Coherency of Generators for Inter-Area Modes Using Digital Filter Bank and Principal Component Analysis

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***Abstract*—In the present work, coherent generators participating in an inter-area mode are identified from time synchronized phasor measurements. In the first step, Zolotarev polynomial based filter bank (ZPBFB) is applied on the real power data of all generators obtained from phasor measurement units (PMUs) to decompose the given signal into monocomponents. The monocomponents are then given to eigen realization algorithm (ERA) for modal frequency and damping. The decomposed signals of those generators participating in a particular inter-area mode are given to principal component analysis (PCA) for identifying generator coherency. The proposed method is demonstrated on IEEE two-area test system.**

***Index Terms*—Zolotarev polynomial, phasor measurement unit (PMU), inter-area oscillations, coherency, eigen realization algorithm (ERA), principal component analysis (PCA).**

I. Introduction

WITH the evolution of deregulated electrical energy markets and the extension of large inter-connected power systems, many tie lines are heavily stressed through large power transfer [1]. Hence, the power systems are prone to operate close to their maximum transmission capacity and stability limits [2], thus increasing the possibility of inter-area oscillations [3]. These oscillations involve swinging of two or more coherent groups of generators against each other [4]–[6].

The coherent groups of generators are independent of the size of the disturbance. So, coherency can be determined by linearizing the system [7], [8]. Generator coherency is analyzed from simplified linear time domain simulations of the power systems [9]. Continuation method has been proposed in [10] to identify generator coherency for varying operating conditions. Modal coherency approach [11], [12] is based on the linearized power system model. Slow coherency based on singular perturbation and time scale decomposition is proposed in [13] for power system model reduction. But, all these methods require access to entire power system dynamic data, which may not always be available, involving a number of utilities.

The recent advances in wide-area measurement systems (WAMS) and the use of synchronized phasor measurement units (PMUs) have made monitoring of power systems dynamics in real-time a promising aspect to enhance and maintain power systems stability while obviating the need for power system dynamic data. In literature, few methods are available for identifying coherent groups of inter-area modes from measured signals. Hilbert-Huang transform (HHT) [14] is based on determining the modes present in the system from the measured signals. HHT cannot identify the modes properly if closely spaced and may lead to coherent groups being obtained are for an artificial mode. Wavelet phase difference (WPD) approach [15] decomposes the signal into different frequency bands for coherency, but for closely spaced modes coherency is not realized. Fast Fourier transform (FFT) based techniques [16]–[19] have been used for generator coherency but they assume stationary and linear model of the system which is not justified in highly non-linear power systems. Coherent generators are clustered using PCA in [20], but this does not indicate the coherency for inter-area modes. The authors proposed a method for identifying generator coherency of inter-area modes using PCA.

In the present work, Zolotarev polynomial based filter bank proposed in [21] is adopted for identifying inter-area modes present in this system. ZPBFB is preferred as this method can identify closely spaced modes. The inter-area modes present in the system are identified using ZPBFB. This method decomposes the time-domain signal into monocomponents. Then the monocomponents representing the inter-area mode of those generators participating in a particular mode are subjected to PCA for modal coherency i.e., group of generators oscillating against another group of generators.

The rest of the paper is organized as following. Zolotarev polynomials are briefly reviewed in Section II. The design of ZPBFB for identifying inter-area modes is discussed in Section III. PCA is briefly discussed in Section IV. The proposed method is demonstrated on two-area test system in Section V. Finally, the conclusions are presented in Section VI.

II. Zolotarev polynomials

Zolotarev polynomials $Z_{p,q}(w,k)$ are an extension of Chebyshev polynomials [22]. The Zolotarev polynomial of

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order $n = p+q$ is shown in Fig.1. The band edges w_p, w_s and the position w_m shown in Fig.1 are given by [23]

$$w_p = 2sn^2\left(\frac{q}{n}K(k),k\right) - 1 \tag{1}$$

$$w_s = 1 - 2sn^2\left(\frac{p}{n}K(k),k\right) \tag{2}$$

$$w_m = w_s + 2\frac{sn\left(\frac{p}{n}K(k),k\right)cn\left(\frac{p}{n}K(k),k\right)}{dn\left(\frac{p}{n}K(k),k\right)}Z\left(\frac{p}{n}K(k),k\right) \tag{3}$$

