

Optimal Bidding Strategies of GENCO under Uncertain Information of Rivals using CVaR

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Abstract—Due to transmission and other constraints, competitive electricity markets are oligopoly instead of perfectly competitive. Hence, Gencos are required to bid strategically to get more profit but the rivals complete information is not available. A strategic producer is considered who trades in day-ahead electricity market with imperfect information of rivals obtained from the historical data. Using this information, an optimization framework is developed to devise optimal bids for maximum profit with minimum risk. Conditional value at risk (CVaR) is used for the risk due to uncertain information of rival's bids. The proposed method has been tested on a six generator example and optimal solution is obtained with efficient frontier between expected losses and CVaR.

Index Terms—Bidding strategies, Conditional value at risk, electricity market, expected loss, market clearing price.

I. INTRODUCTION

Global deregulation in the electricity power industry in most of the world has introduced the concept of a competitive electricity market. For successful market operation, it must provide a satisfactory balance between economic efficiency and security of supply. Electric power has various complex characteristics which makes the design of electricity markets difficult. Electricity cannot be stored and transmission is limited by physical and reliability constraints. It should be consumed instantly after its production. In new deregulated environment, electricity is traded similar to other commodities. However, electricity prices are substantially volatile than other commodity prices, because of its peculiar characteristics. Confronted with this severe price volatility, market participants need to find ways to protect their benefits i.e. to manage risks involved in the market.

In competitive electricity market, generators companies can sell their energy through bilateral contract, spot market, and real time market. The risk involved in these markets are different (i.e. low to high respectively). Hence, the first objective for participants is to optimize their portfolios which distribute their energy into different markets. Bilateral market (or forward market) is an over-the-table market, where prices are decided by mutual contracts and negotiation. Hence, risk involved in this market is almost zero. Risk with the real time market is very high, but we cannot prevent it by taking any

action because it is almost in the real time and participants don't have permission to change their decisions. So, we have considered the day-ahead market for our study and risk involved with this market.

In spot market, Gencos and consumers submit their supply and demand bids. Market Operator run market clearing process on these bids and provide the hourly MCPs and power quantities awarded to market participates. The profit of Gencos will depend on the successful scheduling of its units, bidding strategies, and on the hourly MCP. Bidding strategy of any generator is the tuples of energy price, different cost associated with generator and power limits. Hence, the bidding problem for generators can be defined as optimization of bidding strategies such that it will successfully scheduled into market and maximize its profit. But, the profit of a generator is also depended on the bidding strategies of other generators which is not known. While optimizing the bidding strategies of a generator, we have to consider the bidding behavior of rivals, which can be predicted by historical data.

The optimal bidding strategy problem was first addressed in [1] based on a conceptual optimal bidding model and a dynamic programming based approach was developed, for England-Wales type electricity markets. Each supplier is required to submit a constant price for each block of generation. System demand variations and unit commitment costs were considered in the model. Cooperative and non-cooperative game theory based approaches are proposed to analyze strategic behavior of suppliers in [2], [3]. In [4], this problem is described as a two-level optimization procedure. At the top level a centralized economic dispatch is employed to determine the market clearing price, the production and demand levels of all generators and consumers, and at the lower level a self-unit commitment based on a parametric dynamic programming with an embedded variable bidding parameter is used by each supplier to determine a profitable bid. An implicit assumption is that each supplier has complete information about rivals so that a centralized economic dispatch can be carried out by each supplier in designing bidding strategy. In [5], a two-level optimization procedure for building bidding strategies was presented in which market participants try to maximize their profits under the constraints that an independent system

operator (ISO) determines their dispatches and market price utilizing a transparent optimal power flow (OPF) program with an objective of maximizing social welfare. It is assumed that each participant has an estimated value for the bid from each of the other participants.

Strategic gaming model was developed for profit maximization subjected to anticipated reactions by rivals [6]. It is formulated by Mathematical Program with Equilibrium Constraints (MPEC). Probability distribution function was considered to predict other participant's behavior [7]–[9]. In [7], Lagrangian relaxation based method is used for optimal self-scheduling. Monte-Carlo simulation and analytical optimization methods are used to solve stochastic bidding problem with demand is considered as elastic in nature [8]. A two-level optimization with consideration of individuals risk profile has been solved [9]. The expected profit is calculated by Monte-Carlo and optimal bidding strategy is derived by GA.

