Performance analysis and comparison of load flow methods in a practical distribution system

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***Abstract*—The power sector is in its fast growing phase to meet the growing demand for electricity. With the integration of Distributed Energy Resources (DER) in the Distribution Network (DN), the conventional network has been upgraded from a passive network to an active network. With high penetration of DER the network experiences high R/X ratios, unpredicted variation of source with continuously variation of loads. With all these challenges power grid is committed to provide good quality power to the consumers and maintain the stability of the network. Hence, to analyse the performance of the network, load flow studies are very important. This paper compares the performance characteristics of Newton-Raphson (NR) method and Forward / Backwards Sweep (FBS) method with a variable load pattern. The performance of the methods are evaluated in an IEEE 13 node test feeder. The performance characteristics are evaluated for their different convergence criteria by increasing the R/X ratio, by varying the tolerance values and analysing the voltages at break points. Based on the case studies the results show that FBS method is more preferable for Distribution System.**

I. INTRODUCTION

The power system network comprises of generation, transmission and distribution. Many researchers have developed the load flow algorithm for transmission system, but the load flow analysis for the distribution system is still in its development stage. This is because of the characteristics of the distribution system being highly ill conditioned, high R/X ratio and more penetration of unpredictable Distributed Energy Resources (DER). The conventional Newton-Raphson (NR) load flow method and fast decoupled load flow methods were not suitable for Distribution Network (DN). First to solve the load flow for a DN ladder network theory was introduced [1]. A simple and efficient method to solve the load flow using algebraic expressions were presented [2]. To solve the weakly meshed transmission using active and reactive power flow variables was introduced, this has reduced the computational burden [3]. To analyse the real time performance of a three phase practical distribution system an extension of compensation based power flow method was introduced [4]. To increase the convergence efficiency decoupled load flow method was proposed [5]. Extensive modelling of load flow was done to make suitable for the distribution automation environment [6]. All the above load flow analysis provided a solution for three phase balanced system and was not suitable for unbalanced system. Direct load flow analysis for a three phase unbalanced system was proposed using simple matrix multiplication, which has reduced the time consumption for LU decomposition and Jacobian matrix substitution [7]. To avoid sequential numbering and to reduce the networks with minimum data preparation an efficient load flow solution was proposed [8]. When transformers are considered for load flow analysis, singularity problem arises, to overcome this a new FBS algorithm was introduced [9]. In DN, most of the loads are voltage dependent static loads with line charging capacitance. A new load flow technique was proposed which can handle static loads and line charging capacitance. This method updated polynomial in forward sweep and updated voltage using Kirchhoff's voltage law in the backward sweep [10]. Further to improve the computational efficiency, the FBS method was improved by using Kirchhoff's current law (KCL) in backward sweep and Kirchhoff's voltage law (KVL) in the forward sweep [11]. To include variable loads and shunt admittance equivalent injection current method was proposed. In this method decomposing the system into symmetrical components is not needed and can be substituted directly [12]. When RES were integrated in the network FBS method was modified to handle PV nodes [13].A hybrid NR and FBS method were proposed for ungrounded DN. The system was divided into mainline system and multitap system. The mainline system was handled using NR method and the multitap system was handled using FBS method [14]. To solve a load flow for a network which has multiple three phase AC and DC sections, modified load flow method was proposed to be used for micro grids [15]. The conventional power flow solutions are not appropriate for micro grids as there is no slack bus. Hence a modified NR method was proposed to solve the power flow for micro grids [16]. Convergence ability of distribution power flow methods were compared with voltage dependent loads [17]. Further load flow analysis with distributed generation was compared in [18]. To validate the voltage sweep flow method, comparison were done with other conventional methods [19]. In this method FBS algorithm is compared with NR load flow method to analyse the performance characteristics and conclude a suitable load flow method for a practical distribution

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system. The results are validated in an IEEE 13 node test feeder. The second section describes the formation of forward / backward sweep load flow method and the third section briefs the Newton-Raphson load flow method. The fourth section elaborates the features of IEEE 13 node test feeder and the fifth section explores the results and discussions. The results conclude that FBS method is more suitable for solving load flow of a distribution network with penetration of Distributed Energy Resources.

