# Waveform Relaxation Based Hybrid Simulation of Power Systems

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***Abstract*—Hybrid simulation is performed by integrating two types of power system simulators - an Electro-Mechanical Transient (Transient Stability) Simulator and an Electro-Magnetic Transient (EMT) Simulator. The interfacing of two programs which are running simultaneously with different time-steps has some challenges, especially if the programs have been developed independently. In this paper, the application of the Waveform Relaxation (WR) method is proposed for performing hybrid simulations. Although it is an iterative method, WR seems naturally suited for hybrid simulation, as it does not rely on synchronized interfacing between the two programs while the simulations are running. The simulations can be run independently and they can transfer the output waveforms at the end of each simulation run. In this paper, we present and evaluate this concept and illustrate it with examples.**

***Index Terms*—Hybrid Simulation, Transient Stability Simulator, Electro Magnetic Transient Simulator, Waveform Relaxation Method.**

## I. Introduction

Two types of simulation tools are commonly used for power system transient analysis - Transient Stability (TS) programs and Electro Magnetic Transient (EMT) programs. Since the phenomena of interest in a TS simulation are low frequency oscillations (typically power swings of frequency lower than 2 Hz), loss of synchronism, voltage regulator and governor action etc., the system models are of the "low frequency type". The electrical network is assumed to be in quasi-sinusoidal steady-state, allowing the use of algebraic phasor relationships to describe its behaviour. The resulting differential-algebraic equations are solved using time steps ranging typically from 1 to 10 ms. On the other hand, EMT programmes are meant for capturing the fast transient behaviour of the systems. Detailed high frequency modeling of system components is necessary, and the time-step is typically lower than 50 $\mu$s. The simulated system is usually a small portion of the entire power system. The rest of the system represented by approximate frequency dependent equivalents at the boundaries. This is done to limit the model size (which translates to higher simulation speed) and to limit the detailed data requirements to the relevant portion of the network only.

Although TS and EMT programmes are normally used to simulate phenomena over different time scales, there may be reasons to consider the use of both tools. Take the example of an HVDC link in a large power system. For a TS simulation, a simplified model of an HVDC link may be used (a response type model [1]). This modeling may however, fall short of capturing some important aspects which may significantly change the outcome of the simulation. For example, if a disturbance like a fault occurs near the inverter end, it may trigger commutation failure. If it persists, then the current in the link may be reduced to zero automatically by the controller and ramped-up again after a few cycles. These events may significantly affect the electro-mechanical behaviour of the system. These aspects can be captured accurately by a simulation which models the switching of the individual devices, i.e. an EMT program, while the large system can be efficiently simulated by a TS program.

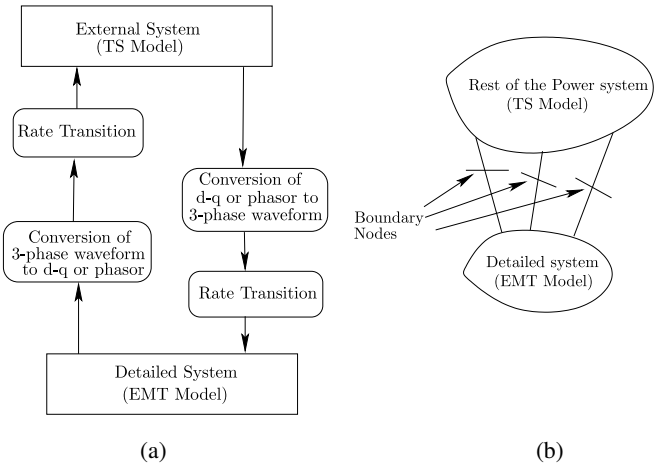


Fig. 1. (a) Multi Rate Hybrid Simulation (b) Interface Between Detailed System and External System

Therefore, a hybrid simulation program which combines a TS simulation and a EMT simulation is needed. The EMT program models a small part of the system in detail (like the HVDC link [2] and a few elements of the surrounding ac system). The rest of the system is modeled using a TS program. The TS program output, e.g., voltage phasors from the larger system at the *boundary nodes*, which are updated at the time steps corresponding to the TS simulation, are given

