# An Investigation on the Numerical Ill-conditioning of Hybrid State Estimators

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***Abstract*—Convergence problems and loss of accuracy due to numerical ill-conditioning is a frequently encountered problem in power system state estimation. It has been experienced by the researchers that the presence of PMU measurements along with the conventional measurements deteriorates the numerical conditioning of the so-called hybrid state estimator. This paper describes a simple technique to analyze the possible causes of ill-conditioning for different measurement configurations in terms of the condition number of the state estimation gain matrix. Approximate formulation of condition number of the gain matrix in presence of PMU with some simplifying assumptions is applied on a 10-bus radial network. This analysis is further extended for IEEE 14-bus system. The difference in weights assigned to the conventional and PMU measurements has been found to be the main cause of numerical ill-conditioning.**

***Keywords—condition number; gain matrix; phasor measurement unit; hybrid state estimator***

## I. INTRODUCTION

State estimator (SE) is an important function for wide area monitoring and control systems (WAMCS). The accurate and convergent SE is very important for the control centre. Numerical ill-conditioning of the estimation problem due to various reasons is usually the cause for degradation of the SE performance. Weighted least squares (WLS) state estimation is widely used by utilities for estimating the states of the system by processing the measurements collected from the supervisory control and data acquisition (SCADA) system. Conventional SCADA measurements consist of the real and reactive power injections, real and reactive power flows, bus voltage magnitudes, and line current magnitudes.

The state estimation gain matrix plays an important role in deciding the numerical behavior of the SE. Numerical behavior of the gain matrix is usually evaluated in terms of its condition number. The formation of the gain matrix depends on the network information, measurement configurations, and their weight assignment scheme, and hence, they may pose a conditioning problem to SCADA based SE [1]-[3]. To overcome the numerical problems, researchers have suggested either eliminating the cause of ill-conditioning or developing robust methods to provide an efficient and convergent solution. A number of robust techniques have been proposed in the literature, such as Peters and Wilkinson method [4], orthogonal decomposition [5]-[6], Levenberg-Marquardt algorithm [7], and regularization method [8]. The numerical ill-conditioning emanated from the power system network may be eliminated by network modeling [2] or avoiding the cause at the time of system commissioning, if possible. By careful selection of slack bus, the conditioning may be improved [7]. Numerical problems due to measurement configuration (measurement value, type, location and its weight), should be alleviated by analyzing the condition number. Condition number for the conventional measurements was analyzed and available in the literature [4], [9]. It has been the experience of power system engineers that the injection measurements are more detrimental to the numerical performance of the SE, compared to the flow measurements.

With the unique ability to measure the phase angles with time-synchronization of the measurements with respect to global positioning system (GPS) clock, the application of phasor measurement units (PMUs) is becoming very popular in power system control centres. The PMU can provide measurements of the voltage phasor, frequency, and the rate of change of frequency at a bus, and current phasors through the lines incident to that bus [10]. Due to the high cost and communication infrastructure requirements associated with the PMUs, as of now PMUs are being installed in power systems in an incremental fashion [11]. Two approaches have been adopted to combine the PMU with the conventional measurements in SE. The first approach is a *post-processing* algorithm that will 'mix' the PMU measurements with the outputs of the existing state estimator, and estimate the states of the system [12]. The later approach handles both conventional and PMU measurements at the same time in a *pre-processing* manner [11] and is usually termed as hybrid state estimator (HSE).

With the application of PMU along with the conventional measurements, the condition number analysis [4], [9] must be revisited with respect to PMU voltage and current phasor measurements. In this paper, rectangular form of the PMU current measurement is preferred to the polar form, as the jacobian matrix in case of polar current may become very large or undefined for certain operating conditions. This paper investigates the cause of numerical ill-conditioning of the HSEs due to the presence of PMU measurements. The differential weights assigned to the PMU and the conventional measurements are found to be the root cause for such numerical ill-conditioning.

