Unbalanced Load Flow Analysis for Distribution Network with Solar PV Integration

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Abstract—The global climatic change is one of the key issues faced by all countries today. Conventional power plants are emitting unwanted gases to the environment, which is the main reason for the climatic change. Many countries have changed their policies and have extended their support for green emissions. Large scale investments have been planned towards installation of Distributed Energy Resources (DER) in the Distribution Network (DN). Among the various DER solar power is more predictable source of energy. This paper emphasizes the impact of solar PV integration in a practical distribution system under continuously variable load pattern. The load flow analysis is performed through Forward / Backward Sweep (FBS) algorithm which is more suitable for a radial system. To get the dynamic behaviour of the system time series analysis is performed. The effectiveness of integration of solar PV is validated in an IEEE 123 node test feeder. The results reveal the voltage support attained in the network through integration of solar PV at selected locations.

ACRONYMS

\[ I_0 = \text{leakage or reverse saturation current, A} \]
\[ q = \text{electron charge} = 1.602x10^{-19}\text{C} \]
\[ V = \text{solar cell voltage, V} \]
\[ k = \text{Boltzman constant} = 1.380658x10^{-23}, \text{WSK}^{-1} \]
\[ T = \text{cell temperature, K} \]
\[ T_{ref} = \text{reference temperature, 301.18°C} \]
\[ R_s = \text{series cell resistance, } \Omega \]
\[ R_{sh} = \text{shunt cell resistance, } \Omega \]
\[ I_{ph} = \text{photo current, A} \]
\[ I_{scr} = \text{cell short circuit current at } T_r, \text{A} \]
\[ S = \text{real solar radiation, } (W/m^2) \]
\[ C_T = \text{temperature co-efficient, } (A/K) \]

I. INTRODUCTION

The distribution system is facing many challenges due to the high penetration of DER in the network. The Renewable Energy Resources (RES) are unpredictable in nature, hence the network experiences instability problems, harmonics issues, voltage imbalance etc. Many researches are under progress to maintain the power system network stability and to give quality power to the consumers. To analyse the performance of DN, load flow solutions are performed. The voltage solutions is the important criteria to analyse the load flow in the DN. Initially load flow solutions were performed only for radial networks. To perform load flow analysis for weekly meshed system load flow analysis was proposed [1]. Newton-Raphson method and Gauss Siedel load flow methods were not suitable for distribution systems. Hence a ladder iterative technique was introduced, which had improved the convergence characteristics [2]. Then to include the non-linear loads, capacitors and transformers an adaptive distributed power flow solution based on compensation method was proposed [3]. Further without LU composition and Jacobian inversion a direct load flow technique was introduced [4]. Many techniques, sequential numbering is necessary. To avoid this an efficient load flow method was proposed and the computational burden has reduced to a greater extent [5]. This method was comprised of a simple equation to compute the voltage magnitude and was capable of handling composite load modelling. Singularity problem arises when transformer is included in the load flow analysis. To overcome this a three phase load flow method for unbalanced radial system was proposed [6]. Further a linear power flow method was introduced to handle the voltage dependent loads [7]. To have an efficient smart distribution analysis the various development in the load flow analysis were reviewed [8]. Among various load flow techniques FBS was concluded as the best method owing to its convergence capability [9]. Further to handle a network with multiple sources three phase unbalanced power flow using PQ and PV models were suggested [10]. An extension of FBS method to handle PV nodes was introduced and the convergence characteristics was improved in [11]. Using real number matrix operations an improved hybrid load flow calculation algorithm was introduced, to have relatively speed convergence ability [12]. An extensible electrical model was proposed to analyse the impact of high penetration of RES [13]. While integrating RES in the DN, the economic analysis should be performed. To assess the incentive regulation and to determine the right incentive decision a new technique was proposed [14]. Like wise when solar PV is integrated, long term analysis should be done through extensive solar PV modelling [15]. With the data supplied by the manufacturer a simple solar PV modelling was introduced [16]. To reduce the number of parameters,
convergence problems and computational cost a new modelling of solar PV was proposed [17]. When there is high penetration of solar PV in the DN, the operation of on load tap changers are affected and leads to sudden voltage rise in the network [18]. To avoid this sizing and location of solar PV should be properly computed. This will reduce the distribution power losses and maintain the voltage profile of the network [19]. A modified teaching learning based algorithm is used to determine the optimal placement and sizing of DER [20]. When DER is integrated in the DN the bidirectional power flows. This power flow varies for every minute. Time series analysis is carried out to analyse the sequence of operation of the DN and its dynamic behaviour is computed [21]. To analyse the huge data while integrating the solar PV and advanced intelligent computational toll has been developed [22]. In this paper, the impact on solar PV integration is analysed using time series load flow analysis. The load flow is carried out using FBS method under continuously varying load pattern. The method is tested in IEEE 123 node test feeder. The second section describes the formation of forward / backward sweep load flow method and the third section briefs the solar PV array connected to a pulse width modulated three phase inverter. It also gives a brief introduction on time series analysis. The fourth section elaborates the features of IEEE 123 node test feeder and the fifth section explores the results and discussions. The results conclude the greater impact of solar PV integration in the distribution system.