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to the EMT program. The EMT output, e.g., current injection from the smaller system at the boundary nodes, is fed to the TS program at the down-sampled instants [3]–[5]. This is depicted in Fig. 1(a). The following aspects have to be determined prior to carrying out a hybrid simulation:

1) Partitioning into the TS and EMT zones of study, i.e., selection of the boundary nodes (See Fig. 1(b)). To ensure accuracy, the high-frequency interaction of the TS sub-system with the EMT sub-system may need to be captured. It is expected that if the EMT program embraces a large-enough part of the system, then the need to separately model the high-frequency interactions with the TS sub-system is reduced. This is based on the observation that high frequency transients generally are localized, and their observability reduces as one moves further away from the location where the transient is initiated. Alternatively, Frequency Dependent Network Equivalents [2], [5] may be used at the boundary nodes, which reduces the need to embrace a large part of the sub-system for the EMT model.
2) Transferring data between the TS and EMT programs: A conventional hybrid simulation involves synchronized execution of the TS and EMT programs, since data is exchanged while the simulations are running. There are several protocols for exchange of data [5], [6]. If the TS and EMT programs have been developed independently, additional programming effort is required for this interfacing.

This paper proposes Waveform Relaxation (WR) as an alternative hybrid simulation method, which is iterative unlike the conventional method. The TS and EMT runs are done sequentially and the exchange of simulation output is done after the entire run of the TS or EMT simulation. If the simulations converge, then the response is practically the same as the conventional hybrid simulation. The WR-hybrid simulation offers the following advantages over a conventional hybrid simulation:

1) Since the output waveforms are exchanged only after the TS or EMT simulation is completed, the data storage and transfer may be done through files, which is very convenient.
2) Access to the inner workings of either the TS or EMT programs is not necessary, which means that any off-the-shelf TS or EMT simulation tools may be used with practically no additional integration effort.
3) Access to the entire output waveform at the end of the simulation allows for accurate interpolation (when up-sampling the TS output) and anti-aliasing with delay compensation (when down-sampling the EMT output).
4) There is greater flexibility in choosing the step sizes and numerical integration methods (including variable step methods) for efficient EMT and/or TS simulations.

Waveform Relaxation was originally proposed as a method of parallelizing the numerical integration of very large systems (e.g., VLSI) [7], [8]. It was also used for parallelizing transient stability simulations of large power systems [9]. Recently it has has also been proposed as a low cost alternative to real-time digital simulation based hardware-in-loop simulation [10], [11]. Since the method is iterative, the main issue with WR is of convergence. Nevertheless, the other advantages motivate us to study its application for hybrid simulation.

In this paper we illustrate the use of WR for the hybrid simulation problem using a simple case study of a single machine connected to an infinite bus by a Thyristor Controlled Series Compensated line, as well as a realistic multi-machine system with an embedded HVDC link. Feasibility of using an approximate low frequency model of the EMT simulation in the TS program for faster convergence [12] is evaluated. The case studies demonstrate the practical feasibility of this method for hybrid simulation.

The paper is organized as follows: A brief description of the WR method is presented in Section II. In Section III, we present examples of WR based hybrid simulation. The method for improving convergence is presented in Section IV.

## II. Waveform Relaxation Method

The scheme is based on the Waveform Relaxation algorithm [7] for solving differential equations. This is illustrated using a simple case of two coupled first-order differential equations:

$$\dot{x}_1 = f_1(x_1, x_2) \tag{1}$$

$$\dot{x}_2 = f_2(x_1, x_2) \tag{2}$$

$$x_1(0) = x_{01},\ x_2(0) = x_{02} \tag{3}$$

The basic idea of a WR algorithm applied to this system is to first guess the waveform $x_2(t)$ in the time window $[0, T]$ and solve (1) as a one dimensional differential equation using a numerical integration method. The solution thus obtained for $x_1(t)$, can be substituted into (2) which will then reduce that equation to a first-order differential equation. This differential equation is now solved using a numerical integration method to obtain $x_2(t)$. Equation (1) is then solved again using the new solution for $x_2(t)$. This is done iteratively till $x_1(t)$ and $x_2(t)$ converge in the time window $[0, T]$. The waveforms in each WR iteration have to start from the initial conditions given in (3). The algorithm can been seen as an analogue of the Gauss-Seidel technique for solving non-linear algebraic equations. Unlike the problem of solving nonlinear algebraic equations, the unknowns are waveforms rather than real variables. In this sense, the algorithms are techniques for time-domain decoupling of differential equations. The algorithm given above is referred to as the Gauss-Seidel WR (GSWR) algorithm.