This paper is organized as follows. In section II, the SE model and the assumptions used for the analysis of its

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numerical ill-conditioning problem are illustrated. Section III finds the approximate condition numbers of the SE gain matrix for a radial system observed through PMUs only. In section IV, the effect of PMU current measurement combined with the conventional measurements is studied for radial systems. The analysis of condition number is extended to non-radial IEEE test systems in section V. Section VI concludes the paper.

## II. State Estimation Model

The measurements collected from the network are processed through WLS SE. The state of the system at the ($k$+1)th iteration steps can be expressed as:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{G}^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\mathbf{x})] \tag{1}$$

where $\mathbf{x}^{(k)}$ denotes the state vector consisting of voltage angle and magnitudes at the $k$th iteration; $\mathbf{H}$ is the measurement jacobian matrix; $\mathbf{G} = \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}$ is the gain matrix; and $\mathbf{R}$ is the measurement error covariance matrix.

From (1), it is evident that the accuracy as well as convergence of the estimator depends on the inversion of the gain matrix. It is very difficult to invert a numerically ill-conditioned matrix. Hence, techniques were developed for inverting a matrix, and still research is going on. The condition number of the gain matrix for conventional WLS SE is often used as an indicator for the numerical conditioning of the problem. The condition number of any non-singular square matrix $\mathbf{A}$ can be defined as [4]:

$$\mathrm{Cond}\ (\mathbf{A}) = ||\mathbf{A}||\ ||\mathbf{A}^{-1}|| \tag{2}$$

where $||.||$ represents a matrix norm. The numerical ill-conditioning of any matrix is characterized by its high condition number. The second order norm is typically used for the state estimation problem. Higher order norms are neglected mainly due to large computational effort. In this paper, first order norm is used to simplify the analysis of the condition number of the gain matrix [13].

The condition number for any measurement configuration is evaluated by the norm of the gain matrix and its inverse. The simplifying assumptions considered for this analysis are mentioned below [4], [9].

1. Voltages at all buses are assumed to be equal to 1 pu, and the difference between phase angles of the bus voltages at the end of a line is assumed to be equal to zero, i.e., $\theta_i - \theta_j \approx 0$, where $\theta_i$ and $\theta_j$ are the voltage angle of the bus $i$ and $j$ at the ends of a transmission line joining the two buses.
2. Reactance to resistance ratio of the lines is assumed to be high; hence line resistances are neglected.
3. All line susceptance magnitudes are taken to be equal to $y$.
4. All shunt branches connected to the lines as well as the buses are neglected.

There is a lot of research work available in the literature for the condition number analysis in presence of conventional measurements. This analysis is extended to find the root cause of numerical conditioning in presence of PMUs in the network. In the next section, condition number analysis is presented when the whole power system network is completely observed through PMU measurements alone.

## III. Condition Number Analysis

The PMU provides the measurement of the voltage phasor as well as the current phasor. In this section, the condition number analysis due to PMU measurement is performed on a10-bus radial system, assuming bus 1 as the slack bus. For this analytical study of the condition number, three cases comprising of different PMU measurement configurations are considered:

- **Case I**: Test system is completely observed by 10 PMU voltage phasor measurements.
- **Case II**: Test system is completely observed by 9 PMU current phasor measurements. Each $i$th PMU current is flowing from bus $i$ to bus ($i$+1).
- **Case III**: 10 PMUs consisting of 10 voltage as well as 18 current phasor measurements are part of the measurements of the test system.