II. FORWARD / BACKWARD SWEEP ALGORITHM

FORMULATION

\[
P_{2} = \frac{V_{1}V_{2}}{Z} \cos(\theta_{2} - (\delta_{1} + \delta_{2})) - \frac{V_{2}^{2}}{Z} \cos(\theta_{2})
\]

\[
Q_{2} = \frac{V_{1}V_{2}}{Z} \sin(\theta_{2} - (\delta_{1} + \delta_{2})) - \frac{V_{2}^{2}}{Z} \sin(\theta_{2})
\]

where

- \(V_{1}\) is the voltage magnitude of node 1
- \(V_{2}\) is the voltage magnitude of node 2
- \(\delta_{1}\) is the angle of \(V_{1}\)
- \(\delta_{2}\) is the angle of \(V_{2}\)

\(Z\) is the magnitude of line impedance.

\(\theta_{2}\) is the angle of line impedance.

From the equations 1 and 2

\[
\cos(\theta_{2} - (\delta_{1} + \delta_{2})) = \frac{P_{2}Z}{V_{1}V_{2}} + \frac{V_{2}}{V_{1}} \cos(\theta_{2})
\]

\[
\sin(\theta_{2} - (\delta_{1} + \delta_{2})) = \frac{P_{2}Z}{V_{1}V_{2}} + \frac{V_{2}}{V_{1}} \sin(\theta_{2})
\]

Squaring and adding equations 3 and 4 will give the value 1 using trigonometric identity

\[
\sin^{2}\theta + \cos^{2}\theta = 1
\]

The maximum real root of the equation giving the node 2 voltage magnitude

\[
V_{2}^{2} + 2V_{2}^{2}(P_{2}R + Q_{2}X) - (V_{1}^{2}V_{2}^{2} + (P_{2}^{2} + Q_{2}^{2})Z^{2}) = 0
\]

In terms of power:

\[
V_{2} = \sqrt{V_{1}^{2} - 2(P_{1}R + Q_{1}X) + \frac{((P_{1}^{2} + Q_{1}^{2})Z_{2})}{V_{1}^{2}}}
\]

Using KVL in Fig.1

\[
V_{1} = V_{2} + I_{1}Z
\]

\[
V_{2} = V_{1} + I_{1}Z
\]

Equations 6, 7, 8 and 9 are frequently used to calculate the nodal voltages in Forward / Backward sweep algorithm.

A. FBS - Step by step procedure

1. Initially backward sweep of the network is proceeded upstream starting from end node proceeding towards the source node.
2. End node voltage value is assumed and the corresponding nodal voltages are calculated from tail end of the feeder towards the source node by applying Kirchhoff’s Voltage Law (KVL) and Kirchhoff’s Current Law (KCL).
3. The calculated voltage is compared with the source voltage and if the convergence is within limits then the process is stopped else proceed in the forward sweep in downstream.
4. In downstream the nodal currents and voltage drop are calculated from the source node to the end node using KCL and KVL.
5. At the end node, the voltage is compared with the reference voltage if the convergence is within limits the first iteration stops, else the new voltage is updated at the end node.
and proceeded with backward sweep and forward sweep till convergence is achieved.
6. In backward sweep, the branch current from the tail end to the sub station end is calculated. The maximum real and reactive power mismatch and voltage mismatch is taken as convergence criteria.

III. SOLAR PV MODELING

Solar PV consists of a set of cells which can be connected either in series or parallel. Solar cells convert light energy into electrical energy. Solar PV array is a combination of a current source, parallel diode which in turn is parallel with a shunt resistance. This entire set is in series with a resistance. The diode converts the light energy into electrical energy and produces a photovoltaic current proportional to the solar radiation. Fig. 2 gives the solar PV array with 6 pulse PWM inverter.

