Transmission Embedded Cost Allocation using Proportional Nucleolus based Game Theoretic Approach in a Restructured Power Market

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***Abstract*—In deregulated electricity markets there is a strong need for effective allocation of transmission embedded costs to market participants. The conventional usage based methods such as MW-Mile and ZCF methods which are currently employed in market scenario may fail to send right economic signals. Hence in this paper, cooperative game theory based approaches are demonstrated. The existing game theory based approaches like Nucleolus and Shapley Value methods are found to be inefficient for transmission embedded cost allocation, due to their own pros and cons. Therefore Proportional Nucleolus (P-N) method which is also a cooperative game theory approach is proposed in this paper to overcome the drawbacks of aforementioned methods. All the methods presented in this paper are tested on IEEE 14 bus system, and a comparative study was carried out with the obtained results.**

Keywords—core; coalition; cooperative game; imputation; Nucleolus; payoff; Proportional Nucleolus; Shapley.

I. INTRODUCTION

In the present day power markets, the issue of allocating fixed costs to the market participants is of great significance. Fixed costs make up the largest part of transmission charges; hence there is a great demand for a fair and effective allocation of these costs to the market participants [1, 2, 3]. Different allocation schemes have been formulated in recent years based on the "natural economic use" of the transmission system [1],[10],[12],[17-18]. In [19] three variations of the MW-Mile method for pricing counter flows are investigated for the cost allocation method. But it failed in providing incentives to users of the grid who causes counter flows. Therefore all the conventional usage based methods like MW-Mile method, Zero counter flow (ZCF) method are advantageous from an engineering point of view, but they may fail to send right economic signals. The fixed cost allocation [4] is a typical case where cooperation between the agents produces incentives and economies of scale. These benefits can in turn be shared among the network participants. Thus the cooperative game theory (C.G.T) provides interesting concepts, methods and models that may be used when assessing the interaction of different agents in competitive markets and in the solution of conflicts that arise in that interaction, such as those of the electricity markets [12]. In particular, cooperative game theory arises as a most convenient tool to solve cost allocation problems [12]. The solution methodologies of cooperative game theory behave well in terms of fairness, efficiency, stability, and qualities required for the correct allocation of transmission costs [13]. C.G.T also suggests reasonable fixed cost allocations that may be economically efficient as well as advantageous from engineering point of view [11]. Hence in this paper three C.G.T methods namely Nucleolus, Shapley and Proportional Nucleolus are attempted for transmission fixed cost allocation problem. Proportional Nucleolus method is proposed in this paper which is already in use for transmission loss cost allocation [18] problem.

In this paper, section II gives the basic introduction of Game theory. Section III discusses about different methods of C.G.T. Section IV presents usage based methods for transmission fixed cost allocation. Section V details about general algorithm of C.G.T and presents a case study on IEEE 14 bus system. Section VI concludes the paper.

II. GAME THEORY

Game Theory is the formal study of decision making where several players must make choices that potentially affects the choice of other players. The Game theory is a kind of mathematical analysis designed to predict what the outcome or to predict the most likely result of a dispute between two individuals [17]. Game theory deals with any problem in which each player's strategy depends on what other players do. It is assumed that the rationality of all players is of common knowledge. A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. Payoff is nothing but the payment received at the end of game. It is mainly employed in power systems to prevent collusion due to market power i.e. discourage collusions that could minimize payoff [5]. The game theory methodologies can be used to identify non-competitive situations (from market co-coordinator point of view) and minimize the risks in price decisions (from participant's point).