A robust self-scheduling methodology based on conditional value-at-risk (CVaR) permits to manage risk under price uncertainty in [10]. Market price based unit commitment (PBUC) problem is focused in [11]. It was described how the market risks in operations can be measured and managed with a set of prices for fuel and electricity and the volatilities in those prices in a geometric Brownian motion price process. To account for price volatility, self-scheduling problems that include profit and risk simultaneously are considered in [12]–[14]. In [15], price uncertainty is modeled using fuzzy numbers in a procedure for maximizing a Gencos profit. The integration of day-ahead optimal bidding problem with virtual power plant auction has developed to decide optimal sale/purchase bids for thermal and generic units [16]. The risk for bidding strategies was quantified in a day ahead market with respect to expected profit [17]. An efficient frontier is evaluated to assess risk associated with a particular expected payoff. Uncertainty in power generation and consumed are also considered in bid optimization problems. A robust unit commitment is proposed in [18] to accommodate the renewable generation uncertainties. Optimal bidding strategies for independent power producers are devised using PBUC considering price and wind power output uncertainties [19]. The problem is formulated as a two-stage stochastic problem with chance constraints to ensure wind power utilization. A security constraints unit commitment problem is discussed [20] in which generation and real time demand variations are modeled as nodal net injection uncertainty.

The objective in this paper is to devised optimal bidding strategies for a generator when rivals bidding parameters are not known. This can be modeled as probability density function of bidding parameters obtained from historical data. Conditional Value-at-Risk will be used for the risk measurement which is not been used with uncertain rivals behavior. The profit maximization and market clearing algorithm, both will be solved as a single objective function which is previously solved in sequential manner. Demand elasticity is not considered under risk constrained bidding of generators.

The paper is organized as follows. The profit maximization

problem is formulated in Section II along with market clearing algorithm. The solution methodology by minimizing expected losses with CVaR risk constraint is presented in Section III. In Section IV, results on an example of a six generator test system are shown and concluded in Section V.

II. PROBLEM FORMULATION

Several independent Gencos compete into a pool market and submit bids to the market operator. Market operator runs auction mechanism on these bids and provide generation schedules along with the uniform MCP. The profit of any strategic generation company will depends on the bidding parameters of own as well as its rivals, which are not known to each other. In this papers, we have used the linear supply bids. These linear bids are the tuple of linear supply function and generation production limits. In perfect competitive market, the linear supply function should be marginal incremental cost of generator for maximum profit.

Let us consider, there are N independent generators those bid into pool market. The marginal incremental cost of i^{th} supplier at any time t is $MC_i(t) = a_i^0 P_i(t) + b_i^0$, where a_i^0 and b_i^0 are marginal cost coefficients and $P_i(t)$ is the generated power of supplier at the time interval t . A strategic supplier will offer either its marginal cost or higher cost than marginal into the market. Let the linear offer curve of i^{th} supplier is $OC_i(t) = a_i P_i(t) + b_i$, where a_i and b_i are bid coefficients. The minimum and maximum generation capacity of Gencos are $P_i(t)^{min}$ and $P_i(t)^{max}$ respectively. Other unit constraints i.e. startup cost, shutdown cost, ramp up/down limits, minimum up/down time limits, are not considered in this paper for simplicity. Now, the profit π_i of i^{th} Genco is written as:

$$\pi_i = \sum_{t=1}^{24} MCP(t) \cdot P_i(t) - Cost_i(P_i(t)) \quad (1)$$

where $MCP(t)$ is Market Clearing Price at time t which is obtained from auction. We consider an electricity market in which participants submit their bids into electricity auction for next day in day-ahead fashion. The market operator runs the market clearing algorithm by setting the market clearing price and provides the generation/consumption schedules to the market participants. The market clearing algorithm is formulated as

$$\min J = \sum_{t=1}^{24} \sum_{i=1}^N OC_i(t) P_i(t) \quad (2)$$

$$\text{s.t. } \sum_{i=1}^N P_i(t) = Pd(t) \quad \forall t \quad (3)$$

$$P_i^{min}(t) \leq P_i(t) \leq P_i^{max}(t) \quad \forall t, i \quad (4)$$

Here, $Pd(t)$ is the load demand at time t . The startup/shutdown costs are not included in uniform MCP of energy. It is charged according to turning ON/OFF status of generators. The profit maximization problem of any strategic generator i is to optimize (1) w.r.t. (2) to (4). It has been

solved using a two level optimization procedure in which at first level MCP, power schedules of generation and demand is determined by a centralized classical economic dispatch. At second level, profit maximization or loss minimization problem is solved to determine the profitable bids.