The mathematical equation of the current is given by

\[ I = I_{ph} - I_0 \left[ \exp \left( q \frac{(V + IR_s)}{AKT} \right) - 1 \right] - \frac{(V + IR_s)}{R_{sh}} \]  \hspace{1cm} (10)

\[ I_{ph} = \left( \frac{S}{S_{ref}} \right) \left( I_{ph,ref} + C_T(T - T_{ref}) \right) \]  \hspace{1cm} (11)

\[ I_0 = I_{0r} \left( \frac{T}{T_r} \right)^3 \exp \left( \frac{qE_g}{k_A} \left[ \frac{1}{T_r} - \frac{1}{T} \right] \right) \]  \hspace{1cm} (12)

where \( I_{0r} \) refers to the diode saturation current under standard test conditions, \( E_g \) is the energy band of the cell semi-conductor (eV) depending on the material used. Solar Radiation and cell temperature is given by

\[ I_{ph} = \left| I_{scr} + k_i(T - T_r) \right| \frac{S}{K_0} \]  \hspace{1cm} (13)

where \( K_i \) is the short circuit current temperature co-efficient.

The PV array produces 250 KW at 394 V(DC) when the solar illumination called as solar insolation is 100KW per cm² at an ambient temperature of 28 degree centigrade. The output of the PV array is connected to a three phase, six pulse - Pulse Width Modulated (PWM) inverter through a maximum power point tracker (MPPT) and DC to DC converter. The variable dc output from the solar PV array is controlled by the DC to DC converter which is usually a chopper circuit, which produces constant output voltage irrespective of the solar variation. Six pulse inverter has six thyristor switches usually IGBT’s (Insulated Gate Bipolar Transistor), separated as positive group and negative group three IGBT’s in each group. At any instant, one IGBT from the positive group and other from the negative group conducts. Thus a pulsating alternative supply is obtained at the load. The output of the inverter is controlled by varying the firing angle through which the IGBT’s conduction is controlled. The output is modulated using pulse width modulation technique. \( V_{PV} \) is the voltage across the PV array and \( I_{DC} \) is the current flowing from the PV array. When the conduction is through IGBT’s 1 and 6 the output is \( E_{AB} \) like wise we get \( E_{AC} \) and \( E_{BC} \) based on the pairs of IGBT’s conduction. The operation of three phase inverter can be defined using eight modes of operation depending upon the status of the switch in each mode of operation. The output equations of the inverter are given in the equations 14, 15 and 16.

\[ E_{AB} = \sum_{n=1,3,5}^{\alpha} \frac{4V_s}{n \Pi} \sin \left( \frac{n \Pi}{3} \sin \left( \omega t + \frac{\Pi}{6} \right) \right) \]  \hspace{1cm} (14)

\[ E_{BC} = \sum_{n=1,3,5}^{\alpha} \frac{4V_s}{n \Pi} \sin \left( \frac{n \Pi}{3} \sin \left( \omega t - \frac{\Pi}{2} \right) \right) \]  \hspace{1cm} (15)

\[ E_{CA} = \sum_{n=1,3,5}^{\alpha} \frac{4V_s}{n \Pi} \sin \left( \frac{n \Pi}{3} \sin \left( \omega t - \frac{\Pi}{6} \right) \right) \]  \hspace{1cm} (16)

A. Time Series Analysis

The analysis of data with respect to time at different intervals leads to statistical complications during modelling a process. The introduction of sampling has made easy to analyze the experimental data at different time intervals, but the traditional statistical methods which uses many assumptions of data have limitations to handle sampling data. Time series analysis gives the correlation between mathematical modelling and statistical analysis. It provides a proper illustration of a process of adopting the classical statistical methodology. Time series analysis is a careful analysis of recorded data over a time of both time domain approach and frequency domain approach. It is a collection of random variables over a time. It can be considered as a sequence of random variables \( a_1, a_2, a_3, \ldots \) where \( a_1 \) refers to the value of the random variable at time \( t_1 \) and \( a_2 \) is the value of the random value at time \( t_2 \). Time series analysis in a DN is a consecutive power flow analysis performed in steps of time for a set of load and generation profiles. It is used to study the dynamic behavior of the power system and helps in grid planning and operation, when DER are included in the DN. The execution time is more when compared to the steady state analysis. But it is very useful while carrying out analysis with integration of
DER like Solar or Wind, where the source variation is based on the weather data of that particular location. Exact analysis could be performed only when the simulation is done for a year. Thus time series analysis provides detailed analysis of the power system network, during the integration of DER.

