Z Bus Computation for Networks having mutually coupled elements using uncoupled equivalents of coupled groups

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Abstract—This paper presents a new algorithm for the formation and modification of bus impedance matrix of networks having mutually coupled elements. The key idea in the proposed algorithm is obtaining the uncoupled equivalent circuit of a coupled group. Using such equivalents, the bus impedance matrix is formed using the well-known and extremely simple $Z_{BUS}$ building algorithm applicable for uncoupled networks. A new algorithm to obtain the elements of such uncoupled equivalent circuits is proposed. This idea is extended for modifying the bus impedance matrix to account for changes in the network due to addition/deletion/modification of a mutually coupled element. The proposed methods are illustrated using examples.

Index Terms—Power system analysis; Bus impedance matrix; mutual coupling; Fault analysis

I. INTRODUCTION

Elements of the bus impedance matrix($Z_{BUS}$) are used in many power system analysis situations. Algorithms for computation of $Z_{BUS}$ for networks having no mutually coupled elements are available since the 60’s. Among these methods, the $Z_{BUS}$ bus building algorithm introduced by Brown et al. [1] is widely known. Saxena et al. [2] proposed an implementation for this $Z_{BUS}$ algorithm by suitably choosing the sequence for the addition of elements. But these methods are applicable only for networks that do not have any mutually coupled elements. The $Z_{BUS}$ building algorithm for networks with mutually coupled elements proposed in [3] is widely known and is found in most of the text books [4], [5]. In this method, mutually coupled elements are added one at a time. This scheme requires inversion of a partial primitive impedance matrix at every step and also the algorithm is quite complex as compared to that in [1]. Another method discussed in [6] uses the matrix inversion lemma to include a mutually coupled group in one step. However, this method requires a large number of matrix operations and sometimes requires the introduction of dummy elements before introducing a group of elements (and their subsequent deletion).

An interesting approach has been suggested in [7] where it is shown by considering some small examples that the bus impedance matrix of circuits with mutually coupled elements can be obtained using the method in [1] if one could find an uncoupled equivalent for a mutual coupled group. The uncoupled equivalents for the examples in [7] have been obtained by long hand calculations and no general procedure is proposed in [7] to find such uncoupled equivalent.

The aim of this paper is to show that the uncoupled equivalent for mutually coupled group of elements can be obtained by adopting [8] a relatively new algorithm for the formation of $Y_{BUS}$ of circuits having mutually coupled elements. We show here how the algorithm in [8] can be suitably modified for this purpose.

In several applications, the network keeps changing marginally from a base case configuration for which $Z_{BUS}$ is known and the $Z_{BUS}$ of these modified configuration would be needed. For such occasions, instead of reconstructing the $Z_{BUS}$ from the beginning, techniques are shown in [9] for removal and changes in self impedance of a line belonging to a mutually coupled group. There is no such direct general method available in the literature to account for changes in both self and mutual value of a network element. In this paper we also propose a general approach to compute the effect of the all types of network changes on the $Z_{BUS}$ including situations of the self and mutual impedances of an element changing simultaneously by extending the scheme proposed here for the formation of the $Z_{BUS}$ matrix.

II. THE PROPOSED METHOD

The algorithm in [8] facilitates the computation of $Y_{BUS}$ of networks with mutually coupled elements in a very simple way. The algorithm requires only the primitive admittance matrix of the network. In any network having mutually coupled elements, one can easily identify one or more groups of mutually coupled elements among the many uncoupled elements. The primitive admittance matrix is computed by inverting the primitive impedance matrix of individual groups of coupled elements. The full matrix need not to be inverted simultaneously.