Where, $sn\left(\frac{p}{n}K(k),k\right)$, $cn\left(\frac{p}{n}K(k),k\right)$ and $dn\left(\frac{p}{n}K(k),k\right)$ are Jacobi's elliptic functions [24] and $K(k)$ is the complete elliptic integral of first kind with Jacobi's elliptic modulus k. Chebyshev polynomials of first kind T_m are used to represent Zolotarev polynomials [23] as

$$Z_{p,q}(w) = \sum_{m=0}^{n} a(m)T_m(w) \tag{4}$$

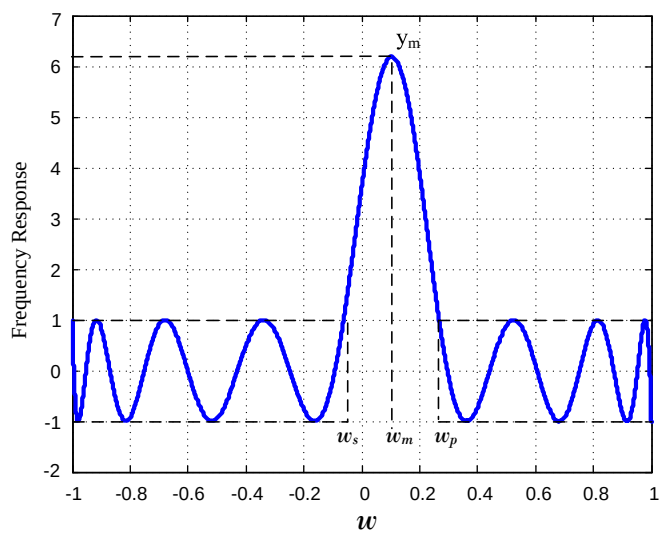

Fig. 1. Zolotarev polynomial $Z_{9,6}(w)$ with $k = 0.6417$, $w_s = -0.0625$, $w_m = 0.1038$, $w_p = 0.2672$ and $y_m = 6.1875$.

III. Zolotarev Polynomial filter bank for identifying inter-area oscillations

The design of Zolotarev polynomial based filter bank for identifying inter-area oscillations is discussed in detail in [21] which is adopted in the present work. A brief review on the design of ZPBFB is given:

1) Compute elliptic modulus k for all $n-1$ polynomials with n being the order of the polynomial.
2) Compute the band edges.
3) Evaluate the coefficients a of (4) as discussed in [23]. Construct Zolotarev polynomial using (4).
4) The frequency responses of all inner filters after normalizing the Zolotarev polynomials are given by

$$H_p(e^{j\omega T}) = \frac{1+Z_{p,q}(e^{j\omega T},k_p)}{M_s(1+y_m)}, \qquad p = 1 \ldots n-1 \tag{5}$$

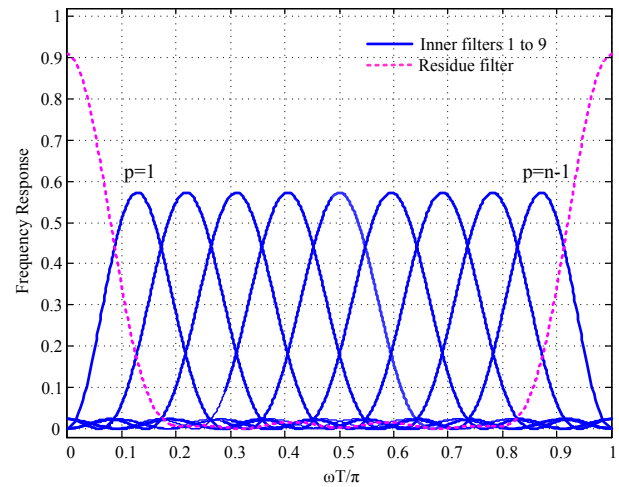

Fig. 2. Frequency responses of inner filters, $H_p(e^{j\omega T})$ of Zolotarev polynomials $Z_{p,n-p}(w,k_p)$ of order $n = 10$ with p varying from 1 to $n-1$ and the frequency response of residual filter $R(e^{j\omega T})$.

The frequency responses of $n-1$ inner filters are shown in Fig. 2.

