A Lyapunov Exponent based Method for Online Transient Stability Assessment

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Abstract—The post fault transient stability of the power system is assessed in this paper using Maximum Lyapunov Exponent (MLE). The phase angles, reported by the PMUs at the generator buses are used for estimating the MLE of the system. The sign and the magnitude of the MLE serve as indicators to the stability of the system. The method utilized for estimation of the MLE is independent of the system model and is solely based on the measurement data of the system states, which can be obtained from the Phasor Measurement Units (PMUs). Different combination of fault duration and location are simulated to analyze the system stability using MLE on 3 machine 9 bus system and 10 machine New England (NE) 39 bus system.

Keywords—Transient stability, online detection, Lyapunov Exponent, PMU measurements

I. INTRODUCTION

The power system network is possibly one of the most complex man made dynamical systems. It often experiences instability under sufficiently large disturbances. The assessment of the system stability is critical for reliable operation of a power system following disturbances. The early detection of the instability enables the operator to prevent major outages by taking preventive control actions. The modern power systems operate under competitive market environment. In such a scenario, the power flow in the lines are dictated by the complex market dynamics, which also leads to the increase in the power system oscillations, pushing the operating point to its stability limit. The large scale deployment of Phasor Measurement Unit (PMUs) in the power system network has made it possible to estimate the bus voltage and branch current phasors, which are sent to the Phasor Data Concentrator (PDCs) located in the control centers at a rate of upto one phasor per cycle. The high refresh rates of PMUs are suitable for predicting online the impending instability.

The classical stability analysis for power system applications is performed by modeling them in the form of Differential and Algebraic Equations (DAE) in a simulation framework running on personal computer (PC). The stability of the simulated power system network is evaluated for a pre-selected set of contingencies and preventive control actions are pre-computed and stored as a set of lookup tables for the power system operators. However, the disturbances, experienced in the power system have uncertainty in magnitude and location, which might not have been envisaged in the pre-determined set of contingencies. The inability of the operator to assess the stability online for a new contingency situation often drives the power system to cascaded tripping and large scale blackout.

The online determination of the stability boundary for the power system transient studies was proposed as early as in [1], in which the estimation of Lyapunov energy function was carried out by using two simplifying assumptions, but the application was restricted to only small test systems. Extended equal area criteria based direct identification of the transient stability was proposed by Pavella et. al. [2], which used the first swing stability concept. The advent of synchrophasor technology opened a new paradigm of measurement based online stability prediction. The Lyapunov Exponents (LEs) have been used for the identification of unstable swing by Liu et. al. [3] [4] using real time PMU data. The multilayer perceptron and support vector machine based transient stability analysis have been proposed for large power system by Moulin et. al. [5]. Jonsson et. al. [6] had proposed a real time clustering of coherently swinging generators using Fourier analysis. Critical cutset based detection of loss of synchronism of swinging generators, based on computation of energy function, had been proposed by Padiyar et. al. [7]. Real time estimation of the stability margin using concept of Ball of Concave Surface was proposed by Sun et. al. [8]. The stability prediction for the post fault condition of the power system using LEs have been proposed in [9][10]. Determination of the voltage stability using Maximum Lyapunov Exponent (MLE) has been proposed in [11] for real time application. The estimation of LEs is performed by forming the state space equations of the power system, from which the variational equation is derived. The method of integrating the variational equation for the post fault power system conditions depends on the size of the Jacobian matrix of the state space equation.

The sign and the magnitude of the MLE are the indicating criteria for the divergence of the state space trajectories. This paper has proposed a measurement based estimation of the MLE using the PMU data without forming the variational equation of the state space representation. The effectiveness of the proposed method for assessing the transient stability is demonstrated through simulation of faults at different locations.
in 3 machine 9 bus system and 10 machine 39 bus New England (NE) system [12].

II. ESTIMATION OF MLE FROM PMU DATA

A large power system consists of several generators, interconnected by long transmission lines. The steady state operation of the power system is characterized by the synchronous operation of all the rotating machines in the network. The simulations have been carried out using PSCAD/EMTDC utilizing the built in synchronous machine model. The state space representation of a \(i^{th}\) synchronous machine in a power system network, having \(m\) generators, at \(k^{th}\) time instant is expressed as follows.

\[
\dot{\delta}^i_k = a_k \Delta \omega^k
\]

where, \(\delta^i\) is the \(i^{th}\) generator internal voltage in radian, \(a_k\) is the synchronous speed in radian/s, \(\Delta \omega^k\) is the \(i^{th}\) generator per unit speed deviation from the synchronous speed, \(\delta^i = (\delta^i_1 \cdots \delta^i_m)^T\) and \(h\) is a nonlinear function describing the dynamics of the \(i^{th}\) generator. The state vector of the system at \(k^{th}\) time instant is denoted by \(X^k = (\delta^1_k \cdots \delta^m_k, \Delta \omega^1_k \cdots \Delta \omega^m_k)\).

