LMP in Single Auction Model of a Restructured Power System using Bat Algorithm

1Murali Matcha, 2Matam Sailaja Kumari, Member, IEEE, and 3Maheswarapu Sydulu, Member, IEEE
1Department of Electrical Engineering, N.I.S.T Brahmmapur, Odisha, India
2,3Department of Electrical Engineering, N.I.T Warangal, Telangana, India
1murali233.nitw@gmail.com

Abstract — In restructured electricity markets, an effective transmission pricing method is required to address transmission issues and to generate correct economic signals. Transmission line constraints can result in variations in energy prices throughout the network. These prices depend on generator bids, load levels and transmission network limitations. A congestion charge is incurred when the system is constrained by physical limits. Locational Marginal Pricing (LMP) has become popular method in restructured power markets to address the congestion charges. This paper presents LMP computation with LP, GA and proposed Bat algorithm (BA) under three different loss cases in single auction model. In single auction model only suppliers submit the bids and the load is assumed to be inelastic. Fixed and Linear bids are considered for generators. All the methods are tested on IEEE 14 bus system, New England 39 bus system and Indian 75 bus system. Linear bids with BA show most optimal fuel cost compared to all other methods.

Keywords — Delivery factors; Fixed bid; Generation shift factors; Genetic Algorithm; Linear bid; Bat algorithm

I. INTRODUCTION

In April 2003 White Paper [3] the U.S. Federal Energy Regulatory Commission (FERC) proposed a market design for common adoption by U.S. wholesale power markets. The electric power industry has undergone deregulation around the world, a core tenet of which is to build an open-access, unambiguous and fair electricity markets. Proper and fair pricing of real power is an important issue in this competitive market. Core features of a market design include; a two settlement system consisting of a day-ahead market supported by a real-time market to ensure continual balancing of supply and demand for power; and grid congestion management by means of nodal pricing.

Under a deregulated electricity market environment, transmission networks hold a vital role in supporting the transaction between producers and consumers. One drawback of transmission constraint is congestion. Congestion occurs when transmission lines or transformers operate at or above its thermal limits and this prevents the system operators from dispatching additional power from a specific generator. Congestion can result in an overall increase in the cost of power delivery. Presently there are two pricing structures [2] that are being used in a competitive energy market to account for congestion: the uniform pricing method i.e., market clearing price (MCP) and the nonuniform pricing method i.e., LMP. In the first method, all generators are paid the same price (MCP) based on the bid of the marginal generator that would be dispatched in the absence of congestion. The second method has been the basic approach in power markets to calculate the LMPs and to manage transmission congestion. LMP is the additional cost for providing one additional MW at certain node. Buyers pay ISO, based on their nodal price for the dispatched energy. The ISO pays sellers, based on their respective nodal prices. The difference in LMPs between two adjacent buses is the congestion cost which arises when the energy is transferred from one location (injection) to the other location (withdrawal). Marginal losses represent incremental changes in system losses due to incremental demand changes. Incremental losses yield additional costs which are referred to as the cost of marginal losses [5]. Thus LMP is the summation of the costs of marginal energy, marginal losses and congestion. Therefore LMP at bus B is stated as follows:

\[ LMP_B = LMP_{energy} + LMP_{cong} + LMP_{loss} \] (1)

In this decomposition model, LMP congestion component at bus B, i.e., LMP_{cong} remains invariant w.r.t different reference buses, and the combination of the other two components, i.e., LMP_{energy} + LMP_{loss}, is also reference-independent. But each of LMP_{energy} or LMP_{loss} is still reference-dependent [12].

In market planning and simulation, DC model is desired due to its robustness and speed. DCOPF is broadly employed by a number of industrial LMP simulators, such as ABB’s Gid ViewTM, GE’s MAPSTM, Siemens’ Promod IVR, and Power World [4], [1]. Several papers have reported different models for nodal price calculation. Reference [5] presented different methods and properties on LMP calculations based on DCOPF with and without loss. Refs. [7], [9] gives a systematic description on how the LMP’s are produced; it also described both the modelling and implementation challenges and solutions. Reference [8] described ACOPF based LMP calculation considering distributed loss. Reference [6] demonstrated an iterative DCOPF based algorithm with lossless model, considering marginal losses, and with fictitious nodal demand model to calculate nodal price. All these 3 models are solved with linear programming.

