
<tr>
<td>Case II</td>
</tr>
</tbody>
</table>

A. Big-O polynomial for Case I

With the help of the Table 2. the total execution time for a particular control theory to generate the ideal reference current using a particular processor is represented as its Big-O polynomial, given as under.

- Big-O (ISC):
  \[ T_{total} = \frac{T}{t_{d}} \times T_{ADD} + 7 \times T_{SUB} + 11 \times T_{MUL} + 5 \times T_{DIV} \]
- Big-O (p-q):
  \[ T_{total} = (\frac{T}{t_{d}} + 22) \times T_{ADD} + 2 \times T_{SUB} + 37 \times T_{MUL} + 5 \times T_{DIV} \]
• Big-O (d-q):
\[
T_{total} = \left( \frac{2T}{t_d} + 12 \right) \times T_{ADD} + \left( \frac{2T}{t_d} + 3 \right) \times T_{SUB} + 22 \times T_{MUL} + T_{DIV}
\]  
\[
(19)
\]

B. Big-O polynomial for Case II

With the help of the Table ??, the total execution time for a particular control theory to generate the ideal reference current using a particular processor is represented as its Big-O polynomial, given as under.

• Big-O (ISC):
\[
T_{total} = \frac{5T}{t_d} \times T_{ADD} + \left( \frac{10T}{t_d} + 15 \right) \times T_{SUB} + 24 \times T_{MUL} + 11 \times T_{DIV}
\]  
\[
(20)
\]

• Big-O (p-q):
\[
T_{total} = \left( \frac{5T}{t_d} + 31 \right) \times T_{ADD} + \left( \frac{10T}{t_d} + 10 \right) \times T_{SUB} + 50 \times T_{MUL} + 12 \times T_{DIV}
\]  
\[
(21)
\]

• Big-O (d-q):
\[
T_{total} = \left( \frac{2T}{t_d} + 12 \right) \times T_{ADD} + \left( \frac{2T}{t_d} + 3 \right) \times T_{SUB} + 22 \times T_{MUL} + T_{DIV}
\]  
\[
(22)
\]

V. RESULTS AND DISCUSSIONS

In order to analyze the performance of the control theories in terms of their execution speed, a case study has been performed on TI processor - TMS320F28377S using Code Composer Studio (CCS) software. The results obtained by using the CCS on TMS320F28377S are summarized as Table IV

![Fig. 2. Computational efficiency of control theories under Balanced source voltage conditions](image2)

The results obtained in the MATLAB are plotted as Fig. ??, ??, ?? and ??, The nature of the computational efficiency vs discretization time for different control theories is plotted in Fig. ??, ??, ??, Please note that number of points in a cycle are

![Fig. 3. Computational efficiency of control theories under Unbalanced source voltage conditions](image3)

<table>
<thead>
<tr>
<th>Operations</th>
<th>Instruction Cycles</th>
<th>Execution time per cycle (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADD</td>
<td>6 clocks</td>
<td>$T_{ADD}$30</td>
</tr>
<tr>
<td>SUB</td>
<td>6 clocks</td>
<td>$T_{SUB}$30</td>
</tr>
<tr>
<td>MUL</td>
<td>6 clocks</td>
<td>$T_{MUL}$30</td>
</tr>
<tr>
<td>DIV</td>
<td>236 clocks</td>
<td>$T_{DIV}$1180</td>
</tr>
</tbody>
</table>

![Fig. 4. Memory allocation for control theories under Balanced source voltage conditions](image4)

![Fig. 5. Memory allocation for control theories under Unbalanced source voltage conditions](image5)
changed based on sampling time $t_d$. The intersection of these curves infers following.

- **Case I:** Below the discretization time of 168 ns, the ISC theory takes the least computational time, followed by p-q and d-q theories. Between the discretization time of 168 ns to 212 ns, d-q theory proves to be much faster than the p-q theory, while ISC remains the fastest. However, for any discretization greater than 212 ns, the d-q theory proves to be best in terms of computational efficiency, while ISC theory is second best in computational performance. In case of memory allocation under balanced voltage source, Fig. ?? infers that ISC theory has the lowest memory requirement. Memory requirement for implementing p-q theory is close to ISC theory. However, in case of d-q theory the memory requirement is highest.

