Evaluation of Locational Marginal Pricing of Electricity under peak and off-peak load conditions

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***Abstract*—The pricing of electricity based on the Locational Marginal Price (LMP) is slowly gaining popularity in Electricity Markets all around the world. In this paper, a modified optimal power flow (OPF) is considered which has the objective of minimising the cost of generation of both real and reactive power from which the LMP of real and reactive power has been obtained. When a shunt capacitor is present in the system, its investment cost is modelled as equivalent generation cost and included into the objective function. Analysis is made on the technical and economic aspects of power system under two load conditions. A novel finding has been made that during off-peak conditions, the LMPs obtained from modified OPF gives a total revenue which is less than the cost of generation, that is, it puts the generators into financial losses. In such cases, the Transmission System Operator (TSO) and the generator should bear the loss in some proportion. Also, this calls for formulation of an effective algorithm for accurate pricing of real and reactive power during off-peak load conditions.**

I. Introduction

Pricing of real and reactive power on the basis of marginal costs was first proposed by Schweppe et.al. [1] as early as 1982 based on the concept of micro-economics. In [2] real-time pricing of reactive power is done considering a price dependent demand. It explains how the marginal cost pricing method is more efficient than the traditional power factor penalty method and how the cost burden can equitably be shared by all customers. Paper [3] gives a clear and detailed algorithm on reactive power planning (mainly allocation and sizing of capacitors) based on Cost Benefit Analysis and Benefit to Cost Ratio. They also showed how effective planning and capacitor installation can avoid exorbitantly high prices of reactive power as obtained in [2]. A detailed theory on technical and economic issues of providing reactive power support has been explained in [4]. Real time pricing based on the objective of maximising the social welfare has been discussed in [5]. Paper [6] separates the reactive power costs as explicit and implicit costs from generation and transmission sources and does reactive power dispatch with the objective of minimising the costs. The effect of considering the real and reactive power generation costs along with investment costs of capacitors on the marginal costs is studied by [7] by solving the OPF using sequential quadratic programming.

In recent years, the shadow prices of the binding constraints in the optimization problem are also being interpreted for better economic operation and security of the power system. In [8] the LMPs were found using Interior point method combined with branch and bound. It provides a perspective of viewing reactive power from a point of local voltage control.

Since the advent of electricity power markets, lot of importance has been given to the pricing of real power. The reactive power costs were always ignored even though they provide the most important function of system security and improved system operation. Reactive power management involves the control of generator voltages, variable transformer tap settings, switchable shunt capacitor and reactor banks along with the allocation of new shunt capacitor and reactor banks in a way that best achieves a reduction in system losses and/or voltage control [9].

According to the draft of Indian Electricity Grid Code 2010 [10], reactive power support is still considered as an ancillary service. Reactive power compensation should be provided locally by the State Transmission Utilities and users connected to the Inter-state Transmission Systems (ISTS) in the low-voltage systems close to the load points to avoid exchange of reactive power to/from ISTS. In case there is exchange of reactive power between the states, it is priced as follows:

1) When voltage at metering point is below 97%, the Regional Entity except generating station pays for VAr drawal and gets paid for VAr injection.
2) When voltage at metering point is above 103%, the Regional Entity except generating station gets paid for VAr drawal and pays for VAr injection.

The charge for VArh is at the rate of 250 Rs./MVArh (3.7 dollar/MVArh) and the rate shall be escalated by 2.5 Rs./MVArh (0.037 dollars/MVArh) per year thereafter, unless otherwise revised by the commission. It also states that all the generating stations should absorb/generate reactive power within their capability limits as per the instruction of Regional Load Dispatch Centre, without sacrificing the generation of real power at that point of time and the generating stations will not be paid for the above service.Hence, reactive power support provided by generators in generating stations is not being valued as a service.

In this paper, study has been done on how the LMPs, transmission line losses and line loading vary at two different load conditions. The economic aspects such as Congestion rent and Generator Revenue are also computed and their effects are

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analysed. The major contribution of this paper, lies in pointing out that the generators will be losing money during light load conditions with LMP scheme of modified OPF.