III. SOLUTION METHODOLOGY

The Gencos profit maximization problem can be solved as minimization of expected losses under risk constrained scenario. The risk involved in this problem due to uncertainties of rivals behavior has been modeled using a constraint on CVaR value. CVaR is used because it is a more consistent measure than VaR and it also provides the losses beyond the specified threshold value. VaR can be unstable and unmanageable for scenario based computations [21].

Let the bidding price of rivals can be predicted from historical data with probability distribution according to its density function with a mean μ and standard deviation σ . A Monte-Carlo simulation can be adopted to solve the problem having M samples. Let a collection of M vectors with $C^1(t), C^2(t), \dots, C^m(t), \dots, C^M(t)$, is generated at each time instants. Vector C consists the uncertain bidding parameters of rivals. This information is feed to market clearing algorithm (i.e. $OC_i(t) \in C^m(t)$) and $MCP^m(t)$ & $P_i^m(t)$ are obtained. Now, the hourly revenue obtained for i^{th} generator for every sample is:

$$R_i^m(t) = MCP^m(t) \cdot P_i^m(t) \quad (5)$$

The hourly loss $L_i^m(t)$ can be obtained from difference of cost and revenue for each hour. The daily losses L_i^m are the sum of hourly loss over the day for every sample.

$$L_i^m(t) = Cost_i(t) - R_i^m(t) \quad (6)$$

$$L_i^m = \sum_{t=1}^{24} L_i^m(t) \quad (7)$$

Hence, the expected value of the daily loss can be expressed as:

$$EL_i = \frac{1}{M} \sum_{m=1}^M L_i^m \quad (8)$$

The expectation of daily loss is dependent various variables of the market clearing process, hence dependent on the bidding parameters of rivals C , power output of generators $P_i(t)$ and market clearing price $MCP(t)$ as well as its own bidding strategy. To obtain the strategic bidding parameter, an optimization problem can be formed as the minimization of expectation of daily loss (i.e. $EL_i(x, C)$) with constraint on CVaR.

$$\min EL_i(x, C) \quad (9)$$

$$CVaR_\beta(x, C) \leq K_\beta \quad (10)$$

$$g(x) \leq 0 \quad (11)$$

where K_β is a constant and $g(x)$ is a system of functions that defines the feasibility region for he vector x . In this problem, x

vector have the power output of generator and strategic bidding parameter.

In [21], technique for optimizing portfolios with a risk constraint on CVaR is provided. The characterization of CVaR in terms of a function is defined as follows:

$$F_\beta(x, C, \alpha) = \alpha + (1 - \beta)^{-1} \int [L(x, C) - \alpha]^+ p(C) dC \quad (12)$$

where α is threshold value of loss, β is confidence level, $p(C)$ is probability of $L(x, C)$, and $[L(x, C) - \alpha]^+ = \max(0, L(x, C) - \alpha)$. The minimization of the function $F_\beta(x, C, \alpha)$ w.r.t. α gives the CVaR value.

$$CVaR_\beta(x, C) = \min_{\alpha} F_\beta(x, C, \alpha) \quad (13)$$

IV. RESULTS

The proposed method is simulated in MATLAB. Data for the test system example is taken from [8], where six producers are competing in an electricity market. The production cost coefficients and power generation limits of all six producers are given in Table I. The load demand is assumed to be given constant and equals to 600MW.

TABLE I
COST COEFFICIENTS AND OUTPUT LIMITS OF GENERATORS

No.	b_i	c_i	P_i^{min}	P_i^{max}
1	2	0.00375	20	160
2	1.75	0.0175	15	180
3	1	0.0625	10	120
4	3.25	0.00834	10	100
5	3	0.025	10	130
6	3	0.025	10	130

In perfectly competitive market, each producer offer its energy bids on its marginal cost for maximum benefit. Hence, we assume that each producer will bid either marginal cost function or higher than its marginal cost function. A strategic producer can estimate its rival's bidding parameters from historical data using available mathematical methods. Lets assume that second generator is the strategic generators whose bids we will optimize. It has the information of its rival's bidding parameters which are assumed to have an joint normal distribution specified same as in [8]. The expectation values (μ) and standard deviation (σ) of j^{th} rival's bidding parameters (a_j & b_j) are specified to make these fall into the domains $[1.05b_j, 1.35b_j] = [\mu_j^b - 4\sigma_j^b, \mu_j^b + 4\sigma_j^b]$ and $[1.05c_j, 1.35c_j] = [\mu_j^c - 4\sigma_j^c, \mu_j^c + 4\sigma_j^c]$, respectively with a probability of 0.9999. Hence,

$$\begin{aligned} \mu_j^b &= 1.2b_j, & \mu_j^c &= 1.2c_j \\ 4\sigma_j^b &= 0.15b_j & \Rightarrow & \sigma_j^b = 0.15b_j/4 \\ 4\sigma_j^c &= 0.15c_j & \Rightarrow & \sigma_j^c = 0.15c_j/4 \\ \rho_j &= -0.1, & j &= 1, 3, 4, 5, 6 \end{aligned} \quad (14)$$