- **Case II:** In case of unbalanced voltage source, Fig. ?? infers that the computational efficiency of d-q is highest, while that of ISC and p-q is nearly same, but between both of them ISC takes the least computational time than p-q. Fig. ?? highlights that the memory requirement for implementing d-q theory is least against ISC and p-q theory, while after around 500 ns of discretization time, the difference is marginal in all the three theories.

**VI. SIMULATION STUDIES**

With an aim to verify the goals of compensation achieved and to compare the transient performance, a simulation study is carried out by implementing the theories on a time varying load using the Simulink (MATLAB). Step changes in load are introduced symmetrically each after 0.5 ms and in a worst case one of the phase is open circuited at 1.5 ms to analyze the transient performance.

**B. Simulation Results**

The results obtained by performing transient analysis on ideal DSTATCOM using the control theories is presented in Fig. ??

**TABLE V**

**SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Source Parameters</th>
<th>Vrms(L-L)= 400 V, 50Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load Parameters</td>
<td>Unbalanced Load (phases a, b and c), 20+15.8 Ω 30+20 Ω and 454+j18.85Ω</td>
</tr>
<tr>
<td></td>
<td>at 50 Hz Non Linear Loads, Three phase diode bridge with load of $R_L=30Ω$, $L_L=40$ mH</td>
</tr>
</tbody>
</table>

**TABLE VI**

**TRANSIENT ANALYSIS WITH TIME VARYING LOAD**

<table>
<thead>
<tr>
<th>Control Theory</th>
<th>Linear unbalanced</th>
<th>Non-Linear</th>
<th>Short circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISC</td>
<td>≥ 10 ms</td>
<td>≥ 10 ms</td>
<td>≥ 10 ms</td>
</tr>
<tr>
<td>p-q</td>
<td>≥ 10 ms</td>
<td>≥ 10 ms</td>
<td>≥ 10 ms</td>
</tr>
<tr>
<td>d-q</td>
<td>≥ 13 ms</td>
<td>≥ 10 ms</td>
<td>≥ 10 ms</td>
</tr>
</tbody>
</table>

**A. System conditions and Simulation parameters**

The system under consideration is an ideal DSTATCOM shown in Fig. ?? is a three phase four wire system with source voltages balanced and sinusoidal. Rest of the simulation parameters are tabulated as Table ??.

**Fig. 6.** Schematic diagram of an Ideal DSTATCOM based three-phase four-wire compensated system

**Fig. 7.** Transient response of time varying load with d-q theory, Load Current

**Fig. 8.** Transient response of time varying load with d-q theory, response to linear unbalanced load

The performance of the theories with a time varying load has been compared in terms of their response time given in Table VI to track the sudden step change and it was found that the transient response of the theories was instantaneous with a delay time equivalent to half cycle ($T/2$) i.e. 10 ms.
The transient performance of the theories was more or less same with a delay varying from 10ms to 13ms, indicating as one of the inherent delay of implementing MAF. However, the delay period due to the time taken in tracking the fundamental component will actually attribute in the equivalent amount of power dissipation in the inverter switches, when realized practically and therefore will result in certain percentage of power loss per load cycle.

VII. Conclusion

In this paper the author analyzed the basic formulation of three control theories under different source conditions to bring out a generic comparison in terms of computational time and memory used in APF control. A comparative analysis using Big-O polynomial is derived to comment upon their efficiency in time and space. A case study is also performed to examine the performance of the theories on TMS320F28377S processor using CCS software. The simulation studies to compare transient performance of the given theories are also performed. This study assists in optimization of selected signal processing algorithms related to specific applications for mitigating power quality problems on the lines of embedded system designing.

ACKNOWLEDGMENT

The authors would like to thank Indian Institute of Technology Madras and Indian Army. This work is supported by the Ministry of Science and Technology, DST India, under the project grant SB/S3/EECE/056/2015.

REFERENCES