In this paper a detailed explanation about without loss, concentrated loss, and distributed loss cases for LMP computation is given. In without loss case, system losses are ignored. In concentrated loss case, total system loss is supplied by the slack bus which creates a burden on the slack bus. To eliminate a large mismatch at the slack bus, loss is distributed to all buses as an extra load in distributed loss case.

In this paper DCOPF is used for all the three loss cases and is solved with LP approach, GA, and with proposed Bat algorithm.
algorithm. In [6] LP approach is used for all the three loss cases with piecewise linear cost curves but it does not give actual marginal cost of generation. To address this drawback, linear bids are used under three loss cases with GA and BA. The linear bid curve introduces non-linearities in the problem, however, it is a more realistic representation of price than that of a fixed price for power. The results of BA are compared with LP and GA for all the 3 cases. Section II presents single auction model. Section III discusses the problem formulation for LMP calculation using delivery factors for 3 loss cases. Section IV presents implementation of GA and proposed Bat algorithm. Section V gives a case study of Indian 75 bus system [11]. Section VI presents results and discussion. Section VII concludes the paper.

II. SINGLE AUCTION MODEL

In this model, the consumer power demand is insensitive to variation in prices and remains constant as shown in Fig. 1. The consumer is ready to accept a specified amount of power at the prevailing market price. The major part of the Discom’s demand bid is price taking, which is required to meet its essential daily services to residential, domestic, and industrial loads.

![Fig. 1. Single auction model](image)

In the Fig. 1, when there is no congestion in the system the aggregate supplier marginal cost curve and consumer inelastic demand intersects at a point ‘λ’ which is called Market Clearing Price. Supplier surplus exists. When congestion occurs, due to addition of congestion price and loss price MCP becomes LMP leading to merchandising surplus or ISO surplus. Then the total social surplus is the sum of supplier surplus and ISO surplus.

III. PROBLEM FORMULATION FOR LMP CALCULATION

In this paper for computation of LMP’s, DCOPF problem is solved with LP – fixed bids, GA – fixed and linear bids, and Bat algorithm – linear bids of generators under all the three loss cases. Active power generations of the generators except slack generation are considered as variables for optimization. The obtained PG’s are used in calculation of LMP for the congested transmission system. Generation Shift factors (GSF) have been used for the calculation of transmission line flows. Delivery factors (DF) at buses have been used to include the impact of marginal losses on nodal price.

A. Generation Shift Factor

Generation shift factor is the ratio of change in power flow of line ‘k’ to change in injection of power at bus ‘i’. GSF coefficient can be computed as

$$ GSF_{k,i} = \frac{B_{a,b}^{\text{inv}} - B_{a,b}^{\text{inv}}}{X_k} \times \frac{1}{|X_k|} $$

where $B^{\text{inv}} = \text{inverse of B (the imaginary part of Y bus matrix),}$ $X_k = \text{reactance of line k,}$ $a$ is sending bus and $b$ is receiving bus of line $k$

B. Delivery Factor

The key to consider marginal loss price is the marginal loss factor, or just loss factor (LF) for simplicity, and the marginal delivery factor, or just delivery factor (DF). Mathematically, they can be written as

It is defined as

$$ DF_i = 1 - LF_i = 1 - \frac{P_{loss}}{\partial P_i} $$

where $LF_i = \text{loss factor at bus ‘i’}$

$$ P_{loss} = \sum_{k=1}^{N} F_k \times R_k $$

where $F_k = \text{line flow of line k , } R_k = \text{resistance of line k,}$

$$ P_i = P_{Gi} - P_{Di} = \text{injection at bus i}$

GSF_{k,i} = \text{generation shift factor to line ‘k’ from bus ‘i’}. $The loss factor (LF_i) at the i^\text{th} bus may be viewed as the change of total system loss with respect to a 1 MW increase in injection at that bus.