During the steady state operation, the right side of (1) is zero. The system deviates from the steady state value on the occurrence of a fault, following which the state vector converges to a stable equilibrium point provided that the system is transiently stable under the post fault scenario. The state vector diverges and leads to instability if the post fault system is transiently unstable.

The MLE is a definitive indicator for the stability of nonlinear systems. The Wolf’s [13] method has been generally used for the estimation of the MLE for the power system applications. The Jacobian matrix from the state space model of the multi machine power system is formed to estimate the evolution of the state vectors’ trajectories. The LEs estimate the exponential rate of convergence and divergence of the nearby trajectories in state space. The logarithm of the distance between the nearest neighbors in the phase plane are also constrained by the mean period, which is considered as 0.0167s in this work. This constraint is considered so that the nearest neighbors occur on the separate orbits of the attractor. The dynamics of the system is tracked by monitoring the distance of the nearest neighbor points evolved after \(j^{th}\) time step. The distance between the nearest neighbor after \(j^{th}\) time step is denoted by \(d^j\).

\[
\Phi_{j,k} = \frac{1}{N_k} \sum_{k=1}^{N_k} \left\| X^{i+k} - X^{i+j} \right\|
\]

According to Rosenstein [14], the MLE of the system is calculated from the change in the distance of the nearest neighbor along the locus of the state space trajectory. The logarithm of the distance between the nearest neighboring points are averaged over each point in time series data of a finite window length. The logarithm of the average distance between the nearest neighbor at \(j^{th}\) time instant is related to the MLE, as given below.

\[
\ln(d^j(j)) = \ln(d^j(0)) + \lambda_i j \Delta t
\]

where, \(\lambda_i\) is the MLE and \(\Delta t\) is the sampling time of the time series data. This work \(\frac{1}{\Delta t}\) is \(60\), which is the data frame transmission rate of the PMU considered in the power system under study. The MLE at the time \(\frac{j}{\Delta t}\) is given by rearranging the expression given in (2).

III. SIMULATION RESULTS

The MLE estimated from the time series data of the power system has been discussed in the previous section. The magnitude of the speed deviation vectors in a power system network followed by a contingency is observed to be insignificant as compared to the phase angle of the generators. Hence, the phase angle of the generator buses are only considered in this work for the state vectors and stored for consecutive 65 phasors, which is used as initial nearest neighbors. The MLE is estimated and reported at the time instant of the next phasors arriving after the initial 65 phasors. The accumulation of the initial 65 phasors starts soon after the fault is cleared in the system.

The proposed method for estimation of the MLE are used for predicting the transient stability for different cases of fault in 3 machine 9 bus system and 10 machine NE 39 bus system. The fault of different duration and location are created in both the systems for transient stability prediction.

The single line diagram of the 3 machine 9 bus system is shown in Fig. 1, in which the generators are connected at buses 1, 2 and 3, whereas, the loads are connected at buses 5, 7 and 9. The PMUs are assumed to be placed at the generator buses for obtaining the phasors of the voltage signal. The phase angle of the voltage signal at the generator buses are measured and aligned with the time base for the estimation of.
the MLE. A three phase to ground fault is created between mid-point of the line joining buses 6 and 7 having fault resistance of 0.01 ohm at 1s. The system is observed to become unstable when the fault is cleared at 1.3s as the phase angle of the bus 2 diverges from the phase angle of the buses 1 and 3, which swing together as shown in Fig. 2. The same fault is also cleared at 1.29s resulting in retaining the stability of the system. The phase angles of all the three generator buses swing together when fault is cleared at 1.29s, as shown in Fig 3.

The MLE estimation starts at the arrival of the phasor after the initial 65 phasors are buffered following the fault clearing. The evolution of the MLE starts after around 1s (corresponding to 65 initial phasors) after the fault is cleared, which is shown in Fig 4 for the two cases of the fault clearing time.

The MLE is observed to be positive for the diverging phase angles, leading to the unstable system that resulted from the clearing of fault at 1.30s. The evolution of the MLE over time for the fault clearing at 1.29s is observed to be negative, which is a case of system retaining stability after the fault is cleared at 1.29s.

The three phase to ground fault is also created at the mid-point of the line joining buses 5 and 6 having fault resistance of 0.01 ohm, at 1s. The fault is cleared at 1.49s, which makes the system unstable as the phase angle of the bus 2 diverges from the phase angle of the buses 1 and 3, which swings together as shown in Fig 5. The reduction of the fault duration from 0.49s to 0.48s retains the stability of the system and the phase angle of the buses 1, 2 and 3 swing together, as shown in Fig 6. The evolution of the respective MLE for fault clearing at 1.49s and at 1.48s are shown in Fig. 7. The MLE is observed to be positive for the fault clearing at 1.49s indicating instability of the system, whereas the negative MLE for the fault cleared at 1.48s indicate the stable system.
MLE for a fault of 0.49s and 0.48s at line joining buses 5 and 6.