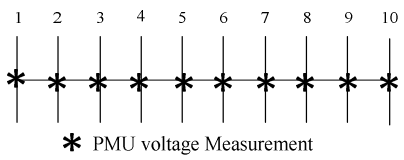


Fig. 1. 10 bus radial system (10 PMU voltage measurements)

Measurement configuration for the case I, where the voltage phasor measurements are available at all the buses, is shown in Fig. 1. Considering bus 1 as the slack bus, the measurement jacobian for this case is expressed as:

$$\mathbf{H}_{\text{case I}} = \begin{pmatrix} \mathbf{H}_\theta & 0 \\ 0 & \mathbf{H}_{\mathrm{V}} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_9 & 0 \\ 0 & \mathbf{I}_{10} \end{pmatrix} \tag{3}$$

where $\mathbf{H}_\theta$ and $\mathbf{H}_{\mathrm{V}}$ are the Jacobian matrices for the voltage angle and magnitude measurements, respectively. $\mathbf{I}_9$ & $\mathbf{I}_{10}$ are unity matrices of size 9x9 and 10x10 respectively. With the jacobian expressed in (3), and assuming the variance in the PMU measurements to be $R_{\mathrm{p}}$, the gain matrix and its inverse can be formulated as:

$$\mathbf{G}_{\text{case I}} = \left(1/R_p\right)\mathbf{I}_{19} \tag{4}$$

$$\mathbf{G}_{\text{case I}}^{-1} = R_p\mathbf{I}_{19} \tag{5}$$

The condition number of the above gain matrix is found to be unity by using (2), (4) and (5). Hence, if the power system network is observed by PMU voltage phasor measurements alone, there is no conditioning issue for the state estimator.

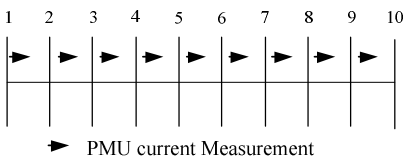


Fig. 2. 10 bus radial system (9 PMU current measurements)

For the measurement configuration in Case II, the current phasor measurements are available at all branches of 10-bus radial system, shown in Fig. 2. The current flowing through branch $i$ (from bus $i$ to bus $i$+1) can be expressed in rectangular form as:

$$\mathbf{I}_{i,\ \mathrm{real}} = y\left[\mathrm{V}_i\ \sin\ \theta_i - \mathrm{V}_{i+1}\ \sin\ \theta_{i+1}\right] \tag{6}$$

$$\mathbf{I}_{i,\ \mathrm{imag}} = y\left[-\mathrm{V}_i\ \cos\theta_i + \mathrm{V}_{i+1}\ \cos\theta_{i+1}\right] \tag{7}$$

where $\mathbf{I}_{i,\ \mathrm{real}}$ and $\mathbf{I}_{i,\ \mathrm{imag}}$ are the real and reactive part of the $i$th current phasor measured by PMU; $\overline{\mathrm{V}}_i = \mathrm{V}_i\angle\theta_i$ is the voltage phasor of the $i$th bus. Considering the assumptions listed in Section II, the gain matrix for this case can be written as:

$$\mathbf{G}_{\mathrm{case\ II}} = \begin{pmatrix} \mathbf{G}_{\theta\ \mathrm{case\ II}} & 0 \\ 0 & \mathbf{G}_{\mathrm{V\ caseII}} \end{pmatrix} \tag{8}$$

where $\mathbf{G}_{\theta\ \mathrm{case\ II}}$ and $\mathbf{G}_{\mathrm{V\ case\ II}}$ are the sub matrices corresponding to the voltage angle and magnitude respectively. The submatrix $\mathbf{G}_{\theta\ \mathrm{case\ II}}$ and its inverse can be expressed as:

$$\mathbf{G}_{\theta\ \mathrm{case\ II}} = \frac{y^2}{R_\mathrm{p}} \begin{bmatrix} 2 & -1 & 0 & - & - & 0 & 0 & 0 \\ -1 & 2 & -1 & - & - & 0 & 0 & 0 \\ 0 & -1 & 2 & - & - & 0 & 0 & 0 \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & - & - & 2 & -1 & 0 \\ 0 & 0 & 0 & - & - & -1 & 2 & -1 \\ 0 & 0 & 0 & - & - & 0 & -1 & 1 \end{bmatrix}_{9\mathrm{x}9} \tag{9}$$