If $x_t$ is the $t^{th}$ time step in Matrix M then the $t^{th}$ time step in the output matrix $Y$ is given by

$$Y_t = O_{pf}(x_t) \tag{17}$$

Equation 17 is solved using any one of the power flow solution method. Power flow through time series analysis is carried out for $t=1,2,...T$.

IV. SYSTEM DESCRIPTION

The IEEE 123 Node test feeder is taken for demonstrating the power flow and realizing the impact of solar PV integration. The IEEE 123 test feeder is a practical test system with a source voltage of 115 KV and a nominal voltage of 4.16 KV. This feeder is taken for time series analysis as this feeder is a lengthy feeder and has voltage drop problems. This voltage drop problems are solved by adding four step type regulators at necessary locations. It consists of a mesh network with enough inter connectivity switches, which can be optimized as per the requirement. The feeder consists of overhead and underground lines with various phasing. It has a combination of all loads such PQ, constant current and constant impedance loads and is unbalanced. It has four inline shunt capacitors to maintain the voltage profile. It is a highly unbalanced system with three phase and single phase laterals with all the loads being spot loads. The system has an inline distribution transformer with a nominal voltage of 4.16 KV for a small span of the network. It has 85 spot loads, out of which 15 loads are constant impedance loads, 13 loads are constant current loads and 57 loads are PQ loads. The network has two regulators with 24 three phase laterals and 10 single phase laterals. The feeder is well behaved and does not have a convergence problem. The power flow is carried out using Newton - Raphson load flow analysis. For running the power flow the line to neutral voltage is considered. At the substation the line to neutral voltage is 66.397 KV and the voltage at the distribution transformer end is 2401.847 V.

V. RESULTS AND DISCUSSIONS

The load flow analysis is carried out in the IEEE 123 node test feeder through FBS method. The load flow analysis is carried out with and without connecting solar PV in the feeder. Fig. 3 and Fig. 4 shows the real and reactive power flow in the feeder before integrating the solar PV in the network.

The load flow is performed for 24 hours time step with different load pattern. Fig. 5 shows the variable load pattern applied to the load nodes of the IEEE 123 test feeder.

From the load flow analysis the voltage mismatch is analysed and nodes are identified where the voltage mismatch is beyond the specified limit ($\pm 5$ percentage of nominal voltage). The nodes with more voltage mismatch are identified for integrating solar PV. In phase A nodes 35, 42, 49, 51, 52, 60, 63 and 111 are identified. In phase B nodes 49, 39 and 56 are identified and in phase C nodes 49,50,66 and 104 are identified for solar PV integration. Fig. 6 shows the IEEE 123 test feeder line diagram in which different nodes are identified for solar PV integration.

Fig. 7 and Fig. 8 shows the real and reactive power flow after the integration of solar PV in the feeder. Fig. 9 illustrate the comparison between voltage in phase A before and after integration of solar PV. From the results it is observed that if the solar PV is integrated at proper locations there will be a greater impact and the voltage mismatch is reduced to a greater extent.

Table I gives the comparison between load voltages in phase A before and after integration of solar PV. It is observed that load voltage magnitude has increased considerably after integration of solar PV in selective nodes.

VI. CONCLUSION

In this paper, the impact of the solar PV when integrated with a practical distribution system is presented. The time series analysis carried out provides the dynamic behaviour of the system. The load flow is run through Forward / Backward sweep algorithm and the results have been validated in a practical distribution system. Based on the voltage solutions
### TABLE I

Phase A Load Voltages Comparison with and without Integration of Solar PV

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<th>After Solar (v)</th>
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**Fig. 6.** IEEE 123 test feeder nodes identified for solar PV integration

**Fig. 7.** IEEE 123 test feeder real power with solar PV

**Fig. 8.** IEEE 123 test feeder reactive power before and after solar PV

**Fig. 9.** IEEE 123 test feeder phase A voltage comparison with solar PV

Nodes were identified and solar PV were integrated suitably. It is noted that there is a greater impact while integrating the
solar PV. It is also observed that the node voltage magnitude has been increased considerably when solar PV is integrated. This paper describes the impact of solar PV integration in the time domain for the distribution system with change in load demand.

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