The WR method can be used for higher order systems as well. In such systems, the partitioning into sub-systems can be done in several ways. The WR method is performed by numerically solving each sub-system independently and iteratively as described for the second order example. In practice, the solution for the differential equations is obtained numerically by discretization with a stable numerical integration method (Trapezoidal method or Backward Euler method being commonly used). The rate of convergence and stability

of the simulation is affected by how the system is partitioned and the numerical method used for discretization [8].

## III. WR Based Hybrid Simulation

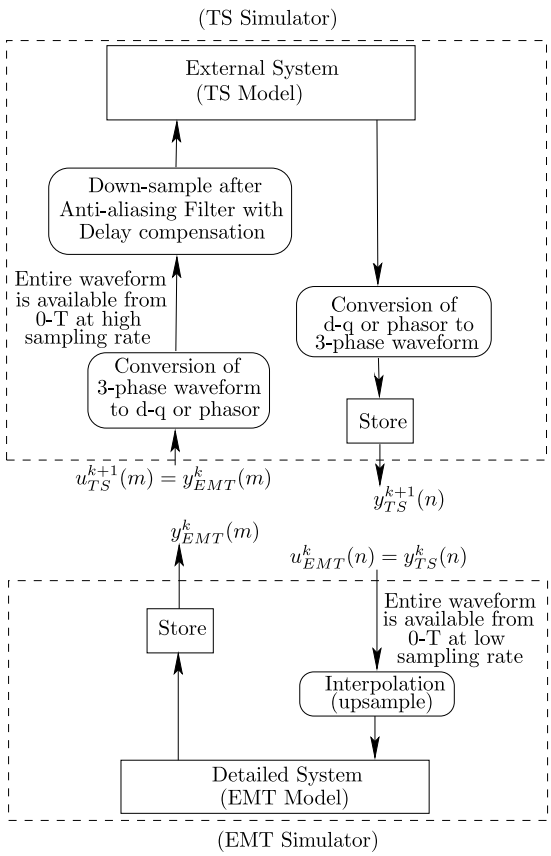


Fig. 2. Hybrid WR simulation

A schematic for WR based hybrid simulation is shown in Fig. 2. Here the data has to be transferred at the end of the complete TS simulation or the EMT simulation. As both the simulators use different time steps, down-sampling and up-sampling needs to be performed in every iteration, as discussed in the introduction. Since the entire waveform is available, delays due to anti-aliasing can be accurately compensated as the entire waveform is available for processing at the end of the simulation interval. Similarly, since the entire waveform in the interval is available, interpolation for up-sampling may be conveniently done. The iterative nature of the WR method (which may be slow to converge) is the main drawback.

### A. Example I: SMIB System with a TCSC

The method is illustrated using a Single Machine Infinite Bus (SMIB) system with a Thyristor Controlled Series Compensator (TCSC) which has a modulation controller for damping of power swings. For hybrid simulation, the swing equations (equations representing the classical model of the generator) is simulated by a transient stability (TS) program (time step $h$ = 10 ms, Runge-Kutta $4^{th}$ order method), while the transmission line and detailed model of TCSC is simulated using an EMT programme (time step $h$ = 50 $\mu$s, Runge-Kutta method). The system is shown in Fig. 3. The EMT and TS models are shown in Figs. 4 and 6 respectively, while the TCSC controller details are shown in Figs. 5.