II. Motivation

Utilities are concerned about price inflation due to shortage of fuel supply and the necessity to meet the ever increasing demand. One approach in addressing this issue is by effective reactive power management. There can be instances when the generator has to sacrifice its real power generation to provide reactive power for maintaining the system security. In such cases, the generator is deprived of selling the real power during that time block. Including it as an opportunity cost considering the capability curve of the generator is quite complex. Hence, in this approach the cost of generation of reactive power is approximated as in [7]. This paper is unique, as it studies the technical aspects and gives importance to the economic issues that arise when power dispatch is done using LMPs obtained from the modified OPF.

III. Problem Formulation

A. Assumptions

1) Generators, apart from capacitors, are also capable of supplying reactive power (generator capability curve not considered, only minimum and maximum limit on reactive power constraint considered).
2) The load is assumed to remain constant during the optimization process.
3) The entire rating of the generator is available for dispatch and there are no reserve requirements met by the generator.

B. Objective function

Modified OPF is formulated for two cases. In Case 1, there is no shunt capacitor present in the system and the objective function (OF) is given by equation (1). In case 2, a continuously variable shunt capacitor of 100 MVA is present at bus 4 in the system and the OF includes the cost of generation of reactive power from capacitors given in equation (4).

1) Case 1: The OF of modified OPF is represented as:

$$Min\ F_1 = \sum_{i \in G} [F_{pi}(P_{gi}) + F_{qi}(Q_{gi})] \tag{1}$$

where F_1 is the sum of fuel cost of generation of real power and reactive power of all the generators. $F_{pi}(P_{gi})$ is the fuel cost of generation of real power of ith generator. $F_{qi}(Q_{gi})$ is the fuel cost of generation of reactive power of ith generator. G is the total no. of generators in the system.
Fuel cost of generation of real power is represented as

$$F_{pi}(P_{gi}) = a_{pi}{P_{gi}}^2 + b_{pi}P_{gi} + c_{pi} \qquad i \in G \tag{2}$$

Conventionally, the fuel costs given by the manufacturer in $\$/h$ is the cost of production of only real power. The cost of production of reactive power is often neglected. Under heavily loaded or poor power factor condition the generators will be supplying reactive power to the system which may bring down their real power production. Hence, they lose the opportunity of selling the real power due to reactive power production. The real and reactive power output of a generator is given by the generator capability curve. The capability of the generator to produce real/reactive power is limited by armature current, field current and under-excitation. The value lost by a generator for providing reactive power depends on the real time dispatch and the price of real power at that instant. Hence, calculating this lost opportunity cost is quite complex. For simplicity, the cost of generation of reactive power can be represented like this

$$F_{qi}(Q_{gi}) = [F_{pi}(S_{gi,max}) - F_{pi}(\sqrt{(S_{gi,max})^2 - (Q_{gi})^2})] \times k \tag{3}$$

where $S_{gi,max}$ is the nominal apparent power of the generator and k is the profit rate of real power generation usually between 5 to 10%. Here $S_{gi,max} = P_{gi,max}$ is assumed.

2) Case 2: When shunt capacitors are present in the system to provide reactive power support or improve voltage profile, the OF for modified OPF is given by:

$$Min\ F_2 = \sum_{i \in G} [F_{pi}(P_{gi}) + F_{qi}(Q_{gi})] + \sum_{j \in C} F_j(Q_{cj}) \tag{4}$$

where F_2 is the sum of fuel cost of generation of real power and reactive power of all the generators and generation cost equivalent to the investment cost of all capacitors. $F_j(Q_{cj})$ is the cost of reactive power generation from shunt capacitors. It is modelled equivalent to the capacitor investment cost return.

$$F_j(Q_{cj}) = Q_{cj} \times \left(\frac{\text{Investment cost (in \$/VAr)}}{\text{(Operating period(in h)} \times u)} \right)$$