The proposed approach requires the determination of an uncoupled equivalent of a coupled group of elements. We first show how this can be done using the method in [8]. Let us consider a network with a group of $m$ mutually coupled elements. Let the number of nodes associated with this group be $n$. Consider an element $y_{ij}$ at the primitive admittance matrix that corresponds to two network elements $i$ and $j$. Let the ‘from’ and ‘to’ nodes of element $i$ be $p$ and $q$ respectively. Similarly the ‘from’ and ‘to’ nodes of element $j$ be $r$ and...
s respectively. The algorithm in [8] considers each non-zero element of the primitive admittance matrix \((y_{ij})\) and updates four elements of bus admittance matrix \(Y_{BUS}\) as follows

\[
\begin{align*}
Y(p, r) &= Y(p, r) + y_{ij} & (1) \\
Y(p, s) &= Y(p, s) - y_{ij} & (2) \\
Y(q, s) &= Y(q, s) + y_{ij} & (3) \\
Y(q, r) &= Y(q, r) - y_{ij} & (4)
\end{align*}
\]

From the above algorithm it is evident that each \(y_{ij}\) element is added and subtracted once from each row of \(Y_{BUS}\) to which they contribute. So the sum of all elements in any row or column of a \(Y_{BUS}\) is zero. This implies that the diagonal element in any row is the negative of sum of all the off diagonal elements.

Now consider another network having elements only with self admittance. Let the self admittance of the element in such a network between say nodes \(a\) and \(b\) (where \(a \neq b\)) be equal to \(-Y(a, b)\) found above. The \(Y_{BUS}\) matrix of such a network would be exactly equal to \(Y_{BUS}\) found by the above algorithm. Therefore this network made up of only uncoupled elements (as defined above) can be considered as the uncoupled equivalent of the \(m\) mutually coupled group of elements.

Hence, finding the uncoupled equivalent of a mutually coupled group of elements requires the determination of the off-diagonal elements of the \(Y_{BUS}\) corresponding to the mutually coupled group. In general power network data consists of some sets of mutually coupled elements and several uncoupled elements. These groups and elements can be handled one by one or they can be handled in one go by finding the uncoupled equivalent for the whole network. We choose the second option here. In order to build the \(Z_{BUS}\) of such network, either the upper or lower off-diagonal elements of \(Y_{BUS}\) are sufficient. In this paper the upper triangular part of a matrix is considered.

### III. Algorithm

The proposed algorithm to find the off diagonal elements of the symmetric \(Y_{BUS}\) is as follows

1) Find the off-diagonal elements of \(Y_{BUS}\) considering all the uncoupled elements
2) Identify all the mutually coupled groups of elements and find their \(y\)-the primitive admittance matrix and for each of the groups repeat step 3 and 4 below
3) Considering each of the diagonal elements \(y_{ii}\) update \(Y_{BUS}\) as follows. Let \(p\) and \(q\) be the ‘from’ and ‘to’ nodes of the \(i^{th}\) branch then if \(p < q\) then \(Y(p, q) = Y(p, q) - y_{ii}\) else \(Y(q, p) = Y(q, p) - y_{ii}\)
4) Considering each of the non zero upper triangular elements \(y_{ij}\) update \(Y_{BUS}\) as follows. Find the two indices \(i\) and \(j\) of the element \(y_{ij}\) and let \(p\) and \(q\) be the ‘from’ and ‘to’ nodes for branch \(i\) and \(r\) and \(s\) be the ‘from’ and ‘to’ nodes for branch \(j\). Update the elements of \(Y_{BUS}\) as follows
   a) if \(p < r\) then \(Y(p, r) = Y(p, r) + y_{ij}\) else \(Y(r, p) = Y(r, p) + y_{ij}\)
   b) if \(p < s\) then \(Y(p, s) = Y(p, s) - y_{ij}\) else \(Y(s, p) = Y(s, p) - y_{ij}\)
5) Using each of the non-zero elements in the upper triangular position of \(Y_{BUS}\), obtain the impedance of the element in the uncoupled equivalent as the reciprocal of the negative of element in \(Y_{BUS}\)

As mentioned earlier, the proposed algorithm can also be implemented without finding all the off-diagonal elements of the \(Y_{BUS}\) of the network together as given above. One can consider each group of mutually coupled elements separately, find their contribution to \(Y_{BUS}\) using the expressions given in step 4 above and include each of the elements of the equivalent found for forming the \(Z_{BUS}\). The advantage of the proposed scheme above is that, if some of the new elements of the uncoupled equivalents turn out to be in parallel with some existing network elements, then their equivalent is automatically obtained and the total computations required for finding the \(Z_{BUS}\) reduces as the number of steps in the \(Z_{BUS}\) building process reduces.