A. Characteristics of a Game

In order to play a game, the players must exhibit the following characteristics [17]:

1. *Symmetry:* Two players, A and B, are symmetrical if all coalitions in which they can participate are met:

$$v(S \cup \{A\}) = v(S \cup \{B\}) \forall S \subset N \ \ such\ that\ \ A, B \notin S \qquad (1)$$

2. *Attractiveness:* One player, A, is more desirable than another player, B, if it satisfies:

$$v(S \cup \{A\}) \geq v(S \cup \{B\}) \forall S \subset N \ \ such\ that\ A, B \notin S \qquad (2)$$

3. *Additivity:* If two or more coalitions agree to co-operate with each other then the payoff with the union can guarantee itself should be at least equal to the sum of their individual payoffs. That is

$$v(S \cup T) \geq v(S) + v(T), \forall S,T \subset N, \ if \ S \cap T = \phi \quad (3)$$

4. *Inessential :* For all the subsets of N if the equality holds in (3), then it is indifferent for the players to form any coalition and the game is called inessential.

$$v(S \cup T) = v(S) + v(T), \forall S,T \subset N, \ if \ S \cap T = \phi \quad (4)$$

In this case the characteristic function 'v' is just additive. For the inessential game it is:

$$v(N) = \sum_{i=1}^{n} v(i) \quad (5)$$

where n is the number of players in the game.

5. *Monotony:* Monotony means that when the characteristic function value v(S) of a coalition increases then the payoff to the members of this coalition is getting larger.

III. Solution Methodologies in Cooperative Game Theory

A. Terminology

Consider a game of N players with a characteristic function v. These players can form 2^N coalitions including the φ (null) coalition. The characteristic function v(S) assigns to each coalition say 'S' is the minimum payoff under any adverse conditions. This can be found by applying max-min criteria to S and (N-S) players. The set of all possible distribution of payoffs to the participants are called Imputations. A payoff vector $y = (y_1, y_2, ...y_n)$ is an imputation if it holds the following two conditions i.e., global rationality given by (6) and individual rationality given by (7).

$$\sum_{i=1}^{n} y_i = v(N) \quad (6)$$

$$y_i \geq v(i), \ i = 1, 2, ... \ n \quad (7)$$

There are numerous methods for the allocation of benefits among the participants or players of a cooperative game. Some of them are briefly described below:

B. The Core

One of the first solutions suggested for cooperative game is the core concept [7]. It is based on domination of imputations. That is, the core of a game is the set of all the imputations that are not dominated over any coalition.

For an imputation to belong to the core, it must satisfy global rationality and coalitional rationality given by (9).

$$\sum_{i=1}^{n} y_i = v(N) \quad (8)$$

$$\sum_{i=1}^{n_s} y_i \geq v(S) \ \ \forall S \subset N \quad (9)$$

where n_s is the number of players in coalition 'S'.

It is clear that the core may include one or more than one imputation or may be even empty. Thus to choose a single solution whenever the core is non empty, Nucleolus concept was introduced in [6, 8].

C. The Nucleolus

It is based on the idea of minimizing the dissatisfaction of the most dissatisfied groups. For a coalition S, measure of its dissatisfaction is the excess e(S):

$$e(S) = v(S) - y(S) \quad (10)$$

where $y(S) = \sum_{i=1}^{n_s} y_i$

Thus the larger the excess, the more dissatisfied the coalition is with this Imputation. Thus it reduces to the following optimization problem.

$$\min e \quad (11)$$

$$y(S) + e \geq v(S) \quad (12)$$

$$y(N) = v(N) \quad (13)$$

One main drawback of Nucleolus is that it is not monotonic i.e., even though the characteristic function *v*(S) of a coalition S is increased, the payoff to the members of this coalition is not affected.

D. The Shapley Value

For the foundation of Shapley value [8-9], [14-15] three axioms have to be settled.

i. Symmetry: $\phi_i(v)$ is independent of the labeling of the players.

$$\phi_{\Pi(i)}(v) = \phi_i(v) \quad (14)$$

ii. Efficiency: The sum of the expectations must be equal to the characteristic functional value for the grand coalition N.

$$\sum_{i=1}^{n} \phi_i(v) = v(N) \quad (15)$$

iii. Additivity: The sum of expectations, for a player, by playing two games with characteristic values v_1 and v_2 must be equal to the value if he played both games together.

$$\phi_i(v_1 + v_2) = \phi_i(v_1) + \phi_i(v_2) \quad (16)$$

Thus the Shapley value which satisfies three axioms is given by

$$\phi_i(v) = \sum_{S, i \in S} \frac{(n_s - 1)!(n - n_s)!}{n!} [v(S) - v(S - \{i\})] \quad (17)$$

Its main advantage is it exhibits monotonicity. However its main disadvantage is that it may or may not lie inside the core.