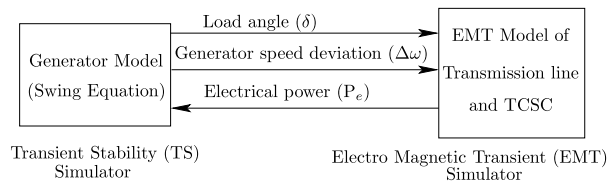


Fig. 3. Schematic of Hybrid Simulation of SMIB system with a TCSC

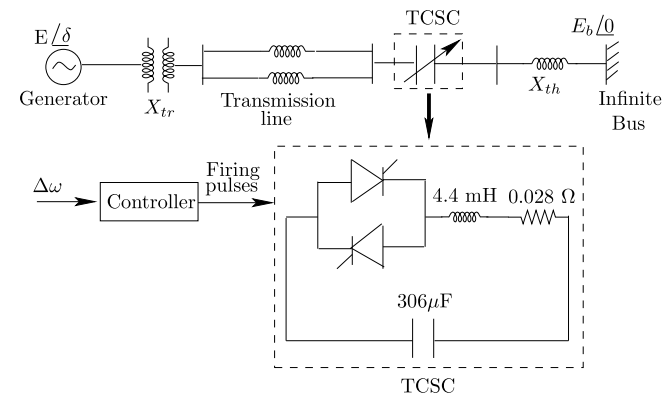


Fig. 4. EMT Model of the SMIB system with a TCSC

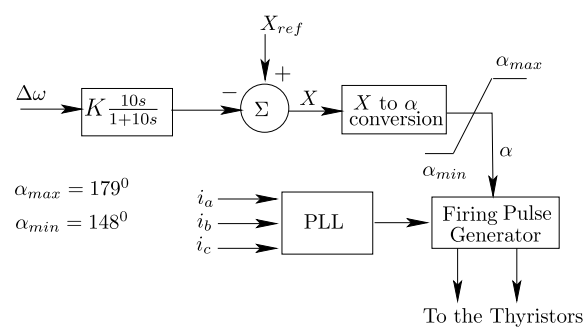


Fig. 5. TCSC controller

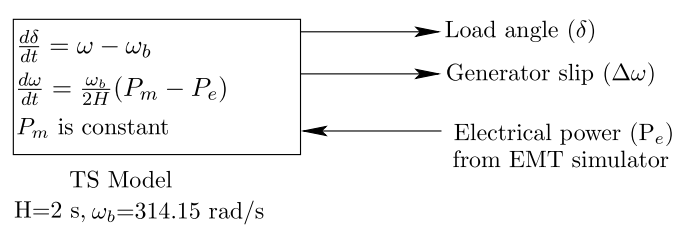


Fig. 6. Schematic of the sub-system simulated using a TS simulator

The 400 kV parallel lines are modeled with distributed line parameters. The TCSC has a parallel resonance at a delay angle $\alpha$ = $147^0$. Below this value it operates in capacitive mode. Generator speed deviation signal $\Delta\omega$ is used to modulate the reactance. The firing angle is calculated from a look-up table. The TCSC has a firing pulse generator synchronised to line current. A $90^o$ shift is then given so that it becomes synchronised with capacitor voltage.

Initial steady state power flow is taken as 510 MW (1.02 pu), corresponding to a phase angle difference $\delta$=$15^o$. One of the transmission lines is tripped at 1.5 s.

The EMT simulation is performed first. In the first iteration, load angle input to the EMT program is $15^o$ and generator slip $\Delta\omega$ is 0 rad/s, throughout the simulation run. The TS program is executed next, using the power $P_e$ obtained from the first EMT run. The load angle generator slip are stored for the EMT simulation in the next WR iteration. As discussed earlier, this process is repeated till convergence[1] is attained.

The generator rotor angles at different WR iterations are shown in Fig. 7 (with damping controller disabled) and in Fig. 8 (damping controller enabled). Waveforms of R-phase current of TCR at various WR iterations are shown in Fig. 9.