II. Newton - Raphson Method

Newton - Raphson method is used to run the power flow as it is suitable for mesh network. NR power flow modeling consists of two sets of equations. The first set of equations describes the current injection from the load into the system. They are split into real and imaginary parts as given below.

$$\triangle I_{rn} = \frac{(P_n)(V_{rn}) + (Q_n)(V_{in})}{(V_{rn})^2 + (V_{in})} - \sum_{k=1}^{m}\sum_{t}(G_{nk}V_r - B_{nk}V_{ir}) \tag{1}$$

$$\triangle I_{in} = \frac{(P_n)(V_{in}) + (Q_n)(V_{rn})}{(V_{rn})^2 + (V_{in})} - \sum_{k=1}^{m}\sum_{t}(G_{rk}V_in - B_{rk}V_r) \tag{2}$$

where
$\triangle I$ is the current injection in the bus
n is the bus number.
t represents all phases connected to the bus.
P is the real power of the load.
Q is the reactive power of the load.
B_{nk} is the susceptance of the n^{th} bus.
G_{nk} is the conductance of the n^{th} bus.
B_{nk} and G_{nk} are elements of the nodal matrix.
If E_n represents voltage of the bus such that
V_{rn} is the real portion of the voltage
V_{in} is the imaginary portion of the voltage

$$E_n = V_{rn} + jV_{in} \tag{3}$$

With the current injections calculated the voltage updates are calculated through:

$$\begin{bmatrix}\triangle I_{rn} \\ \triangle I_{in}\end{bmatrix} = -J^-1\begin{bmatrix}\triangle V_{in} \\ \triangle V_{rn}\end{bmatrix}$$

where J^-1 is the inverse Jacobian.

$$J = \begin{bmatrix}\frac{\delta\triangle I_{in}}{\delta V_{rn}} & \frac{\delta\triangle I_{in}}{\delta V_{in}} \\ \frac{\delta\triangle I_{rn}}{\delta V_{rn}} & \frac{\delta\triangle I_{rn}}{\delta V_{in}}\end{bmatrix} \tag{4}$$

Equations 1,2,3 and 4 are frequently used to calculate the nodal voltages, using current injections into the system through NR method.

III. Forward / Backward Sweep Algorithm

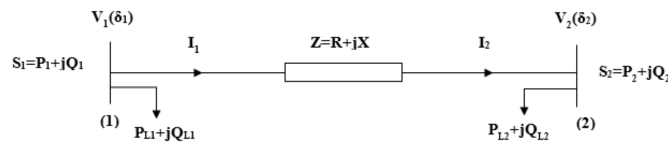

Fig. 1. Distribution Line Model

Let us consider a distribution line model as shown in Fig.1 Two nodes 1 and 2 are taken where as node 1 is the sending end and node 2 is the receiving end. Fig.2 shows the process flow of FBS algorithm in backward sweep and forward sweep. The real and reactive power at node 2 is given by

$$P_2 = \frac{V_1V_2}{Z}Cos(\theta_z - (\delta_1 + \delta_2)) - \frac{V_2^2}{Z}Cos(\theta_z) \tag{5}$$

$$Q_2 = \frac{V_1V_2}{Z}Sin(\theta_z - (\delta_1 + \delta_2)) - \frac{V_2^2}{Z}Sin(\theta_z) \tag{6}$$

where
V_1 is the voltage magnitude of node 1
V_2 is the voltage magnitude of node 2
δ_1 is the angle of V_1
δ_2 is the angle of V_2
Z is the magnitude of line impedance.
θ_z is the angle of line impedance.