E. Proportional Nucleolus

This solution concept coincides with the cases where core is nonempty. It is an important characteristic of extended core solution concept. It gains greater importance, as there are considerable number of games in which the core concept cannot be applied. As the extended core is a multiple valued concept, it is important to find a unique solution among its imputations. Hence Proportional Nucleolus (P-N) method is proposed in this paper for transmission fixed cost allocation problem, which is previously used for transmission loss cost allocation problem [18]. P-N method always chooses an imputation from the extended core in a similar way to the concept of Nucleolus. P-N method differs from the original Nucleolus in the definition of excess concerned with coalitions. It is defined as

$$e(Y:S) = \frac{v(s) - \sum_{i \in s} y_i}{v(S)} \tag{18}$$

The Proportional Nucleolus can expand the core to obtain a unique solution in both cases of empty core and large core. Thus the P-N method provides a better solution to both the extended core and core selection problem. The solution approach for P-N is to solve a linear program of the following problem formation:

$$Min\ e \tag{19}$$

subjected to

$$\sum_{i \in S} y_i \geq v(S)\ (1-e) \tag{20}$$

$$\sum_{i \in N} y_i = v(N) \tag{21}$$

The P-N of a game satisfies the properties namely:

- non emptiness
- single valuedness
- solution belonging to extended core.

P-N solution always lies inside the core and it is always monotonic as the excess value is proportional to the characteristic functional value. Thus it is the most efficient and plausible method among all the discussed game theoretic approaches.

IV. Fixed Cost Allocation using Usage Based Methods

A. The MW-Mile Method

MW-Mile method takes into account the transacted power flow on all transmission lines, it can reflect not only the amount of wheeled energy, but also the path and distance of transfer. However this method does not consider the economies of scale (The cost advantage that arises with increased output of a product. Economies of scale arise because of the inverse relationship between the quantity produced and per-unit fixed costs; i.e. the greater the quantity of a good produced, the lower the per-unit fixed cost because these costs are shared over a larger number of goods) of transmission network facilities and does not argue the stability of the solution.

For each transaction 'i'

$$f_{i,l} = C_l \left| P_{i,l} \right| \tag{22}$$

where, C_l = specific transfer cost of branch 'l'

$f_{i,l}$ =use of branch 'l' by participant 'i'

$P_{i,l}$ = power flow on branch 'l' by participant 'i'

Thus the total network usage for 'nl' number of lines is given by

$$f_i = \sum_{l=1}^{nl} f_{i.,l} \tag{23}$$

Thus the cost allocation by MW-Mile method is given by

$$MWM_i = K * (f_i \Big/ \sum_{j=1}^{n} f_j) \tag{24}$$

where 'K' is the total fixed cost to be allocated.

The drawback of this method is, it does not consider the direction of line flow.

B. The Zero Counter Flow Method

MW-Mile method does not consider the direction of power flow of each transaction. However, it is often argued that power flows having opposite direction from the net flow (the power flow due to all transactions) contribute positive in the system situation by relieving congestions and increasing the Available Transfer Capacity. Using Zero Counter Flow method transmission users are charged or credited based on whether their transactions lead to flows or counter flows with regard to the direction of net flows. The method suggests that if a particular transaction results in flows in the opposite direction of the net flow, then the transaction should be credited. Hence to accommodate this concept, Zero Counter Flow (ZCF) method is introduced. According to this method, the usage of a line by a particular transaction is set to zero if the power flow due to the transaction goes in the opposite direction of the net flow for the line.