C. LMP Calculation with Different Loss Cases

Case 1. Without Losses using DCOPF

In this method the objective function is minimization of total marginal production cost subjected to power balance and line flow constraints. LMP’s are calculated from the obtained generator power outputs. ISO payments to generators, Generator profit, load payment to ISO, ISO profit and system social surplus are also calculated.

The objective function is

$$ \text{Minimize } J = \sum_{i=1}^{N} C_i(s) $$

s.t. $\sum_{j=1}^{N} P_{Gi} = \sum_{j=1}^{N} P_{Di}$

$$ F_k \leq \lim_{k}, k=1, 2 \ldots M $$

$$ P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}}, \text{ i}=1,2,\ldots., N $$

where $N$ is number of buses, $M$ is number of lines, $C_i(s)$ is cost function of the generator ‘i’ i.e. $(C_i(s) = b_i P_{Gi}^2 + c_i P_{Gi})$ in $$/hr,$

$P_{Gi}^{\text{out}}$ is output power of generator at bus i (MW), $P_{Di}$ is the demand at bus i (MW), $F_k$ is the line flow of line k, $\lim_{k}$ is the thermal limit of line k.
Case 2. With Concentrated Loss using DCOPF

In this method also the main objective is minimization of total marginal production cost subject to energy balance and line flow constraints. However, in LMP based electricity markets, system marginal losses have significant impact on the economics of power system operation. So, system marginal losses have to be taken into account for obtaining accurate prices. In this model it is assumed that total system loss is supplied by slack bus generator. The problem is to

\[ \text{Minimize } J = \sum_{i=1}^{N} C_i(s) \]  

s.t. \[ \sum_{i=1}^{N} DF_i \times (P_i) + P_{\text{loss}} = 0 \]  

\[ F_k \leq \text{limit}_{k}, \text{ } k=1,2,\ldots,M \]  

\[ P_{\text{min}} \leq P_{Gi} \leq P_{\text{max}}, \text{ } i=1,2,\ldots,N \]

where \( P_{\text{loss}} \) is the total system loss. \( P_{\text{loss}} \) in (11) is used to offset the doubled average system loss caused by the marginal loss factor (LF) and the marginal delivery factor (DF).

After getting power outputs of generators for the above dispatch, slack bus power is calculated using (7) or (11) and the price at the reference (slack) bus needs to be calculated by substituting slack bus power either in fixed bids or linear bids. At the reference bus, both loss price and congestion price are always zero. Therefore, the price at the reference bus is equal to the energy component.

The decomposition of LMP is shown here

\[ LMP_{\text{energy}} = \lambda = \text{price at the reference bus} \]  

\[ LMP_{\text{loss}} = -\sum_{k=1}^{M} GSF_{k-B} \times \mu_k \]  

where \( \mu_k \) is the constraint cost of line k, defined as:

\[ \mu_k = \frac{\text{change in total cost}}{\text{change in constraint's flow}} \]  

\[ LMP_{\text{loss}} = \lambda \times (DF_B - 1) \]  

(LMP_B = 0 for lossless power system)

After calculating the individual components of LMP using (14-16), total LMP at each bus is calculated using (1).

Case 3. With Distributed Loss using DCOPF

Concentrated loss model addresses the marginal loss price through the delivery factors. However, the line flow constraint in (12) still assumes a lossless network. But the equality constraint in (11) informs that total generation is greater than the total demand by the average system loss. This causes a mismatch at slack bus and this mismatch is absorbed by the system slack bus. If system demand is huge e.g., a few GW then the system loss may be of the order of hundreds of MW and it may not be possible to add all the losses to slack bus. To address this issue, it is necessary that the line losses are to be shared among the buses as extra loads. This paper employs the concept of distributed loss, to represent the losses of the lines connected to a bus. In this method system losses are distributed among all the buses and eliminate the large mismatch at the reference bus. In this approach, loss in each transmission line is divided into two equal halves, and each half is added to respective end buses of that line as an extra load. So for each bus, the total extra load is the sum of halves of line losses which are connected to that bus. The extra load at bus ‘i’ is assumed as \( E_i \), and it is defined as follows:

\[ E_i = \sum_{k=1}^{M_i} \frac{1}{2} \times P^2_k \times R_k \]  

where \( M_i \) is number of lines connected to bus i. The line flow \( F_i \) for this case is calculated as in (18).

\[ F_i = \sum_{k=1}^{M_i} GSF_{k-i} \times (G_i - D_i - E_i) \]

The problem formulation for this case is same as in case 2. After getting power outputs of generators, LMPs’ at all buses are computed. With this approach, the fuel cost can be further reduced than the concentrated loss case and the burden on the slack bus can be eliminated.