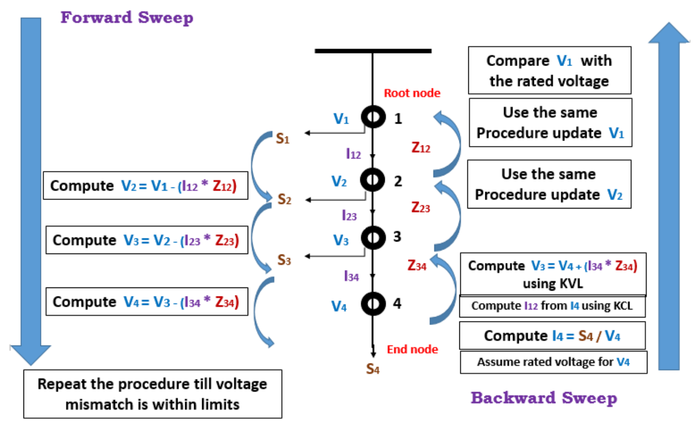

Fig. 2. Process flow of Load Flow analysis using FBS

From the equations 5 and 6

$$Cos(\theta_z - (\delta_1 + \delta_2) = \frac{P_2Z}{V_1V_2} + \frac{V_2}{V_1}Cos(\theta_z) \tag{7}$$

$$Sin(\theta_z - (\delta_1 + \delta_2) = \frac{P_2Z}{V_1V_2} + \frac{V_2}{V_1}Sin(\theta_z) \tag{8}$$

Squaring and adding equations 7 and 8 will give the value 1 using trignometric identity

$$(Sin^2\theta + Cos^2\theta) = 1 \tag{9}$$

The maximum real root of the equation giving the node2 voltage magnitude

$$V_2^2 + 2V_2^2(P_2R + Q_2X) - (V_1^2V_2^2 + (P_2^2 + Q_2^2)Z^2 = 0 \tag{10}$$

In terms of power:

$$V_2 = \sqrt{V_1^2 - 2(P_1R + Q_1X) + \frac{((P_1^2 + Q_1^2)Z^2)}{V_1^2}} \tag{11}$$

Using KVL in Fig.1

$$V_1 = V_2 + I_1Z \tag{12}$$

$$V_2 = V_1 + I_1Z \tag{13}$$

Equations 10,11,12 and 13 are frequently used to calculate the nodal voltages in Forward / Backward sweep algorithm. First Backward sweep of the network is proceeded upstream starting from end node, proceeding towards the source node. End node voltage value is assumed and the corresponding nodal voltages are calculated from tail end of the feeder towards the source node by applying Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL). The calculated voltage is compared with the source voltage and if the convergence is within limits then the process is stopped else proceeded in the forward sweep in downstream. In downstream the nodal currents and voltage drop is calculated from the source node to the end node using KCL and KVL. At the end node, the voltage is compared with the reference voltage if the convergence is within limits the first iteration stops, else the new voltage is updated at the end node and proceeded with backward sweep and forward sweep till convergence is achieved. In backward sweep, the branch current from tail end to the sub station end is calculated. The branch currents are calculated and then using the equation 12 maximum real and reactive power mismatch and voltage mismatch is taken as convergence criteria.

IV. System Description

To compare the performance characteristics of NR load flow method and FBS load flow method we have taken the IEEE 13 node test feeder [20]. Fig. 3 illustrates the features of IEEE 13 node test feeder. The feeder is very small but it has very interesting characteristics. It is very short in length but highly loaded 4.16 KV feeder, with the combination of both overhead lines and underground lines. It has one inline capacitor to maintain the voltage profile. It is a highly unbalanced system with three phase, two phase and single phase laterals. It has both spot loads and distributed loads. Distributed loads are concentrated in the central part of the line segment. The system has an inline distribution transformer with a nominal voltage of 4.16 KV for a small span of the network. It has 8 spot loads and one distributed load. Out of which 2 loads are constant impedance loads, 2 loads are constant current loads and 5 loads are PQ loads. The network has 5 three phase laterals, 3 two phase laterals and 2 single phase laterals. The combination of all features makes IEEE 13 node test feeder more suitable to test the performance characteristics in the Distribution System.