Thus the change for each transaction 'i' is given as

$$f_{i,l} = \begin{cases} C_l P_{i,l} & P_{i,l} > 0 \\ 0 & P_{i,l} \leq 0 \end{cases} \tag{25}$$

Thus the total network usage is given by

$$f_i = \sum_{l=1}^{nl} f_{i.,l} \tag{26}$$

Thus the cost allocation by ZCF is given by

$$ZCF_i = K * (f_i \Big/ \sum_{j=1}^{n} f_j) \tag{27}$$

But this method may fail to send right economic signals, i.e. it is well established from engineering point of view but subsidizes the largest network users with comparatively smaller users due to the counter flows of former. The savings due to counter flows are not allocated as payoffs to participants, which is a major drawback of ZCF method.

Hence to overcome the drawbacks of usage based methods, Game theory based methods are attempted in this paper.

V. Fixed Cost Allocation using Cooperative Game Theory and a Case Study on IEEE 14 Bus System

A. Game Definition

Many of the fixed cost allocation methods are based on the network usage from the side of market participants. The payment R_i allocated to each participant 'i' or player 'i' may be given by one of the following forms:

$$R_i = K * (f_i / \sum_j f_j) \quad (28)$$

$$R_i = f_i b \quad (29)$$

where b= cost of 1 MW power flow

B. General Algorithm

Step1: Consider the number of possible coalitions that can be formed using the players (n) of the game.

Step2: Run DCOPF for each transaction 'i' and then calculate corresponding fixed cost f_i.

Step 3: Calculate characteristic function v(S) of each coalition

$$v(S) = (\sum_{i=1}^{n_s} f_i) - f_s \quad (30)$$

where f_s = usage of the network by coalition S

From (30) it is explicit that the characteristic function represents the savings that can be achieved in case of cooperation. It is obvious that for individual player i, it is v(i) = 0.

Step4: Using Nucleolus, Shapley Value and Proportional Nucleolus methods allocate the savings to all the players i.e., payoffs of the players y_i arose from the solution of the game.

Step 5: These payoffs are resulting in a reduction of f_i for each player:

$$f_i' = \begin{cases} f_i - y_i & if \ f_i > y_i \\ 0 & if \ f_i < y_i \end{cases} \quad (31)$$

where f_i' is the new use of network by player i. If the savings assigned to player i are larger than the original f_i then the f_i' is set at zero. Thus, a player does not have the opportunity to receive money back from the network operator. The reason of making this adjustment is to prevent the misuse of game from the side of players.

Step 6: Calculate the amount that player i has to pay. The cost allocation is done using the given formula

$$R_i = K \frac{f_i'}{\sum_{j=1}^{n} f_j'} \quad (32)$$

When the electricity market operates in an environment of bilateral transactions then each transaction agent or player is responsible to pay a part of power system fixed cost. The formulation of a coalition between some players can be profitable by the existence of counter flows.

C. Case Study on IEEE 14 Bus System

The above algorithm is implemented on IEEE 14 bus system [16]. The loads are aggregated based on their Locational Marginal Prices (LMP) and then 4 transactions are formed in the system. The generator power outputs are obtained by running DCOPF and thus the obtained transactions (Players) are as given in Table I.

TABLE I. Transaction data of IEEE 14 Bus System

Transaction / Player (i)	Load demand (MW)	S (j, k)	B (i)
1	29.3	(1 → 24.070508) , (2 → 5.229492)	2,5
2	142	(1 → 75.247070) , (2 → 66.752930)	3,4
3	30.8	(1 → 19.452344) , (2 → 11.347656)	6,12,13
4	56.9	(1 → 21.694922) , (2 → 35.205078)	9,10,11,14

where S(j,k) = Bus 'j' supplying load 'k' for transaction 'i'. B(i) = Load Buses. In the above table, row 1, the first transaction comprises of a total load of 29.3 MW (buses 2 and 5 are grouped together based on their LMP's evaluated using Power World Simulator). This load is met by both generators with 1st generator generating 24.07 MW where as 2nd generator 5.23 MW.

By running a DCOPF for each transaction, the network usage and characteristic functional values of each coalition are obtained considering counter flows and are presented in table II. The last row shows the grand coalition in which all players are present, which assures maximum savings.