Here ρ_j is the correlation coefficient between b_j and c_j .

Lets the strategic producer decides to fix its bidding parameter b_2 and wants to optimize c_2 . Simulation has been

carried out on 5000 samples of rivals bidding parameters. The expected losses of strategic producer for all samples are normally distributed as in Fig. 1. The bidding parameters which is to be optimized, is varied from marginal value to five times of it because producer will likely to bid above its marginal cost value. The variation of expected losses and CVaR with respect to bidding parameters are drawn in Fig. 2.

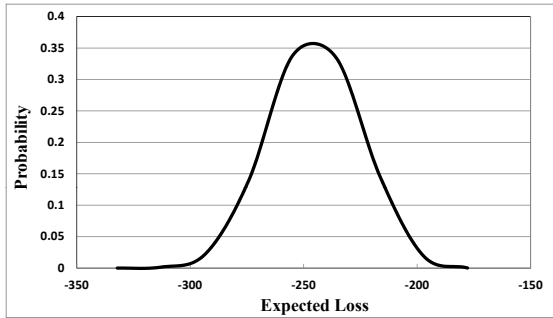


Fig. 1. Distribution of Expected losses of strategic generator

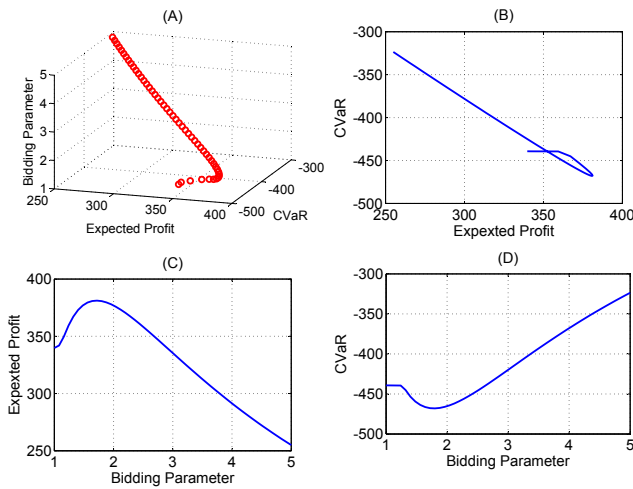


Fig. 2. Variation of Expected profit and CVaR with bidding parameter

It is observed from the results that with increase in the bidding parameter expected loss and CVaR both decreases, *i.e.* expected profit is increasing with minimizing risk. If bidding parameter is further increased (after point C with $k=1.72$), expected profit/loss start decreasing/increasing with CVaR is increasing. Hence, point C is our optimal point and the bidding parameter $c_1 = k \times c_1^0 = 0.0301$ is optimal bid offered by strategic producer.

V. CONCLUSION

A methodology has been developed to devise optimal bids for power producers with uncertain information of its rivals in a day-ahead electricity market. The expected profit is optimized along with risk constraints in which CVaR is considered as risk measure. As CVaR provides a single number, which could encapsulate information of many sensitivity factors like

uncertainty of many rivals. This single value would lead to simplicity of calculation and can easily be implemented by means of computational devices with fast response.

An optimization based method has been presented which minimize the expected losses of strategic producer along with the CVaR. The market participants bids linear supply function and market clearing algorithm has been applied to these bids to get power schedules and MCP for day-ahead market. A six suppliers example has been used for demonstration and it is observed that the method provides optimal bidding strategy with maximum profit and minimum risk. With better estimation of rivals information, more optimal bidding strategy can be devised. Demand side participation and unit commitment constraints are not considered which could be included easily in the market clearing algorithm.