### IV. Illustration

The steps of the proposed technique are illustrated with a small network shown in Fig. 1. The line data of the network is given in Table I. In this example, node 2 is taken as reference.

It is observed that the network has two mutually coupled group of elements. Elements 1, 2 and 4 form Group 1 and elements 6 and 7 form Group 2. The primitive admittance matrix corresponding to these groups are given below. Using the upper triangular elements of these matrices (identified as \(A\) to \(F\) in (5), \(G\) to \(F\) in (6) and the uncoupled elements 3 and 5.)

![Fig. 1. Illustration of Proposed Scheme](image-url)

<table>
<thead>
<tr>
<th>Table I</th>
<th>NETWORK DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element</td>
<td>Nodes From</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>
The computation of upper off diagonal elements of $Y_{BUS}$ to obtain the uncoupled equivalent is done using the elements of $y_{2\text{new}}^d$ and shown in Table IV.

$$y_{2\text{new}}^d = \begin{pmatrix} 5^A & 5^C \\ 5 & 15^B \end{pmatrix}$$ (10)

Now Brown’s $Z_{BUS}$ building algorithm is used for this line data in Table III. Some intermediate results in the step by step $Z_{BUS}$ building are given below considering node 2 as reference node. Starting with the reference node when we add elements $a,d,g$ and $e$, the $Z_{BUS}$ of this partial network would be

We obtain the upper triangular elements of $Y_{BUS}$ matrix using the algorithm in section 3. This is given in Table II. The identifiers A to I are used in (5) and (6) to clearly indicate how each of the elements in $y$ contribute to various elements in $Y_{BUS}$ in Table II.

Comparing Table I and Table II, we see that both of them have 7 elements. In Table I we have two sets of coupled groups, whereas the network corresponding to Table II all elements are uncoupled. It may also be seen that the element between nodes 2 and 5 in Table I is now modified because of one of the new elements of the equivalent circuit gets added to this value as can be seen in Table II. The impedance data of the uncoupled equivalent is given in Table III where impedance of the elements are obtained as the reciprocal of the $Z_{BUS}$.

### TABLE II

<table>
<thead>
<tr>
<th>Nodes From</th>
<th>Off diagonal elements of $Y_{BUS}$ matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2</td>
<td>$-60^A - 30^D = -90$</td>
</tr>
<tr>
<td>1 3</td>
<td>$-20^B - 30^D + 20^E + 10^F = -20$</td>
</tr>
<tr>
<td>3 5</td>
<td>$-10^C + 10^F = 0$</td>
</tr>
<tr>
<td>2 3</td>
<td>$30^D - 20^F = 10$</td>
</tr>
<tr>
<td>2 5</td>
<td>$20^E - 10 = 10$</td>
</tr>
<tr>
<td>1 5</td>
<td>$-20^E - 10^F = -30$</td>
</tr>
<tr>
<td>4 5</td>
<td>$-4^2 - 6^f + 2^H + 2^C = -6$</td>
</tr>
<tr>
<td>2 4</td>
<td>$-8$</td>
</tr>
</tbody>
</table>

### TABLE III

<table>
<thead>
<tr>
<th>Element</th>
<th>From-node</th>
<th>To-node</th>
<th>Self Z-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>3</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>5</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>d</td>
<td>2</td>
<td>3</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>e</td>
<td>2</td>
<td>5</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>f</td>
<td>4</td>
<td>5</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>g</td>
<td>2</td>
<td>4</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Now Brown’s algorithm [1] is used for this line data in Table III. Some intermediate results in the step by step $Z_{BUS}$ building are given below considering node 2 as reference node. Starting with the reference node when we add elements $a,d,g$ and $e$, the $Z_{BUS}$ of this partial network would be

Now element $b$ and then $f$ are added to the partial network to get

### V. MODIFICATION OF $Z_{BUS}$ FOR NETWORK CHANGES

In any network, the changes could be one of the following (i) Addition of an element, (ii) Deletion of an element (iii) Parameter changes of a line. A general method is proposed here to modify the $Z_{BUS}$ matrix to account any of such changes. The discussions here are limited to only changes involving mutually coupled group as the methods for handling changes in uncoupled elements are well known [4], [5].