The effect of the fault duration is also studied on the line joining buses 4 and 5. The three phase to ground fault is created at the mid point of the line joining buses 4 and 5 at 1s for duration of 0.48s and 0.47s. The system is observed to be unstable for the fault clearing at 1.48s from the inception, while with the fault clearing at 1.47s after the inception, the system is observed to be stable. The evolution of the MLE for the two fault clearing times are shown in Fig. 8, where the MLE for the fault clearing at 1.48s is positive and, thus, shows unstable scenario, where as the MLE for fault clearing at 1.47s is negative after remaining slightly positive for a short duration, which is a stable scenario.

The online detection of transient stability by monitoring the evolution of the MLE is also studied for different locations of fault in the NE 39 bus system, shown in Fig 9. The MLE of the system is estimated by considering the phase angle of all the generator buses, assumed to be equipped with PMUs. The three phase to ground fault is simulated at the mid-point of the line joining buses 25 and 26 at 1s, and cleared at 1.35s for the first case and at 1.34s for the second case, leading to instability of the system for the first case. The phase angle of the generator buses are shown in Fig. 10, where the phase angle of the generator bus 38 diverges from the phase angle of the other generator buses swinging together, when the fault is cleared at 1.35s, while the fault clearing at 1.34s leads to the stable scenario, in which all the phase angle of the generator buses swing together, as shown in Fig 11.

The MLE, estimated from the phase angle of the generator buses, is shown in Fig. 12, for both the cases of fault clearing at 1.35s and 1.34s. The MLE estimated for the case of fault clearing at 1.35s is positive as the system becomes unstable whereas the MLE estimated for the fault clearing at 1.34s is negative as the system remains stable. The second scenario studied in the NE 39 bus system, is a line outage between buses 25 and 26, following a three phase to ground fault at the mid-point of the line joining buses 25 and 26 at 1s. The line is discharged at 1.3s and the fault is not cleared till then.

The phase angles of all the generator buses remain synchronized and swing together, which is shown in Fig. 13 (a). The system remains stable even after the outage of the
faulted line and the estimated MLE is negative, which is shown in Fig 13(b), estimated after the faulted line is isolated.

The three phase to ground fault is also created at the midpoint of the line joining buses 28 and 29 at 1s, which is cleared at 1.20s and 1.19s, considered as the two cases. The fault, when cleared at 1.20s, makes the system unstable as the phase angle of the generator bus 38 diverges from the phase angle of the rest of the generator buses in the system. The plot of the phase angle of all the generator buses after the fault is cleared at 1.20s and 1.19s are shown in Fig. 14 and Fig 15, respectively.

The unstable and the stable cases with the two different fault clearing time are detected by the MLE, as shown in Fig. 16. The unstable case is detected by positive value of the MLE and the stable case has negative value of the MLE after remaining slightly positive for a very short duration.

The three phase to ground fault is also created at bus 4 in the NE 39 bus system, which is cleared at 1.34s after the inception of the fault. The stability of the system is not disturbed in this case and the phase angles of all the generator buses remain in synchronism after the fault is cleared. The plot of the MLE is shown in Fig 17, which is observed to be negative after the fault is cleared.

The stability of the NE 39 bus system is also studied by creating a three phase to ground fault at bus 4 and subsequently clearing it 0.14s and 0.13s after its inception. The system has been observed to be unstable when the fault is cleared after 0.14s after its inception. The system remains stable for the case of fault clearing after 0.13s from its inception. The MLE for the case of the fault clearing at 1.14s is observed to be positive indicating the unstable scenario, whereas the case of fault clearing at 1.13s retains the stability of the system and the MLE is negative, as shown in Fig 18.

![Fig. 13.](image1.png)  
(a) Phase angle of generator buses for line outage post fault of 0.3s at line joining buses 25 and 26. (b) MLE for line outage post fault of 0.3s at line joining buses 25 and 26.

![Fig. 14.](image2.png)  
Phase angle of generator buses for a fault of 0.2s at line joining buses 28 and 29.

![Fig. 15.](image3.png)  
Phase angle of generator buses for a fault of 0.19s at line joining buses 28 and 29.

![Fig. 16.](image4.png)  
MLE for a fault of 0.2s and 0.19s at line joining buses 28 and 29.

![Fig. 17.](image5.png)  
MLE for a fault of 0.34s at bus 18.

![Fig. 18.](image6.png)  
MLE for a fault of 0.14s and 0.13s at bus 4.
I. CONCLUSION

The online estimation of Maximum Lyapunov Exponent (MLE) for both 9 bus and NE 39 bus systems have been demonstrated, in this work, for detection of impending transient instability. The faults are created at different locations in the two system and subsequently cleared at different time durations making the system stable for some cases and unstable for the remaining cases. The MLE is estimated from the phase angle of all the generator buses assumed to be measured by the PMUs after the fault is cleared and the transient stability of the system is determined within one second using the sign and the magnitude of the estimated MLE. The early indication of instability can be used by the system operator to initiate emergency controls or may trigger automatic sequence of protection mechanism to save the grid from possible blackout condition.

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