IV. IMPLEMENTATION OF GA AND BAT ALGORITHM

A. Bat Algorithm (BA)

The basic bat algorithm (BA) developed by Xin-She Yang [13]. This algorithm is the combination of PSO and Harmony search algorithms. For the bats in simulations, the following equations define how their positions \( x_i \) and velocities \( v_i \) in a d-dimensional search space are updated. The new solutions \( x_i^{t+1} \) and velocities \( v_i^{t+1} \) at time step \( t \) are given by

\[ f_i = f_{\text{min}} + (f_{\text{max}} - f_{\text{min}}) \beta \]  

\[ v_i^{t+1} = v_i^t + (x_i^t - x^*) \beta \]  

\[ x_i^{t+1} = x_i^t + v_i^{t+1} \]

where \( f_i \) is the ith bat frequency, \( \beta \epsilon [0,1] \) is a random vector drawn from a uniform distribution. Here ‘\( x^* \)’ is the current global best location (solution) which is located after comparing all the solutions among all the ‘n’ bats at each iteration ‘t’. As the product \( \lambda f_i \) is the velocity increment, adjust the velocity change by using \( f_i \) or \( \lambda_i \) while fixing the other factor \( \lambda_i \) (or \( f_i \)), depending on the type of the problem of interest. The present work uses \( f_{\text{min}} = 0 \) and \( f_{\text{max}} = 1 \) (depending on the domain size of the problem of interest). Initially, each bat is randomly assigned a frequency which is drawn uniformly from [\( f_{\text{min}}, f_{\text{max}} \)].

For the local search part, once a solution is selected from among the current best solutions, a new solution for each bat is generated locally using random walk.

\[ x_i^t = x^* + (0.01 \times \text{rand}) \]

where \( x_i^t \) is the current position of solution, \( x^* \) is the current best solution. The in depth details and pseudo code of Bat algorithm are given in [13].
B. Algorithm for LMP Computation using GA and BA

1) Using GA

Step 1: Read no. of buses, no. of lines, slack bus number, and Bus data. Read GA parameters like population size, chromosome length, no. of units, maximum no. of generations, elitism probability, crossover probability, mutation probability, and epsilon. Read a, b, c coefficients; min. and max. limits of generators. Read line data including line thermal limits.

Step 2: Generate randomly power generations of all generators except slack generator and decode them.

Step 3: Calculate Generation shift factors using (2).

Step 4: Calculate initial line flows using (5).

Step 5: Calculate the system loss i.e., $P_{\text{loss}}$ in each line using (4) for case 2 and case 3.

Step 6: For case 3, calculate the extra load at each bus ‘i’ using (17) from initial line flows, and then calculate new line flows using (18).

Step 7: Calculate loss factors and then delivery factors at each bus using (3).

Step 8: Calculate $P_{\text{gen}}$ of slack bus using (7) of case 1 or (11) for cases 2 and 3.

Step 9: Check for line flow limits using (8). If the line limits are violated add penalties to objective function.

Step 10: Check for slack bus power limits using (9). If it violates the limits add penalties to objective function.

Step 11: Calculate the marginal fuel costs of all units with the randomly generated PG’s; calculate the total fuel cost using (6) for all cases and then calculate the fitness function = $100/(1+\text{objective function+ penalties})$.

Step 12: Sort the chromosomes in the descending order of fitness.

Step 13: Is iteration = max. no. of iterations. If yes go to step 18 else go to step 14.

Step 14: If fitness (1) = fitness (psize)$\Rightarrow$ problem converged.

If no go to step 17.

Calculate the energy price of the reference bus either with fixed bids or with linear bids and then calculate the decomposition of LMP using (14), (15) and (16).

Step 15: Calculate ISO payment to generators (generation at bus i * LMP at bus i) and then calculate supplier surplus (ISO payment to generator - fuel cost of generator).