If the investment cost is $\$0.0116/VAr$, operating period is 131400 hours assuming 15 years of operation, u is the usage rate taken as $2/3$. Here Q_{cj} is in MVar. Then,

$$F_j(Q_{cj}) = Q_{cj} \times \$0.1324 \tag{5}$$

C. Constraints

The constraints to be considered are the following:

1) Equality constraints: Real and Reactive power balance at all the buses. N is the total no. of buses in the system.

$$g_{pi} = P_{gi} - P_{di} - \sum_{j \in N} V_i V_j Y_{ij} cos(\delta_j - \delta_i + \theta_{ij}) = 0$$

$$g_{qi} = Q_{gi} - Q_{di} + \sum_{j \in N} V_i V_j Y_{ij} sin(\delta_j - \delta_i + \theta_{ij}) = 0$$

2) Inequality constraints: At any bus i,
Voltage magnitude constraints

$$|V_{i,min}| \leq |V_i| \leq |V_{i,max}| \qquad \text{i=2 to N}$$

Voltage angle constraints

$$\delta_{i,min} \leq \delta_i \leq \delta_{i,max} \qquad \text{i=2 to N}$$

Real power generation constraint

$$P_{gi,min} \leq P_{gi} \leq P_{gi,max} \qquad i \in G$$

Reactive power generation constraints

$$Q_{gi,min} \leq Q_{gi} \leq Q_{gi,max} \qquad i \in G$$

Apparent power flow constraint on the transmission line

$$\sqrt{(P_{ij})^2 + (Q_{ij})^2} \leq S_{ij,max} \qquad i \in N, j \in N, j \neq i$$

Reactive power output constraint of capacitor (This constraint is included only when objective function F_2 is considered).

$$0 \leq Q_{ci} \leq Q_{ci,max} \qquad i \in C$$

3) Lagrangian definition: The Lagrangian function for the above optimization problem can be defined as follows:

$$\begin{aligned} L = & \sum_{i \in G} [F_{pi}(P_{gi}) + F_{qi}(Q_{gi})] + \sum_{j \in C} F_j(Q_{cj}) \\ & - \sum_{i \in N} [\lambda_{pi} g_{pi} + \lambda_{qi} g_{qi}] \\ & + \sum_{i \in N} [\mu_{vi,min}(V_{i,min} - V_i) + \mu_{vi,max}(V_i - V_{i,max})] \\ & + \sum_{i \in N} [\mu_{di,min}(\delta_{i,min} - \delta_i) + \mu_{di,max}(\delta_i - \delta_{i,max})] \\ & + \sum_{i \in G} [\mu_{pi,min}(P_{Gi,min} - P_{Gi}) \\ & + \mu_{pi,max}(P_{Gi} - P_{Gi,max})] \\ & + \sum_{i \in G} [\mu_{qi,min}(Q_{Gi,min} - Q_{Gi}) \\ & + \mu_{qi,max}(Q_{Gi} - Q_{Gi,max})] \\ & + \sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} \eta_{ij}(S_{ij} - S_{ij,max}) \\ & + \sum_{j \in C} [\mu_{cj,min}(Q_{Cj,min} - Q_{Cj}) \\ & + \mu_{cj,max}(Q_{Cj} - Q_{Cj,max})] \end{aligned} \tag{6}$$

IV. Solution Methodology

The above optimization problem with quadratic objective function, linear and non-linear constraints is solved using MATLAB Interior Point Solver (MIPS) [11] and FMINCON of MATLAB optimization toolbox. The convergence time of both the algorithms on the 6-bus system under consideration is reported in Table I. MIPS is found to have a faster convergence in all the cases under consideration. The results obtained are discussed in section VI. Both the methods gave the same result except for the convergence times.