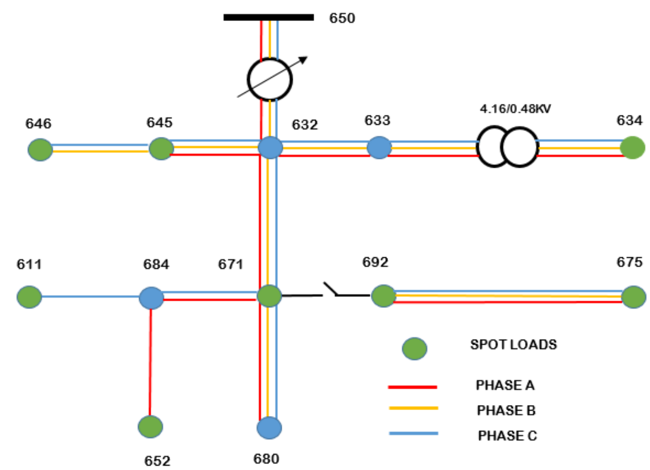

Fig. 3. IEEE 13 node Test Feeder

Criteria	NR Base	NR Solar	FBS Base	FBS Solar
Total Simulation time (sec)	2	5	1	2
Simulation speed (objects. hours /sec)	1.0K	1.3K	2.1K	3.1K
Convergence efficiency - passes / time step	1.00	1.00	1.00	1.00
Simulation rate (* real time)	43200	17280	86400	43200

TABLE I
IEEE 13 node test feeder simulation results

V. Results and Discussions

The load flow analysis is carried out in the IEEE 13 node test feeder through NR method and FBS method. The load flow analysis is carried out with and without connecting solar PV in the feeder. The performance characteristics of NR method and FBS method is analysed by comparing various case studies.

A. case study 1

The simulation is carried out with a continuously variable load pattern. The convergence ability of the system is compared and listed out in table I.

It is observed from table I that the NR method takes more time for simulation and FBS has a higher rate of simulation.

	NR Base			FBS Base		
	A	B	C	A	B	C
Max V	2551.839	2532.866	2566.846	2551.888	2532.953	2566.899
Min V	2362.888	2401.777	2344.949	2363.068	2401.777	2345.388
	NR Solar			FBS Solar		
	A	B	C	A	B	C
Max V	2551.839	2532.592	2566.847	2551.888	2532.62	2566.899
Min V	2363.746	2401.777	2346.02	2363.773	2401.777	2346.071

TABLE II
IEEE 13 NODE TEST FEEDER VOLTAGE VALUES

Eventhough the convergence efficiency is same for both the methods, FBS method is capable of handling more parameters (objects) during simulation.

B. Case study 2

In a radial network the successive voltage difference for each nodes is taken as an important criteria to check the convergence ability of a method. Table II gives the maximum and minimum voltage of IEEE 13 node test feeder for NR method and FBS method with and without integration of solar PV. Fig.4 gives the voltage difference between various nodes and Fig.5 gives the voltage difference between various nodes when solar PV is integrated in the feeder.