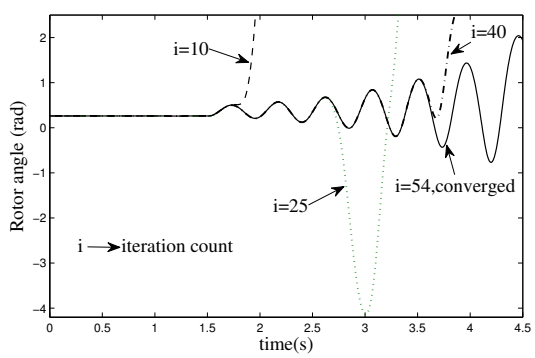


Fig. 7. Converged Rotor angle obtained using the WR based Hybrid Simulation of the SMIB system with a TCSC, with $K = 0$ pu/(rad/s)

### B. Example II: HVDC Link in a Large Grid

We now perform a more realistic simulation of a HVDC link in a large power system. The system data is adapted from the Eastern Region and Western Region (ER-WR) grid of India [10], and the HVDC link located between Chandrapur and Padghe. The 12-pulse HVDC link is modeled in detail along with the filters and converter transformer in the EMT simulation, and rest of the network is modeled for the TS simulation. The ac-side high-voltage buses of the HVDC Link are the boundary buses - the ac grid is modeled as

[1]If $\delta^k$ is the signal used for the convergence check at the $k^{th}$ WR iteration and the convergence criteria used is $\sum_{n=0}^{p}\left[\delta^k(n) - \delta^{k-1}(n)\right]^2 \leq 0.0001$, where $n$ is the time step number of the digital simulation and $p$ is the total number of time steps in the simulation. If $h$ is the simulation time step, then the total simulation time is $T = ph$.

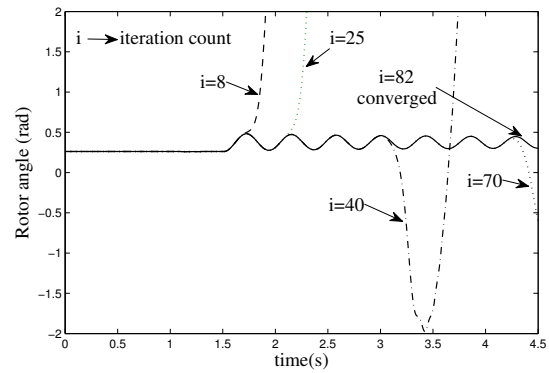


Fig. 8. Converged Rotor angle obtained using the WR based Hybrid Simulation of the SMIB system with a TCSC, with $K = 0.02$ pu/(rad/s)

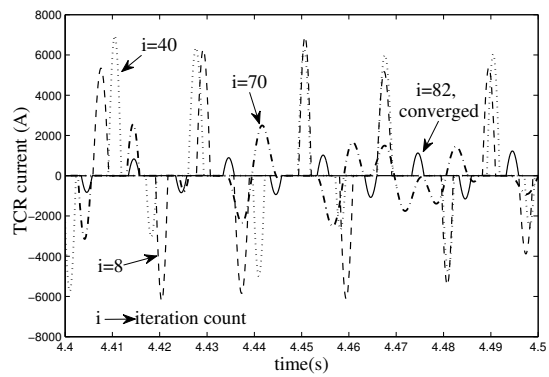


Fig. 9. Converged R-phase current of TCR obtained using the WR based Hybrid Simulation of the SMIB system with a TCSC, with $K = 0.02$ pu/(rad/s)

a voltage source behind a reactance in the EMT program, while the HVDC link is modeled by current injection at the terminals for the TS program. The voltage source values are obtained from the TS program while the current injections are obtained from the EMT program. The HVDC converter and controller model is adapted from 12-pulse HVDC model of [13] `power_hvdc12pulse.mdl`. The HVDC system has following sequence of disturbances:

1) At t = 0.02 s (i.e. when the converters are deblocked), the reference current is ramped to reach the minimum value of 0.1 pu in 0.3 s (0.33 pu/s).
2) At t = 0.4 s, the reference current is ramped from 0.1 pu to 1 pu in 0.18 s (5 pu/s).
3) At the end of this starting sequence (t = 0.58 s), the DC current reaches steady state.
4) At t = 0.7 s, a -0.2 pu step is first applied on the reference current (decrease from 1 pu to 0.8 pu ) and at t=0.8 s, the reference current is reset to its 1 pu original value
5) A Line-Ground fault in the DC side is applied at 1.0 s for 50 ms duration.