V. System Data

A standard 6-bus test system from [12] is considered whose single line diagram is given in Fig.1.The line data, generator real and reactive power limits and cost co-efficients are given in appendix. The co-efficients of reactive power costs are obtained from (3) by approximating the resulting curve as a quadratic polynomial. The voltage values are constrained between 1.07 and 0.95. For analysis purpose, loads are assumed to be present at all the buses and two load conditions, peak and off-peak, are considered. Table II gives the bus data of the system. The peak load is taken as 2.5 times the off-peak load. For Case 1, the analysis is done at off-peak and peak load condition. For case 2, analysis is done only peak load as shunt capacitors are needed only during that condition.

TABLE I
Comparison of convergence time

Parameter (in sec)	Case 1				Case 2	
	Off-peak		Peak		Peak	
	FMIN CON	MIPS	FMIN CON	MIPS	FMIN CON	MIPS
Mean	0.0760	0.0674	0.1047	0.0677	0.0911	0.0679
Median	0.08	0.06	0.09	0.06	0.09	0.06
Mode	0.08	0.06	0.09	0.06	0.09	0.06
Standard Deviation	0.0187	0.0242	0.0721	0.0223	0.0239	0.0359
Minimum time	0.05	0.05	0.08	0.05	0.07	0.05

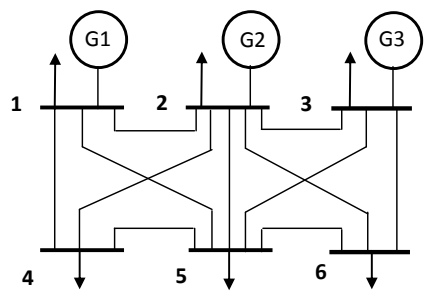

Fig. 1. Single line diagram of 6 bus system

TABLE II
Bus Data under peak and off-peak conditions

Bus No.	Type	V (pu)	Peak load		Off-peak load	
			P_d (MW)	Q_d (MVAr)	P_d (MW)	Q_d (MVAr)
1	Slack	1.07	40	25	16	10
2	PV	1.05	20	12	8	4.8
3	PV	1.05	30	10	12	4
4	PQ	-	100	80	40	32
5	PQ	-	85	50	34	20
6	PQ	-	120	30	48	12

VI. Results and Discussion

The modified OPF for case 1 and case 2 in section III are carried out using MIPS and FMINCON and analysis is done on the following technical and economic aspects:

A. Locational Marginal Prices
B. Transmission losses
C. Line loading
D. Generator revenue

TABLE III
RESULTS

Case	1- Off-peak		1- Peak		2-Peak	
Obj. func.	2454.85		5628.55		5537.75	
V (pu)	V_{min}	Bus	V_{min}	Bus	V_{min}	Bus
	1.044	4	0.95	6	0.989	6
	V_{max}	Bus	V_{max}	Bus	V_{max}	Bus
	1.07	2	1.07	1	1.07	1
Loss(MW and MVAr)	P_{loss}=1.287		P_{loss}=13.842		P_{loss}=9.957	
	Q_{loss}=3.90		Q_{loss}=44.45		Q_{loss}=32.02	
Line loading,S_{ij} (MVA)	Min	Branch	Min	Branch	Min	Branch
	3.415	4	10.186	10	9.375	11
	Max	Branch	Max	Branch	Max	Branch
	29.152	5	60	5,9	60	9
λ_p (from bus 1 to 6)	11.450		13.789		13.515	
	11.449		12.812		14.028	
	11.522		11.879		12.044	
	11.676		17.879		14.525	
	11.698		14.979		14.764	
	11.674		16.845		17.400	
λ_q (from bus 1 to 6)	-0.003		0.443		0.073	
	-0.064		0.285		0.262	
	-0.028		0.126		-0.008	
	0.097		4.091		0.133	
	0.010		1.609		0.587	
	-0.012		2.221		1.054	

A. Locational Marginal Prices

Locational Marginal Prices (LMPs) are defined as the marginal costs of consuming real and reactive power at any particular bus at that instant. Conventionally, the marginal costs of a system vary at different buses and LMPs arise due to transmission line congestion. In case of no congestion, the LMPs are very close to each other. The LMPs consists of 3 components, the marginal costs of generation, congestion rent and the marginal cost of losses [13]. In the modified OPF the LMPs are the co-efficients of equality constraint in equation (6), that is, λ_p and λ_q.