5) Estimate the maximum value by computing the sum of frequency responses of inner filters as

$$M_s = \max\left\{\sum_{p=1}^{n-1}\frac{1+Z_{p,q}(e^{j\omega T},k_p)}{1+y_m}\right\} \tag{6}$$

6) Obtain the impulse response coefficients of all inner filters.
7) The frequency response of the residual filter is given by

$$\begin{aligned} R(e^{j\omega T}) &= O(e^{j\omega T}) - \frac{1}{M_s}\sum_{p=1}^{n-1}\frac{1+Z_{p,q}(e^{j\omega T},k_p)}{(1+y_m)} \\ &= O(e^{j\omega T}) - \frac{1}{M_s}\sum_{p=1}^{n-1}H_p \end{aligned} \tag{7}$$

where, $O(e^{j\omega T})$ is the overall frequency response which is equal to unity.

8) Decompose the residue filter as low pass and high pass filters.
9) From the frequency response of low pass and high pass filters compute their impulse response coefficients.

The measured signal obtained from PMUs is preprocessed with a low pass filter to eliminate all frequencies grater than 1 Hz as inter-area oscillations are in the range of $0-1$ Hz. Then ZPBFB decomposes the time domain signal into monocomponents and the monocomponents are subjected to eigen realization algorithm (ERA) for modal information. An overview of signal decomposition with the Zolotarev polynomial based filter bank is illustrated in Fig. 3.

IV. Principal Component Analysis

Principal component analysis (PCA) is a technique that uses an orthogonal transformation to convert a set of observations

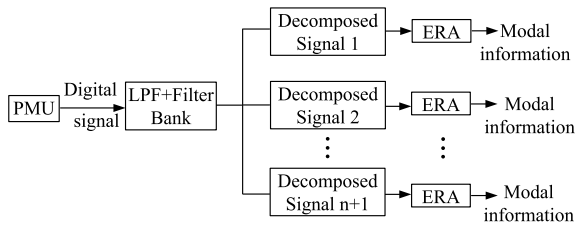

Fig. 3. Zolotarev polynomial based filter bank with ERA for identifying modal information from decomposed signals.

of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables. This transformation is defined in such a way that the first principal component has the largest possible variance and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to the preceding components. The resulting vectors are an uncorrelated orthogonal basis sets.

Let $X_{m\times n}$, be the data matrix which consists of n observations on m variables. New variables are constructed as a weighted averages of original variables. These new variables are termed as principal components. The basic equation of PCA in matrix notation is given by

$$X = YW^T \tag{8}$$

where W is a weight matrix and Y is referred as matrix of scores. The score matrix is calculated from the variance-covariance matrix, S and is given by

$$S = cov(X) = \frac{1}{p-1}X^T X \tag{9}$$

Since S is symmetrical, the eigen vectors of S are orthogonal. Singular value decomposition of $S = UDV^T$ provides the solution to the PCA, where D is a diagonal matrix with eigenvalues of S, U is the eigen vector matrix of S. SVD of X will have right and left eigen vectors same as the left and right eigen vectors of covariance matrix S, but the eigen values of X are the square roots of the eigen values of S.

$$X = UD^{\frac{1}{2}}V^T \tag{10}$$

From Eqs. (8) and (10), the score matrix Y is given as

$$Y = UD^{\frac{1}{2}} \tag{11}$$

From Eqs. (8) and (10), the weight matrix is given as $W^T = V^T$. If two principal components include maximum variance of the data, the scores are represented on a two dimensional plot. Similar clusters have same coordinates on the two-dimensional plane. These clusters are used in identifying coherent generators of an inter-area mode that is a set of coherent generators in an area oscillating against another set of generators in another area.

V. Simulation Studies

Coherent groups of generators oscillating against each other in an inter-area mode are identified with the proposed method and is demonstrated on IEEE two-area test system. The detailed simulation results are presented below.

A. Two-Area System

Two-area system is a 4 generator, 10 bus system with a connected load of 2734 MW. The single line diagram of the system is shown in Fig. 4. The system consists of three tie lines which connects two similar areas. Dynamic data of generators, network and exciter data used in the study are available in [25], [26]. Transient model of the generators with static excitation is considered for analysis. Two case studies are considered for identifying generator coherency with (i) one of the tie-lines between 7-8 tripped (ii) one line between 6-7 and one line between 7-8 are assumed to be out of service. The details of small signal stability analysis are presented in Table I. The system has one inter-area mode with generators $G1$, $G2$ oscillating against generators $G3$, $G4$ in both the cases.