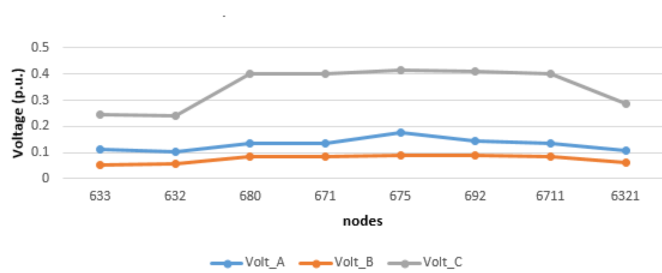

Fig. 4. Voltage difference between nodes

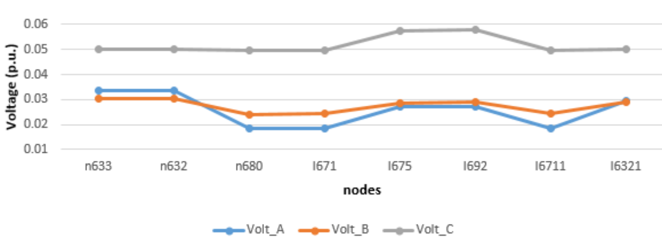

Fig. 5. Voltage difference between nodes with solar PV integration

It is observed that in both the cases the voltage mismatch is more in case of NR method and the convergence is fast in case of FBS method. Fig. 6 illustrates the voltage magnitude at the three break point nodes 632, 684 and 671. The voltage magnitude is high in case of FBS method. Thus for a radial distribution network FBS method is more preferable to perform the load flow analysis.

C. Case study 3

The real power loss in the feeder is compared for NR method and FBS method. From Fig.7 it is observed that the real power loss for FBS method is very less when compared

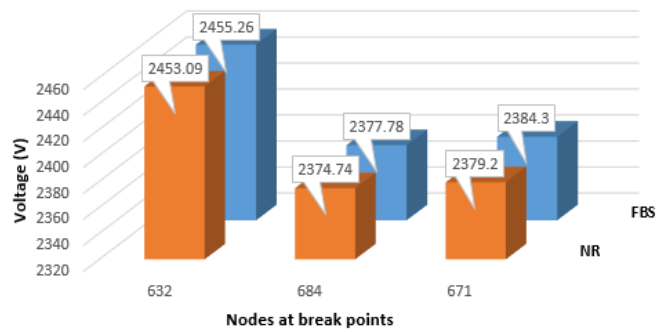

Fig. 6. Voltage Magnitude between at break point nodes

with NR method. As the real power loss is reduced when DER is integrated FBS method is more preferable.

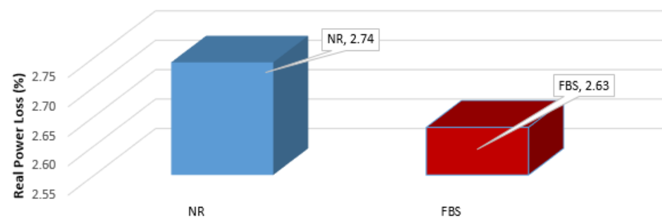

Fig. 7. Real power loss in IEEE 13 node test feeder

D. Case study 4

The convergence criteria is increased in steps and the convergence ability of NR method and FBS method is compared. The characteristics are compared with and without solar PV. Fig.8 illustrates that the FBS method has faster convergence ability than NR method. This shows that FBS method with its sweeping algorithm is more suitable for radial distribution network with the integration of DER.

E. Case study 5

The main important criteria for a distribution network to be ill conditioned is its high R/X ratio. The convergence

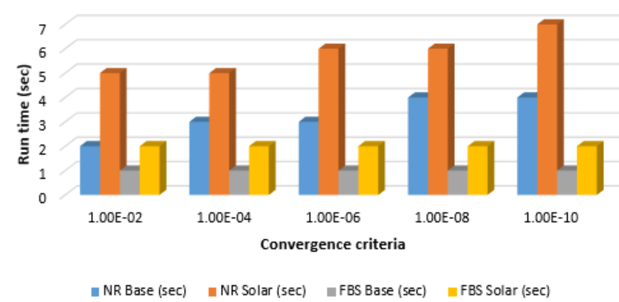

Fig. 8. IEEE 13 node test feeder NR Vs FBS convergence ability

R/X	**NR - conv. Time (sec)**	**FBS conv. Time (sec)**
0.55	2	1
1.1	2	1
2.75	2	1
5	3	2
10	3	2
10.5	5	2
11	5	2
11.5	30	2
11.75	not converged	2
11.8	not converged	2
11.85	not converged	2
11.9	not converged	2
12	not converged	5