During off-peak load in Table III the variation in λ_p between the buses is only 2.15% and λ_q values are almost negligible. Contrarily, during peak load the line flows through branches 5 and 9 are at maximum and hence there is a wide variation in λ_p of 40.32% and λ_q values are also significant. λ at bus 4 and 6 are high because one of the lines connected to them are constrained. λ of bus 4 is higher than that of bus 6 because the load power factor at bus 4 is poor as observed from Fig. 2. The change in λ follow almost the same trend as the change in load as seen in Figs. 2 and 3. In case 2, as seen in Fig. 4, the λ at bus 4 is reduced drastically with the insertion of capacitor at that bus during peak load.

Significance of "Negative λ_q" : It can be seen from Table III that λ_q can become negative at some buses. It signifies that by absorbing reactive power at that bus the objective function (cost of generation) comes down. Therefore, the TSO pays to the load/generator which absorbs reactive power at that bus and collects money from the generators which inject reactive power at that bus.

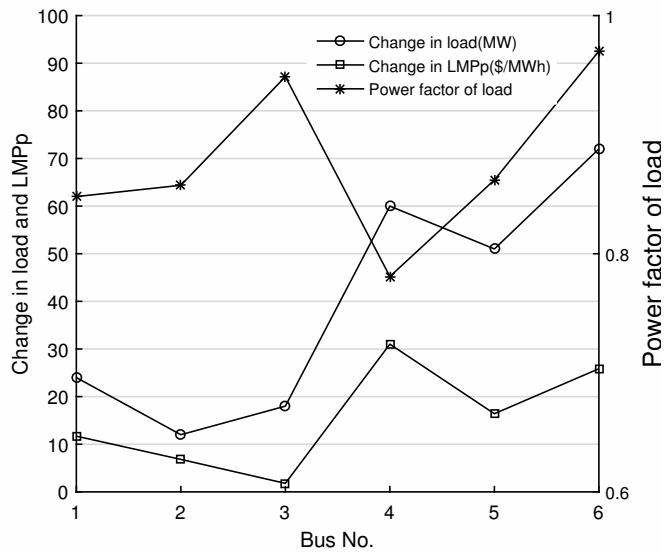

Fig. 2. Comparison of Pd and LMPp during peak and off-peak condition with power factor of load for Case 1

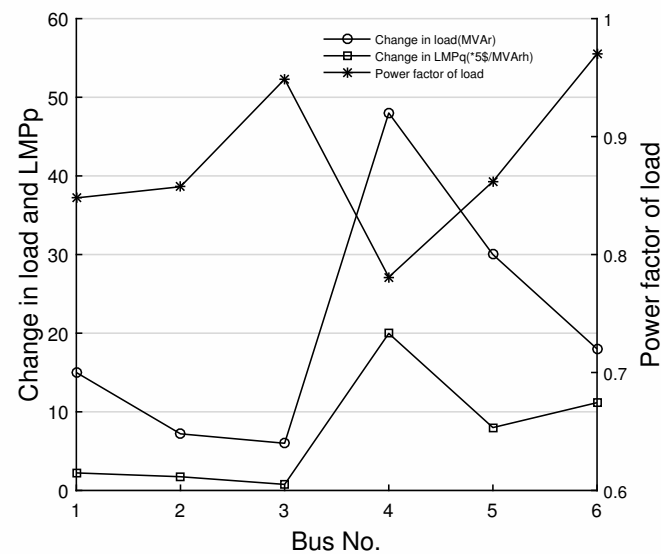

Fig. 3. Comparison of Qd and LMPq during peak and off-peak condition with power factor of load for Case 1