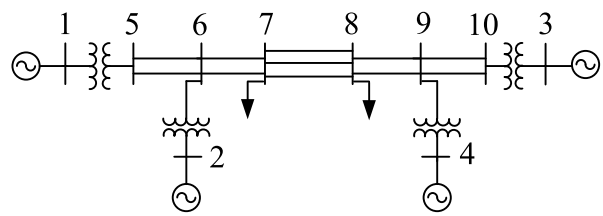

Fig. 4. Single line diagram of two-area system.

TABLE I
Small signal stability analysis of IEEE 2-area, 4 machine system

Case No.	Lines Tripped	Frequency (Hz)	Damping	Coherent Generators Oscillating Against Each other
1	7-8	0.6126	0.0856	(G1, G2) vs (G3, G4)
2	6-7, 7-8	0.5472	0.0742	(G1, G2) vs (G3, G4)

TABLE II
Modal analysis of generator real power signals of 2-area, 4-machine system with ZPBFB and ERA

Real power signal	Case 1		Case 2	
	Frequency (Hz)	Damping	Frequency (Hz)	Damping
$G1$	0.6150	0.0857	0.5489	0.0743
$G2$	0.6150	0.0857	0.5489	0.0743
$G3$	0.6150	0.0857	0.5489	0.0743
$G4$	0.6150	0.0857	0.5489	0.0743

To identify the inter-area modes present in the system using Zolotarev polynomial based filter bank, a 5 percent change in load was simulated at Bus-7 for 1 s long duration in both the

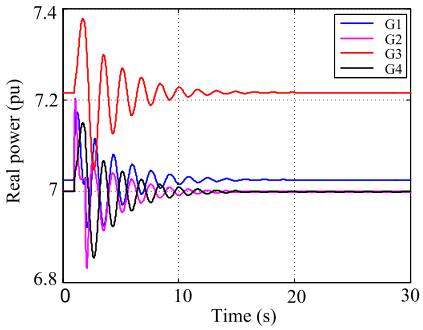

Fig. 5. Real power flow of all generators for a 5 percent change in load at Bus-7 for 1 s with 7-8 removed.

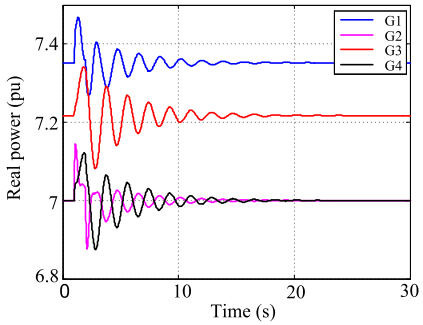

Fig. 6. Real power flow of all generators for a 5 percent change in load at Bus-7 for 1 s with 6-7 and 7-8 removed .

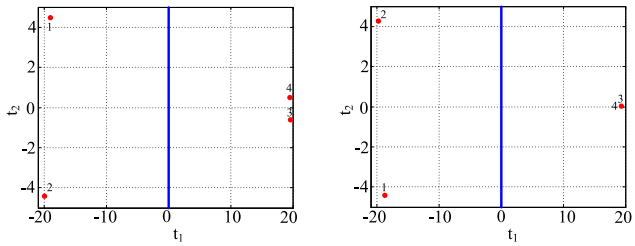

Fig. 7. Generator coherency with load pulse at bus 7 (a) Line $7-8$ removed, (b) Lines $6-7$ and $7-8$ removed.

cases. The real power data of all generators corresponding to both the cases are shown in Figs. 5, 6. The modal information obtained from ZPBFB and ERA is presented in Table II. From Table II, it can be observed that all the generator real power signals are participating in both the inter-area modes. The decomposed signals corresponding to inter-area mode of all generators are given to PCA to identify generator coherency for case (i) and case (ii), respectively. Figs. 7(a), 7(b) represent coherent generator groups for both the cases. Generators with similar X-coordinates form coherent groups. Hence, it can be observed from Figs. 7(a), 7(b) that generators $G1$, $G2$ correspond to one group while generators $G3$, $G4$ correspond to another coherent group. The results demonstrate that generator coherency obtained with the proposed method is same as the coherency obtained from small signal stability analysis.

VI. Conclusion

A methodology for identifying generator coherency from time-synchronized phasor measurements is presented. The inter-area modes present in the system are estimated using ZPBFB and ERA. From ZPBFB, the generators participating in a particular mode are known. The decomposed signals of those generators participating in a particular mode are given to PCA for identifying generator coherent groups. It is observed that the coherent group of generators obtained from small-signal stability and the proposed method are same. However, the advantage with the proposed method is that the identification of coherent groups is in real time.

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