The EMT model of HVDC link is simulated with a time step of 50 $\mu$s and rest of the grid in the TS program is simulated with a time step of 10 ms. A load flow is performed for the rest of the TS network, to start the simulation at steady state. The initial current injections at rectifier and inverter buses are taken to be zero as the startup sequence is to be simulated. The grid voltages obtained at the HVDC buses are in the DQ reference frame [1]. They are transformed to a-b-c reference frame for the EMT simulation. The 3-phase currents measured at rectifier and inverter buses during the EMT simulation are converted back to DQ reference frame and injected at the respective buses for TS simulation. Fig. 11 shows the converged rotor angles in the center-of-inertia frame of reference.

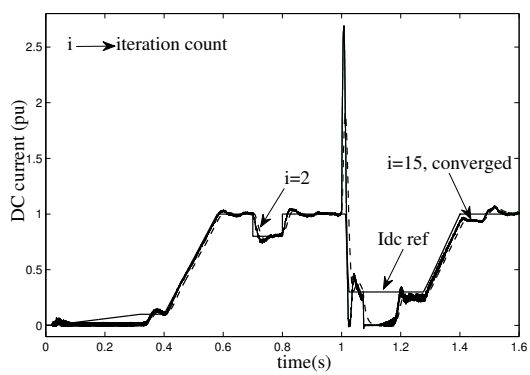


Fig. 10. Converged DC Current obtained using the WR based Hybrid Simulation, for start-up, change in reference and DC line fault in the 12-Pulse HVDC link in the ER-WR Grid.

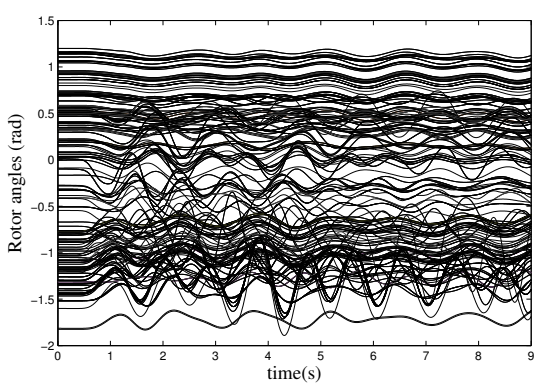


Fig. 11. Converged Rotor angles in the center-of-inertia frame of reference obtained using the WR based Hybrid Simulation, for start-up, change in reference and DC line fault in the 12-Pulse HVDC link in the ER-WR Grid

Convergence is very slow in the SMIB-TCSC system (Example-I). This is not surprising, as the machine is directly connected to the TCSC compensated line, and is strongly coupled to the TCSC controller. On the other hand, the HVDC link in the second example, is of relatively smaller size (1.5 GW) compared to the capacity of the grid (35 GW), which probably explains the faster convergence. In situations like the SMIB-TCSC, there is a need to speed-up the convergence if the WR method is to be a feasible alternative to conventional hybrid simulation. We explore a method for improving convergence in the following section.

## IV. Speeding up Convergence

We now consider the application of the method proposed in [12] for improving convergence. This uses an approximate model of the external sub-system. The schematic of the method (adapted for the SMIB-TCSC example) is shown in Fig. 12. The TS simulation uses the approximate (variable reactance) model of the TCSC, the transmission line and the TCSC controller. This model is a quasi-sinusoidal steady state model which neglects the fast transients and switchings. The approximate model allows us to "come close" to the response which would have resulted from using the detailed EMT model. This prevents wild deviations in the initial iterations which results in slow convergence with the basic WR method. However, this approximate response has to be canceled out eventually and replaced by the accurate EMT response. Therefore, at every time step, the output of the approximate model in the previous iteration is subtracted and the EMT output available at that iteration is used. It is easy to see that when the waveforms converge, the effective output of the approximate models is zero, since $P_{e2}^k = P_{e3}^k$, and only the EMT output determines the solution.