Step 16: Calculate load payments to ISO (demand at bus j * LMP at bus j), and ISO surplus (total load payment to ISO – total ISO payment to generators) for all the 3 loss cases, and then finally calculate social surplus (supplier surplus + ISO surplus) in 3 loss cases & STOP.


iteration = iteration + 1; Go to step 4.

Step 18: STOP and print maximum number of iterations reached.

2) Using BA

Using BA, the algorithm mathematically is similar to GA except the BA operators used. Due to space constraint Bat algorithm for LMP computation cannot be presented here.

V. CASE STUDY ON INDIAN 75 BUS SYSTEM

Test systems used are IEEE 14 bus, New England 39 bus and Indian 75 bus systems to compute LMPs’ under all the 3 loss cases with LP, GA and BA. Only Indian 75 bus system [11] (Uttar Pradesh State Electricity Board data i.e., 400kV, 220kV and 132kV grid data) results are presented in this paper which has 15 generators, 72 lines and 24 transformers. GA parameters used are Population size: 40, Number of bits for each generator in the chromosome: 12, Elitism probability: 0.15, Crossover probability: 0.85, Mutation probability: 0.01, Tolerance: 0.0001. BA parameters used are Population size = 15 (10-25), Loudness parameter (A) = 0.25, Maximum iterations = 500, Pulse rate (r) = 0.5, Frequency min = 0, Frequency max = 0.02, Alpha = 0.9, Gamma = 0.9. In this work for LP approach, POWER WORLD SIMULATOR is used.

For base case loading, 9th line connecting 4-28 buses congested in the direction 4$\rightarrow$28 for all the three loss cases with LP, GA and BA. The corresponding results are listed in table I. The LMPs’ at all buses are also calculated. The fuel cost comparison graphs for cases 1, 2 and 3 is shown in Fig. 2. Figs. 3, 4 and 5 show the LMPs’ comparison for cases 1, 2 and 3. LMP comparison graphs imply that due to congestion in the line 4$\rightarrow$28, LMP at 4th bus (i.e., source node of congested line and also having generator) undergoes a large variation in all the methods for all the 3 loss cases. Fig. 6 shows the convergence characteristics of GA and BA. This case study highlighted that BA approach with linear bids of generators minimizes the total fuel cost of the system and improves the social surplus of the system. Slack generator power in distributed loss model is comparatively reduced than in concentrated loss model. The constraint cost or shadow price of congested line is 3.35 $/MWh for case 1, 3.67 $/MWh for case 2 and 3.7 $/MWh for case 3, which are obtained from Power World Simulation.

From the table I it can be observed that in case 2, with bat linear bids, slack bus generation violated the maximum limit which is the major drawback of case 2 pointed out in this paper. In case 3, with bat linear bids, slack bus generation sets within its limits.

In Fig. 3 with all the methods, the LMPs’ at all the buses except at the 4th bus are same due to the absence of loss price in the LMP components. In Figs. 4 and 5 using all the methods, LMPs’ slightly vary at the buses except at 4th bus due to the presence of loss price in LMP.

![Fig. 2. Fuel cost comparison graph for all the three loss cases of Indian 75 bus system](image-url)
TABLE I.  
ACTIVE POWER GENERATIONS OF GENERATORS FOR INDIAN 75 BUS SYSTEM IN 3 CASES