B. Transmission losses

Transmission loss is very small during off-peak condition as seen in Table III. During peak load, with the insertion of capacitor at bus 4, the real power loss are reduced considerably in lines 1,2 and 3 and reactive power losses are reduced in lines 2 and 3 as seen in Fig. 5. Overall, real power loss is reduced by 28% and reactive power loss by 19% from case 1 to case

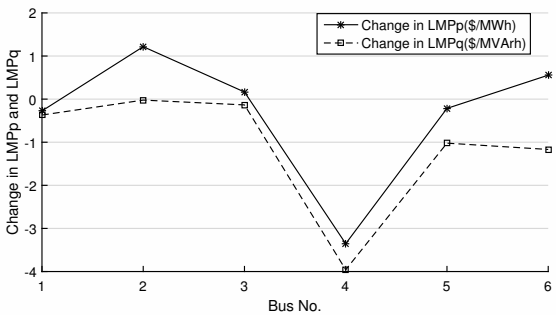

Fig. 4. Change in LMPs from case 1 to case 2 at peak load

2. Reduction in losses means reduced generation and hence reduced fuel costs for same load condition.

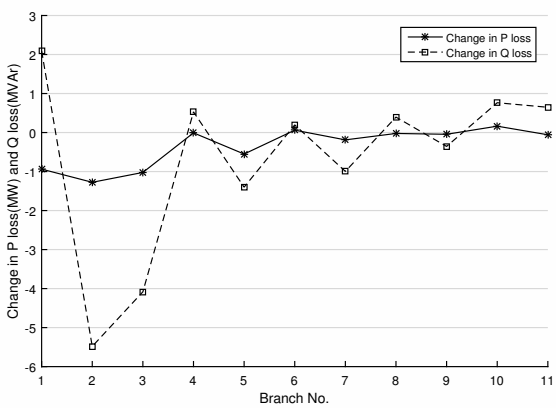

Fig. 5. Change in Losses of lines from case 1 to case 2 at peak load

C. Line Loading

During off-peak, none of the transmission lines are constrained. The variation in real and reactive power loading of the line from case 1 to case 2 is shown in Fig. 6. The MW loading of the line is reduced by 5.75% by installing a capacitor at bus 4. This means that the generator has chances of selling greater MW into the market and earn by doing so. Thus, insertion of capacitor releases some MW loading of the line and aids for greater power to be transacted through the existing lines.

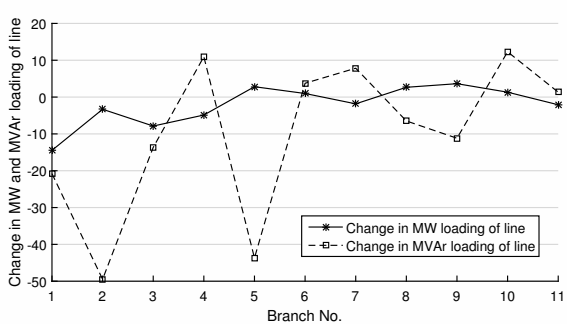

Fig. 6. Change in Line Loading from case 1 to case 2 at peak load

D. Generator revenue

To calculate the generator revenue, the 6-bus system considered is assumed to have the market structure as shown in Fig. 7. The generator revenue is calculated using the equations (7) - (10) and the results are tabulated in Table IV. Note that the generator revenue also includes the cost of losses. The cost of losses has not been explicitly separated here.

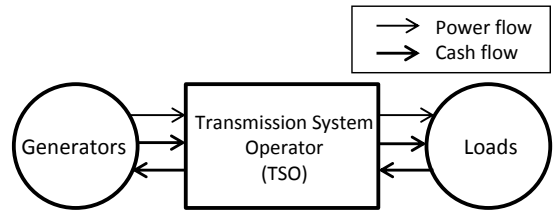

Fig. 7. Market Structure of 6-bus system

$$\textit{Total revenue} = \sum_{i \in N} [\lambda_{pi} P_{di} + \lambda_{qi} Q_{di}] \tag{7}$$

In case of congested line, the congestion rent is given by

$$\textit{Congestion rent} = \sum_{ij \in limit} \eta_{ij} S_{ij,max} \tag{8}$$

$$\textit{Generator revenue=Total revenue-Congestion rent} \tag{9}$$

$$\textit{Generator profit=Generator revenue-Cost of generation} \tag{10}$$

The cost of generation is given by (1) and (4) for Case 1 and 2 respectively.