The low frequency model of TCSC and transmission line for calculation of line flow $P_e$ is available and is shown in Fig. 13. The response of the system in different WR iterations

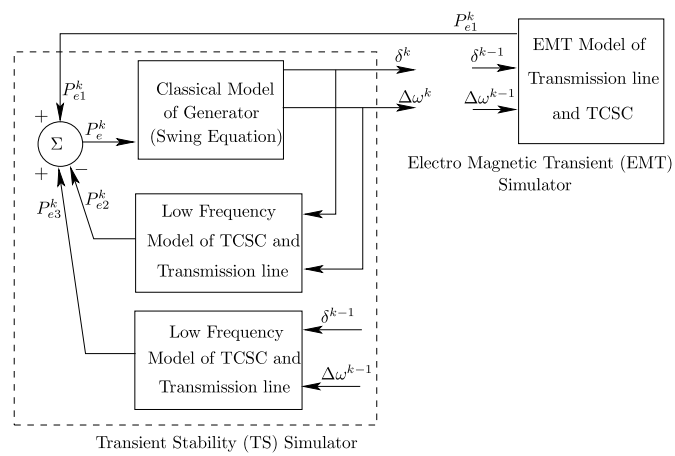


Fig. 12. Modified WR Scheme for hybrid simulation of the SMIB system with a TCSC

with the modified method are shown in Fig. 14. The final convergence is achieved in 20 WR iterations in comparison to 83 WR iterations in the earlier case. Waveforms of R-phase current of TCR at various WR iterations in the time interval 4.44 s - 4.5 s are shown in Fig. 15.

Fortunately, approximate low frequency models (suitable for TS simulations) are available for most network based controllers. Thus this can be a feasible general method for speeding up TS-EMT hybrid simulations.

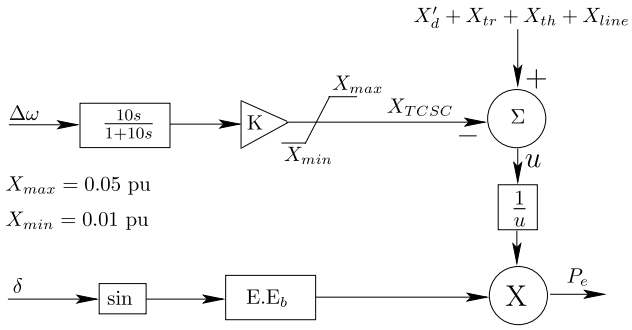


Fig. 13. Low Frequency Model of TCSC, Transmission line, and its controller

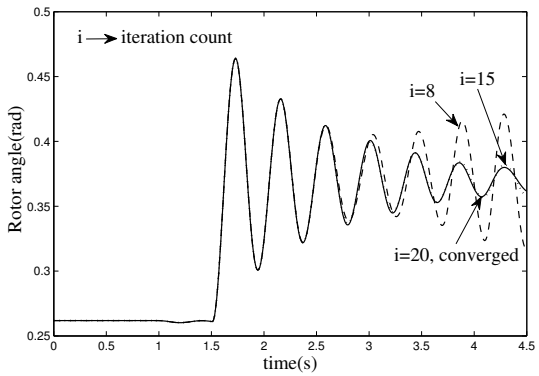


Fig. 14. Converged Rotor angle obtained using the modified WR based Hybrid Simulation of the SMIB system with a TCSC

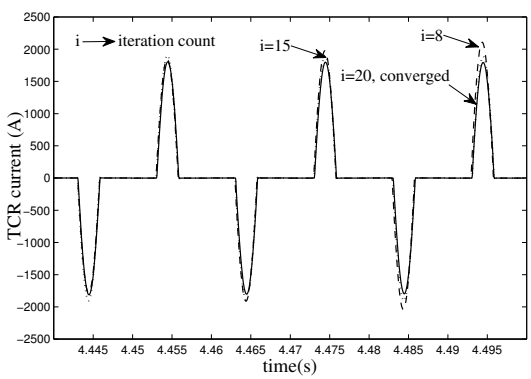


Fig. 15. Converged R-phase current of TCR obtained using the modified WR based Hybrid Simulation of the SMIB system with a TCSC

## V. Conclusion

The application of Waveform Relaxation method for performing hybrid simulations is proposed and illustrated using simple examples. There are several advantages of this approach. Since transient stability and electro-magnetic transient simulations are carried out independently over the entire simulation interval, and data is exchanged only at the end of the entire simulation run, this is very flexible and convenient from the point of view of interfacing.

Slow convergence is a concern for strongly coupled TS and EMT sub-systems. In such situations, a modified WR scheme which uses an approximate TS model of the EMT sub-system is shown to be effective in speeding up convergence.

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