REFERENCES

- [1] A. David, "Competitive bidding in electricity supply," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 140, no. 5, pp. 421–426, Sep 1993.
- [2] R. W. Ferrero, S. M. Shahidehpour, and V. C. Ramesh, "Transaction analysis in deregulated power systems using game theory," *Power Systems, IEEE Transactions on*, vol. 12, no. 3, pp. 1340–1347, Aug 1997.
- [3] R. W. Ferrero, J. F. Rivera, and S. M. Shahidehpour, "Application of games with incomplete information for pricing electricity in deregulated power pools," *Power Systems, IEEE Transactions on*, vol. 13, no. 1, pp. 184–189, Feb 1998.
- [4] C. Li, A. Svoboda, X. Guan, and H. Singh, "Revenue adequate bidding strategies in competitive electricity markets," *Power Systems, IEEE Transactions on*, vol. 14, no. 2, pp. 492–497, May 1999.
- [5] J. Weber and T. Overbye, "A two-level optimization problem for analysis of market bidding strategies," in *Power Engineering Society Summer Meeting, 1999. IEEE*, vol. 2, 1999, pp. 682–687 vol.2.
- [6] B. Hobbs, C. Metzler, and J.-S. Pang, "Strategic gaming analysis for electric power systems: an mpec approach," *Power Systems, IEEE Transactions on*, vol. 15, no. 2, pp. 638–645, May 2000.
- [7] D. Zhang, Y. Wang, and P. Luh, "Optimization based bidding strategies in the deregulated market," *Power Systems, IEEE Transactions on*, vol. 15, no. 3, pp. 981–986, Aug 2000.
- [8] F. Wen and A. David, "Optimal bidding strategies and modeling of imperfect information among competitive generators," *Power Systems, IEEE Transactions on*, vol. 16, no. 1, pp. 15–21, Feb 2001.
- [9] V. Gountis and A. Bakirtzis, "Bidding strategies for electricity producers in a competitive electricity marketplace," *Power Systems, IEEE Transactions on*, vol. 19, no. 1, pp. 356–365, Feb 2004.
- [10] R. Jabr, "Robust self-scheduling under price uncertainty using conditional value-at-risk," *Power Systems, IEEE Transactions on*, vol. 20, no. 4, pp. 1852–1858, Nov 2005.
- [11] M. Denton, A. Palmer, R. Masiello, and P. Skantze, "Managing market risk in energy," *Power Systems, IEEE Transactions on*, vol. 18, no. 2, pp. 494–502, May 2003.
- [12] A. Conejo, F. Nogales, J. Arroyo, and R. Garcia-Bertrand, "Risk-constrained self-scheduling of a thermal power producer," *Power Systems, IEEE Transactions on*, vol. 19, no. 3, pp. 1569–1574, Aug 2004.
- [13] H. Yamin and S. Shahidehpour, "Risk and profit in self-scheduling for genscos," *Power Systems, IEEE Transactions on*, vol. 19, no. 4, pp. 2104–2106, Nov 2004.
- [14] E. Caruso, M. Dicorato, A. Minoia, and M. Trovato, "Supplier risk analysis in the day-ahead electricity market," *Generation, Transmission and Distribution, IEE Proceedings-*, vol. 153, no. 3, pp. 335–342, May 2006.
- [15] H. Yamin, "Fuzzy self-scheduling for genscos," *Power Systems, IEEE Transactions on*, vol. 20, no. 1, pp. 503–505, Feb 2005.
- [16] F. Heredia, M. Rider, and C. Corchero, "Optimal bidding strategies for thermal and generic programming units in the day-ahead electricity market," *Power Systems, IEEE Transactions on*, vol. 25, no. 3, pp. 1504–1518, Aug 2010.

- [17] R. Karandikar, A. Abhyankar, S. Khaparde, and P. Nagaraju, "Assessment of risk involved with bidding strategies of a genco in a day ahead market," *Int. J. Emerg. Elect. Power Syst.*, vol. 1, no. 1, Sep 2004.
- [18] R. Jiang, J. Wang, and Y. Guan, "Robust unit commitment with wind power and pumped storage hydro," *Power Systems, IEEE Transactions on*, vol. 27, no. 2, pp. 800–810, May 2012.
- [19] Q. Wang, J. Wang, and Y. Guan, "Price-based unit commitment with wind power utilization constraints," *Power Systems, IEEE Transactions on*, vol. 28, no. 3, pp. 2718–2726, Aug 2013.
- [20] D. Bertsimas, E. Litvinov, X. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *Power Systems, IEEE Transactions on*, vol. 28, no. 1, pp. 52–63, Feb 2013.
- [21] R. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *J. Risk*, vol. 2, no. 3, pp. 21–41, 2000.