The proposed scheme for modifying the $Z_{BUS}$ for any type of change in a mutually coupled element is as follows. By following the algorithm in section 3, the upper triangular elements of admittance matrix corresponding to the coupled group of the changed group $Y_{NEW}$ is computed. For the same group, from the original data of the existing configuration, the uncoupled equivalent $Y_{OLD}$ is obtained. Using the difference between the upper triangular elements of $Y_{NEW}$ and $Y_{OLD}$, a set of uncoupled elements are obtained. This new set of uncoupled elements are added to the $Z_{BUS}$ matrix of the existing configuration to obtain the $Z_{BUS}$ of the modified system.

To illustrate this procedure, the system used in section 4 whose $Z_{BUS}$ is given (9) is used. Let the self impedance of line 7 change to 0.1 and the mutual coupling between lines 6 and 7 is change to $-0.1$ instead of the values given in Table I. The new primitive admittance matrix of the coupled group of lines 6 and 7 is

$$y_{2\text{new}}^d = \begin{pmatrix} 30^D & 10^C \\ 5 & 15^B \end{pmatrix}$$

The computation of upper off diagonal elements of $Y_{BUS}$ to obtain the uncoupled equivalent is done using the elements of $y_{2\text{new}}^d$ and shown in Table IV.

$$y_{2\text{new}} = \begin{pmatrix} 5^A & 5^C \\ 5 & 15^B \end{pmatrix}$$ (10)
TABLE IV

<table>
<thead>
<tr>
<th>Nodes From</th>
<th>To</th>
<th>Off diagonal elements of $Y_{BUS}$ matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>$-5A - 15B - 5C - 5C = -30$</td>
</tr>
</tbody>
</table>

a line between nodes 4 and 5 with a self impedance of $\frac{1}{z_1}$ is added to the $Z_{BUS}$ in (9). Thus the $Z_{BUS}$ after considering the indicated parameter changes in one of the elements of group 2 is obtained as

$$Z_{BUS} = \begin{bmatrix}
1 & 0.0152 & 0.0304 & 0.0137 & 0.0173 \\
3 & 0.0304 & 0.1608 & 0.0274 & 0.0347 \\
4 & 0.0137 & 0.0274 & 0.0623 & 0.0456 \\
5 & 0.0173 & 0.0347 & 0.0456 & 0.0578
\end{bmatrix} \quad (11)$$

In order to highlight the merits of the proposed scheme, we compare the effort involved in carrying out the same change using the existing technique. Modifying the $Z_{BUS}$ to account for this type of parameter change of an element of a mutually coupled group requires two steps. First a step to delete the existing element and then a step to add a new element. In the context of the example above, first we have to add a mutually coupled element of self $z = -0.2$ mutually coupled to line 6 with a $z_m = 0.1$, as a third element of a mutually coupled group to the $Z_{BUS}$ in (9). In the second step we have to add an element of self $z = 0.1$ connected between nodes 4 and 5 and coupled to element 6 with a $z_m = -0.1$ as a second element of a mutually coupled group. Hence, it can be readily seen that the proposed scheme is much simpler.

VI. CONCLUSION

A new technique for forming $Z_{BUS}$ of networks having mutually coupled elements has been presented. This method makes finding the $Z_{BUS}$ matrix of such networks as simple as that of finding the $Z_{BUS}$ matrix of networks with no mutually coupled elements. The new method is also extended to handle the problem of modifying the $Z_{BUS}$ for incorporating network changes such as addition/deletion/parameter change of mutually coupled elements. The proposed schemes are illustrated with examples and their simplicity is highlighted. The proposed schemes in addition to their simplicity also have significant pedagogical value.

REFERENCES