From table II for coalition 5:

Players 1 and 2 forms coalition.

$f_s = 353.8507$

$f_1 = 31.1526$

$f_2 = 326.3217$

v(S) = $(f_1 + f_2) - f_s$ = (31.1526 + 326.3217) - 353.8507 = 3.6236 €

Similarly v(S) is calculated for each coalition. v(S) is the minimum amount which the coalition can assure itself. v(S) value obtained for grand coalition in table II is the maximum total savings i.e., 68.5833 € which is allocated to players in the game as their payoffs.

In table II network usage values for each coalition are calculated by (26). v(S) values are calculated by (30).

Next is to calculate the minimum and maximum values of payoffs y_i. $y_{i\ \min}$ is taken as v(S) when player 'i' acts alone i.e., zero for all 4 players. $y_{i\ \max}$ is taken as $v(S \cup \{i\}) - v(S)$ [1].

TABLE II. CHARACTERISTIC FUNCTIONAL VALUES OF IEEE 14 BUS SYSTEM

Sl. no.	Coalition	f_s	v(S)
1	1 0 0 0	31.1526	0
2	0 1 0 0	326.3217	0
3	0 0 1 0	133.0911	0
4	0 0 0 1	230.2376	0
5	1 1 0 0	353.8507	3.623
6	1 0 1 0	161.3797	2.864
7	1 0 0 1	258.0073	3.382
8	0 1 1 0	433.6315	25.78
9	0 1 0 1	538.2311	18.32
10	0 0 1 1	320.8602	42.46
11	1 1 1 0	461.7372	28.82
12	1 1 0 1	566.2091	21.50
13	1 0 1 1	348.8808	45.60
14	0 1 1 1	623.9727	65.67
15	1 1 1 1	652.2197	68.58

For player 1:

$$y_{1\ \max} = v(15) - v(14) = 2.9056$$

Similarly for the remaining 3 players maximum limits are determined and are shown in table III.

TABLE III. MAXIMUM LIMIT OF PAYOFFS

Player	y_i max
1	2.9056
2	22.9828
3	47.0805
4	39.7551

The payoffs and the new usage of network of player 'i' obtained by Nucleolus, Shapley Value and Proportional Nucleolus methods are shown in tables IV, V and VI.

From tables IV, V and VI, it is observed that the sum of the payoffs of 4 players is equal to v(S) of grand coalition in table II. That means the payoffs satisfied the condition shown in (6). New usage of network by player 'i' is $f_i^{'}$ and is calculated using (31). From these tables IV, V and VI, the total network usage by all the 4 players is equal to the value obtained for f_i of grand coalition value in table II. This indicates that when the 4 players acting individually the total network usage is 720.803 € whereas when 4 players forms a grand coalition the total network usage is reduced to 652.2197 €. Finally the allocations to players can be computed by (32).

TABLE IV. PAYOFFS AND NEW NETWORK USAGE OF 4 PLAYERS IN NUCLEOLUS METHOD

Player	y_i	$f_i^{'}$
1	1.45	29.7026
2	3.62	322.7017
3	45.63	87.4611
4	17.88	212.3576
total	68.583	652.223

TABLE V. PAYOFFS AND NEW NETWORK USAGE OF 4 PLAYERS IN SHAPLEY VALUE METHOD

Player	ϕ_i	$f_i^{'}$
1	2.3284	28.8242
2	15.3312	310.9905
3	27.2605	105.8306
4	23.6630	206.5746
total	68.5831	652.2199

TABLE VI. PAYOFFS AND NEW NETWORK USAGE OF 4 PLAYERS IN PROPORTIONAL NUCLEOLUS METHOD

Player	y_i	$f_i^{'}$
1	0.0	31.1526
2	3.78	322.5417
3	26.32	106.7711
4	38.48	191.7576
total	68.58	652.223

Next step is to calculate the total fixed cost to be covered by the market participants i.e., K. This cost is calculated by multiplying the power flows with their corresponding line lengths and line costs. Table VII shows the allocation of K = 2773.35 € to four players with all the above discussed methods.