$$\mathbf{G}^{-1}_{\theta\ \mathrm{case\ II}} = \frac{R_\mathrm{p}}{y^2} \begin{bmatrix} 1 & 1 & 1 & - & - & 1 & 1 & 1 \\ 1 & 2 & 2 & - & - & 2 & 2 & 2 \\ 1 & 2 & 3 & - & - & 3 & 3 & 3 \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 1 & 2 & 3 & - & - & 7 & 7 & 7 \\ 1 & 2 & 3 & - & - & 7 & 8 & 8 \\ 1 & 2 & 3 & - & - & 7 & 8 & 9 \end{bmatrix}_{9\mathrm{x}9} \tag{10}$$

The first order norm of $\mathbf{G}_{\theta\ \mathrm{case\ II}}$ and its inverse can be found to be $4y^2/R_\mathrm{p}$ and $45R_\mathrm{p}/y^2$, evaluated from (9) and (10). Since $\mathbf{G}_{\theta\ \mathrm{case\ II}} = \mathbf{G}_{\mathrm{V\ case\ II}}$, the condition number of the gain matrix is equivalent to that of $\mathbf{G}_{\theta\ \mathrm{case\ II}}$ and evaluated as $(4y^2/R_\mathrm{p})\times(45R_\mathrm{p}/y^2) = 180.$

For both the cases, where the 10-bus radial system is observed by either the PMU voltage phasor or the current phasor measurements, the condition number of the gain matrix is independent of the weight assignment as well as the line susceptance $y$, and is found to be 1 and 180.

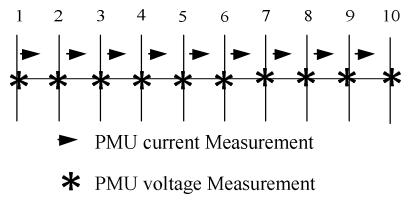


Fig. 3. 10 bus radial system (9 PMU current & 10 PMU voltage measurements)

For case III, the power system network is covered by 10 PMU voltage phasors and 9 PMU current phasor measurements, as shown in Fig. 3. Assuming the variance for both types of measurement as $R_\mathrm{p}$, the gain matrix can be expressed as:

$$\begin{aligned} \mathbf{G}_{\mathrm{case\ III}} &= \frac{1}{R_p}\begin{pmatrix} \mathbf{H}_{\mathrm{case\ I}} \\ \mathbf{H}_{\mathrm{case\ II}} \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{H}_{\mathrm{case\ I}} \\ \mathbf{H}_{\mathrm{case\ II}} \end{pmatrix} \\ &= \frac{1}{R_p}\left(\mathbf{H}^{\mathrm{T}}_{\mathrm{case\ I}}\mathbf{H}_{\mathrm{case\ I}} + \mathbf{H}^{\mathrm{T}}_{\mathrm{case\ II}}\mathbf{H}_{\mathrm{case\ II}}\right) \\ &= \begin{pmatrix} \mathbf{G}_{\theta\ \mathrm{case\ III}} & 0 \\ 0 & \mathbf{G}_{\mathrm{V\ caseIII}} \end{pmatrix} \end{aligned} \tag{11}$$

where $\mathbf{G}_{\theta\ \mathrm{case\ III}}$ and $\mathbf{G}_{\mathrm{V\ case\ III}}$ are the sub matrices corresponding to the voltage angle and magnitude respectively. Considering the admittance $y$ as 1 p.u. for simplification, the sub matrix $\mathbf{G}_{\theta\ \mathrm{case\ III}}$ can be expressed as:

$$\mathbf{G}_{\theta\ \mathrm{case\ III}} = \frac{1}{R_\mathrm{p}} \begin{bmatrix} 3 & -1 & 0 & - & - & 0 & 0 & 0 \\ -1 & 3 & -1 & - & - & 0 & 0 & 0 \\ 0 & -1 & 3 & - & - & 0 & 0 & 0 \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & - & - & 3 & -1 & 0 \\ 0 & 0 & 0 & - & - & -1 & 3 & -1 \\ 0 & 0 & 0 & - & - & 0 & -1 & 2 \end{bmatrix}_{9\mathrm{x}9} \tag{12}$$