TABLE III
CONVERGENCE CHARACTERISTICS COMPARISON FOR NR AND FBS METHOD

N \ R/X	**0.55**	**2.75**	**5**	**7.5**	**10**	**12**
633	2445.46	2444.65	2443.17	2441.47	2438.47	2430.26
630	2551.89	2551.89	2551.89	2551.89	2551.89	2551.89
632	2452.73	2452.18	2451.10	2449.94	2447.87	2442.13
650	2401.78	2401.78	2401.78	2401.78	2401.78	2401.78
680	2378.64	2378.12	2376.85	2375.70	2373.59	2367.76
684	2374.01	2373.49	2372.23	2371.08	2368.97	2363.16
634	275.53	262.81	247.74	227.86	200.29	153.86
652	2364.11	2363.58	2362.34	2361.19	2359.09	2353.31
671	2378.64	2378.12	2376.85	2375.70	2373.59	2367.76
675	2363.06	2362.54	2361.24	2360.08	2357.94	2352.07
692	2378.64	2378.12	2376.85	2375.70	2373.59	2367.76

TABLE IV
FBS - PHASE A VOLTAGE VALUES AT NODES WITH INCREASE IN R/X VALUES

characteristics of NR method and FBS method is tested by increasing the value of R/X ratio in steps.

Table III illustrates that the FBS method withstands with increase in R/X value from 0.55 through 12 but NR method stops converging before 11.75. Table IV gives the phase A voltage values at the nodes for increase in R/X values when load flow is run through FBS method. Where N is the node numbers. Table V gives the phase A voltage values at the nodes for increase in R/X values when load flow is run through NR method. This important criteria for a distribution system reveals that FBS method is the best suitable method for a DN with penetration of DER.

N \ R/X	**0.55**	**2.75**	**5**	**7.5**	**10**	**11.5**
633	2445.35	2444.40	2443.17	2441.24	2437.80	2431.46
630	2551.84	2551.84	2551.84	2551.84	2551.84	2551.83
632	2452.62	2451.97	2451.12	2449.78	2447.41	2442.92
650	2401.78	2401.78	2401.78	2401.78	2401.78	2401.78
680	2378.51	2377.84	2376.97	2375.61	2373.24	2368.50
684	2373.89	2373.22	2372.35	2371.00	2368.63	2363.90
634	275.51	262.71	247.67	227.26	198.52	164.27
652	2364.02	2363.35	2362.48	2361.13	2358.77	2354.04
671	2378.51	2377.84	2376.97	2375.61	2373.24	2368.50
675	2362.89	2362.21	2361.33	2359.96	2357.57	2352.79
692	2378.50	2377.83	2376.96	2375.60	2373.23	2368.49

TABLE V
NR - PHASE A VOLTAGE VALUES AT NODES WITH INCREASE IN R/X VALUES

VI. CONCLUSION

In this paper, the performance characteristics of Newton-Raphson method and Forward/Backward Sweep algorithm is compared when solar PV is integrated. The algorithm is validated in a IEEE 13 node test feeder with various case studies. The performance assessment reveals the voltage mismatch between nodes, voltage magnitude at break points, convergence ability and simulation outputs. It gives a clear observation of the real power losses in the feeder and the change in performance when the value of R/X is increased. From the results it is observed that FBS method is the most preferred load flow analysis method for a radial distribution network with the integration of DER. The computational burden is less for FBS method and hence the convergence ability is more. Owing to the complex calculations like Jacobian matrix inversion and singularity problem when transformers are included in NR method, it is concluded that Forward / Backward sweep method is the best suitable method for a radial distribution system.

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