TABLE IV
RESULTS OF ECONOMICS

Cost/ Revenue	Case 1 Off-peak	Case 1 Peak	Case 2 Peak
Cost of generation	2454.85	5628.55	5537.75
Total revenue	1842.2122	6736.7546	6054.6090
Congestion rent	-	820.44	398.88
Generator profit	-612.6378	287.7646	117.9790

In Table IV, during off-peak condition, the total revenue is found to be less than the cost of generation, that is, the generator is incurring loss in revenue. This can be interpreted in the following ways:

1) Lower LMP will encourage the load to increase their consumption and hence slowly as the demand increases the LMPs will increase and the cost of production will be met.
2) Looking from the perspective of System Operator, there is excessive transmission capacity and installed generation available than required in that time block. Therefore, the difference in total revenue and cost of generation should be borne by the TSO and generator in some proportion.

3) The LMPs obtained using modified OPF during off-peak condition do not meet the cost of generation and hence, new algorithm needs to be developed to accurately price the active and reactive power at buses during off-peak condition.

However, during peak load condition in case 1 and 2 the generator profit is positive.

VII. Conclusion

The following inferences are made from the case studies:

1) LMPs, which are co-efficients of equality constraints in the Lagrangian equation of the modified OPF, are dependent on various factors such as marginal cost of production, line losses, congestion, load at a bus and its power factor.
2) LMPs can be positive or negative.
3) LMPs obtained by modified OPF are found to give loss in revenue at off-peak load condition and give profit during peak load condition.

Thus, on a concluding note:

1) The variations of LMPs for various cases have been studied.
2) A negative LMP would mean incentive for drawing power at that bus and penalty for injecting power at that bus.
3) It is seen that the generators suffer loss at off-peak load condition which necessitates the need for a detailed study on an hourly load curve and see if the overall profit of the generator is positive. Also, this calls for the need of an effective and efficient algorithm for real and reactive power pricing so that the generators do not suffer losses at any load condition.

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Appendix

The details of the 6-bus system under study is given below.

TABLE V
Generator Real Power Constraints and cost co-efficients

Bus No.	P_{min} (MW)	P_{max} (MW)	a_{pi} $(\$/(MW)^2h)$	b_{pi} $(\$/MWh)$	c_{pi} $(\$/h)$
1	50	200	0.00533	11.669	213.1
2	37.5	150	0.0089	10.333	200
3	45	180	0.00741	10.833	240

TABLE VI
Generator Reactive Power Constraints and cost co-efficients

Bus No.	Q_{min} (MVAr)	Q_{max} (MVAr)	a_{qi} $(\$/MVAr^2h)$	b_{qi} $(\$/MVArh)$	c_{qi} $(\$/h)$
1	-100	150	0.0022	-0.0442	0.6056
2	-100	150	0.0041	-0.1462	0
3	-100	120	0.0023	-0.0256	0.2715

TABLE VII
Line data

From Bus	To Bus	R (pu)	X (pu)	B (pu)	$S_{ij,max}$ (MVA)
1	2	0.1	0.2	0.04	100
1	4	0.05	0.2	0.04	100
1	5	0.08	0.3	0.06	100
2	3	0.05	0.25	0.06	60
2	4	0.05	0.1	0.02	60
2	5	0.1	0.3	0.04	60
2	6	0.07	0.2	0.05	60
3	5	0.12	0.26	0.05	60
3	6	0.02	0.1	0.02	60
4	5	0.2	0.4	0.08	60
5	6	0.1	0.3	0.06	60