The first order norm of $\mathbf{G}_{\theta\ \mathrm{case\ III}}$ and its inverse can be found to be 5/ $R_\mathrm{p}$ and 0.9998$R_\mathrm{p}$, evaluated from (12) for unit value of per unit admittance of the line. Since $\mathbf{G}_{\theta\ \mathrm{case\ III}} = \mathbf{G}_{\mathrm{V\ case\ III}}$, the condition number of the gain matrix is equivalent to that of $\mathbf{G}_{\theta\ \mathrm{case\ III}}$ and found to be 4.9988.

In this section, the condition number is formulated on 10-bus radial system for all three cases. There is no numerical ill-conditioning observed, when the measurement system consists of PMUs only. However, it is rare and impractical that the system is covered by one type of measurements only. Hence, in the next section, Condition number is evaluated for a combination of different types of measurements.

## IV. Condition Number Analysis for Hybrid Measurement Model

In the previous section, due to the single type of measurement, the condition number of the gain matrix is evaluated and found to be independent of the measurement variance. When two or more measurement types are present in the power system network, the relative weight is an important factor affecting the numerical conditioning. The effect of the measurement variance on the numerical conditioning of the gain matrix is discussed in this section for the same 10-bus radial system.

The variance in the Conventional (power flow and injection) and PMU (current as well as voltage phasor) measurements are taken as $R_c$ and $R_p$, respectively. The corresponding measurement weights for WLS state estimation are taken as $W_c$ and $W_p$, respectively. The ratio of weight to the PMU as well as the conventional measurements is assumed to be $w$ ($w=W_p/W_c$) throughout this paper. The variances in the measurements can be taken as proportional to the square of the maximum measurement error specified by the measurement device manufacturer [11]. For a typical accuracy level of 1% for the conventional measurements and 0.03% for the PMU current measurements [14], the ratio of the weights assigned to the PMU current measurement and conventional power injection measurement is $(0.01/0.0003)^2 \approx 1111$. For this section, branch admittance $y$ is assumed to be one per unit.

### A. Numerical Conditioning in Presence of Conventional Flow Measurements and the PMUs

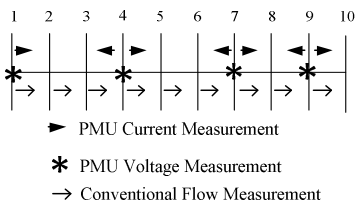


Fig. 4. 10 bus radial system (4 PMUs & 9 conventional flow measurements)

For this subsection, 9 conventional flow measurements along with 4 PMUs (4 voltage and 7 current phasors) are considered, as shown in Fig. 4. Two test cases are considered for the condition number analysis of the gain matrix.

- **Case 1**: 9 conventional flow and 4 PMU voltage phasor (located at bus 1, 4, 7 and 9) measurements are considered, while PMU current phasor measurements being ignored. The approximate expression for the first order norm of the gain matrix and its inverse for a typical $w$ around 1111 can be written as:

$$\begin{aligned} \|\mathbf{G}\| &= W_c\left(4+w\right) \\ \|\mathbf{G}^{-1}\| &\approx 1/W_c \end{aligned} \tag{13}$$

The condition number of the gain matrix is a linear function of $w$, and can be expressed as:

$$cond(\mathbf{G}) \approx 4+w \approx w \tag{14}$$

- **Case 2**: 9 conventional flow and 7 PMU current phasor (located between buses 1-2, 4-3, 4-5, 7-6, 7-8, 9-8 and 9-10) measurements are considered for this case, while PMU voltage phasor measurements being ignored. The approximate expression for the first order norm of the gain matrix and its inverse for a typical $w$ around 1111 can be written as:

$$\begin{aligned} \|\mathbf{G}\| &= 4W_c\left(1+w\right) \\ \|\mathbf{G}^{-1}\| &= \frac{45+13w}{W_c\left(1+w\right)} \end{aligned} \tag{15}$$