<table>
<thead>
<tr>
<th>Generator bus No.</th>
<th>CASE 1</th>
<th>CASE 2</th>
<th>CASE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LP-FB</td>
<td>GA approach</td>
<td>BA-LB</td>
</tr>
<tr>
<td></td>
<td>FB</td>
<td>LB</td>
<td>FB</td>
</tr>
<tr>
<td>1 (slack bus)</td>
<td>684.2</td>
<td>730.05</td>
<td>730.05</td>
</tr>
<tr>
<td>2</td>
<td>360</td>
<td>282.52</td>
<td>282.52</td>
</tr>
<tr>
<td>3</td>
<td>280</td>
<td>279.93</td>
<td>279.93</td>
</tr>
<tr>
<td>4</td>
<td>185</td>
<td>189.87</td>
<td>189.87</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>25.0</td>
<td>25.0</td>
</tr>
<tr>
<td>6</td>
<td>220</td>
<td>219.95</td>
<td>219.95</td>
</tr>
<tr>
<td>7</td>
<td>160</td>
<td>159.96</td>
<td>159.96</td>
</tr>
<tr>
<td>8</td>
<td>180</td>
<td>179.96</td>
<td>179.96</td>
</tr>
<tr>
<td>9</td>
<td>505.92</td>
<td>201.59</td>
<td>201.59</td>
</tr>
<tr>
<td>10</td>
<td>180</td>
<td>179.96</td>
<td>179.96</td>
</tr>
<tr>
<td>11</td>
<td>209</td>
<td>208.95</td>
<td>208.95</td>
</tr>
<tr>
<td>12</td>
<td>775</td>
<td>1106.8</td>
<td>1106.8</td>
</tr>
<tr>
<td>13</td>
<td>1000</td>
<td>999.76</td>
<td>999.76</td>
</tr>
<tr>
<td>14</td>
<td>250</td>
<td>249.94</td>
<td>249.94</td>
</tr>
<tr>
<td>15</td>
<td>554</td>
<td>553.87</td>
<td>553.87</td>
</tr>
</tbody>
</table>

System Loss (MW)                130.247| 63.59 | 63.59 | 88.135| 130.247| 63.58 | 63.58 | 106.516| 130.247| 63.58 |
Fuel cost ($/hr)     56097.0356| 55981.2 | 48528.21| 48141.71| 57347.664| 56696.62| 49237.91| 49175.81| 58070.319| 57376.59| 49557.2 |
ISO payment to Generators ($/hr)  60239.80| 63277.41| 61718.34| 64529.94| 63427.678| 65089.39| 65660.11| 70079.86| 63395.79| 65166.35| 66910.11 |
Generator profit ($/hr)  4142.7644| 7296.2 | 13190.12| 16388.22| 6080.014| 8392.77| 16422.2| 20904.05| 5325.471| 7789.75| 15900.32|
Load payment to ISO ($/hr)  60859.55| 62461.35| 61093.03| 63878.68| 62641.35| 65243.39| 65814.16| 70428.97| 62641.35| 65218.56| 66519.71|
ISO profit ($/hr)        619.75| -636.06| -625.3 | -651.26| -786.328| 153.99| 153.94| 349.11| -754.44| 52.21| 49.84|
Social surplus ($/hr)   4762.51| 6660.14| 12564.82| 15736.96| 5293.686| 8546.76| 16576.14| 21253.16| 4571.031| 7841.96| 15950.17|
iterations       396   | 125    | 396    | 125    | 312    | 57     |
CPU time (sec)     897.928| 227.863| 897.928| 227.863| 703.208| 112.469|
From Fig. 6 it can be observed that, Bat algorithm takes less iterations to converge than GA. Similar kind of results are obtained for IEEE 14 bus and New England 39 bus systems also. But due to space constraint the results of 14 and 39 bus systems are not presented in this paper.

VII. CONCLUSIONS

This paper presents a simple transmission pricing scheme i.e., Locational Marginal Pricing using DCOPF to calculate the transmission variable cost considering three different loss cases in a pool type restructured electricity market. Under all the 3 loss cases, LMPs’ are evaluated at all the buses in a power system. To solve the DCOPF problem under each loss case, LP, GA and Bat algorithm are used. Fixed bids of generators are used with LP and GA. Linear bids of generators are used with GA and BA. Initially comparison is made between LP and GA. It is observed that GA-LB gives the most optimal fuel cost in this comparison. Next for further optimization of fuel cost BA-LB is used. For all the used test systems, the slack bus generator power with distributed loss approach is reduced than with the concentrated loss approach and burden on the slack generator is removed. It is observed that considerable savings in total fuel cost of generators is achieved with BA-LB under all the 3 loss cases. LMPs’ with linear bids are calculated to avoid the non smooth nature of bid curve in fixed bids. In all the used test systems, the distributed loss with BA-LB approach for LMP calculation shows reliable results with optimized fuel cost values.

REFERENCES