TABLE VII. COST ALLOCATION USING VARIOUS METHODS IN IEEE 14 BUS SYSTEM

Player	MWM(€)	ZCF(€)	Nucleolus(€)	Shapley Value(€)	Proportional Nucleolus (€)
1	104.50	119.86	126.28	122.56	121.13
2	1340.23	1255.54	1372.16	1322.38	1331.65
3	528.08	512.07	371.91	450.01	464.96
4	800.52	885.85	902.98	878.39	855.59

Players are in the ascending order of 1,3,4,2 w.r.to demand as can be seen from table I. 3rd player utilizes more network compared to 4th player because 3rd player accounts for higher line lengths than 4th player. 4th player got 902.9843 € by Nucleolus method & 878.3903 € by Shapley value method, whereas by P-N method the cost is still reduced to 855.5968 €. 3rd player got 371.9095 € by Nucleolus method and 450.0096 € by Shapley value method whereas by P-N method this player got 464.9632 €. As 3rd player uses more network, the cost is increased by P-N method. Remaining cost is allocated to other 2 players. The comparison graph is shown in Fig. 1 further supports the results obtained using various methods.

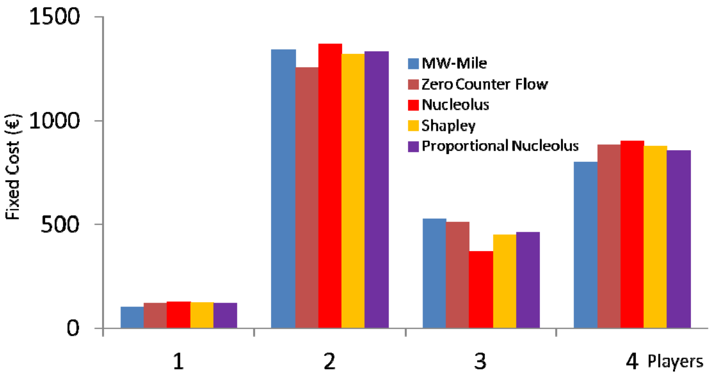

Fig.1. Cost allocation using different methodologies for IEEE 14 bus system

From Fig.1, we can infer that the cost allocated using Game Theory is in tune with other fixed cost allocation methods.

VI. Conclusions

In this paper cooperative game theory is implemented for power system fixed cost allocation in a transaction based market model in an equitable manner and the results obtained are compared with conventional usage based methods like MW-Mile method and Zero counter flow method. The study of the cost allocation is of high importance and it is at the origin of discussion and study all over the world. Since players behave in rational way, the cost allocation problem becomes a matter of conflict. Cooperative game theory is used to deal with such matters of conflict. In MW-Mile method counter flows are not accounted. In ZCF method counter flows are accounted but the savings due to counter flows are not allocated to players which could be achieved with game theory methods. Hence game theory methods give correct economic signals about the allocations of transmission fixed cost to players in the system. In the case of a pool market, concerning the whole system, there is no obstacle for such an implementation. However, negative characteristic function values may arise if the game is played at each system branch. For a bilateral transaction market, the fixed cost allocation can take place in the entire network as well as at each single branch. All the results of IEEE 14 bus system satisfy individual, coalition and global rationalities. Nucleolus is not monotonic, solution always lies within the core if the core is non-empty and it may favor some players only. If the core is empty Nucleolus method cannot produce solution. Shapley value method is monotonic and always assigns a non zero payoff to the players. But the solution with Shapley Value method may or may not lie within the core. Hence Proportional Nucleolus method is proposed for fixed cost allocation problem in this paper to overcome the drawbacks of Nucleolus and Shapley Value methods. P-N method is also monotonic and the solution is always lies within the core for both empty and non empty core cases due to the extended core concept used in P-N method. Due to its inherent property of extended core concept, a better solution is obtained by P-N method in the presented case study.

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