From (15), the condition number of the gain matrix is formulated as:

$$cond(\mathbf{G}) = 4\left(45+13w\right) \approx 52w \tag{16}$$

The condition number of the measurement configuration consisting of 9 conventional flow measurements without any PMUs is found to be 180 irrespective of their weights. For the measurement configuration in Fig.4, the condition number of the gain matrix in case 1 and 2 is linear in terms of $w$ and higher as clear from (14) and (16). The formulation for the condition number of the gain matrix is different for different measurement configurations.

### B. Numerical conditioning in presence of Conventional Injection measurements and PMUs

For this subsection, 10 conventional injection measurements along with 4 PMUs (4 voltage and 7 current phasors) are considered as shown in Fig. 5. Condition number analysis of the gain matrix is performed on two test cases:

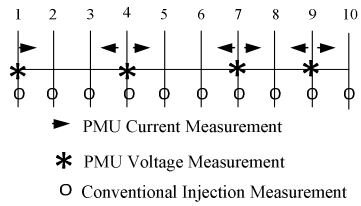


Fig. 5. 10 bus radial system (4 PMUs & 10 conventional injections measurements)

- **Case 1**: 10 conventional injection and 4 PMU voltage phasor (located at bus 1, 4, 7 and 9) measurements are used for the condition number analysis while, PMU current phasor measurements being ignored. The condition number of the gain matrix at a typical weight ratio of 1111, may be approximated as:

$$cond(G) \approx 0.805w \tag{17}$$

- **Case 2**: 10 conventional injection and 7 PMU current phasor (located between buses 1-2, 4-3, 4-5, 7-6, 7-8, 9-8 and 9-10) measurements are considered for this case while, PMU voltage phasor measurements being ignored. The condition number of the gain matrix at a typical weight ratio of 1111, may be approximated as:

$$cond(G) \approx (4w+16)\times 6.5 \approx 26w \tag{18}$$

The condition number of the measurement configuration consisting of 10 conventional injection measurements without any PMUs is found to be 6600 irrespective of their weights. For the measurement configuration in Fig.5, the condition number of the gain matrix in case 1 and 2 is linear in terms of $w$ and higher as clear from (17) and (18). The formulation for the condition number of the gain matrix is different for different measurement configurations.

## V. Condition Number for IEEE 14-Bus System

In the previous Sections, the condition number of the gain matrix is analyzed for a single type of measurement, as well as for combinations of different type of measurements on a radial system. To get a more practical analysis of the condition number, IEEE 14-bus is used as a test system with the assumptions (2-4) described in Section II being relaxed. 14 injections, 20 flows and two PMUs at bus 1 and 6 are considered for the analysis of the condition number. Depending on the PMU current considerations, two test cases are analyzed.

- **Case A** considers injection measurements at all the buses and flow measurements through all the lines, along with PMU measurements corresponding to 2 PMUs at bus 1 and 6 (2 voltages and 6 currents).
- **Case B** considers injection measurements at all the buses and flow measurements through all the branches along with 7 PMU voltages corresponding to 2 PMUs at bus 1 and 6. In this case, all PMU currents are converted into voltages at the other end of the branch.

The maximum measurement error in conventional measurements is assumed to be equal to 1% of the measurement, and that for the PMU current measurement is assumed to be 0.03% of the measurement. The variances in the two types of measurements are computed by assuming uniform probability distribution of the measurement value over the range of manufacturer-specified maximum uncertainty, as shown below [11].

$$\mathbf{R}_i = \frac{(\Delta u_i)^2}{3} \tag{19}$$

where $\Delta u_i$ is the specified maximum uncertainty in the $i$th measurement.

For both the cases, the condition number of the gain matrix is evaluated at flat voltage for different value of the weight ratio, and plotted in Fig. 6. For a typical weight ratio (≈1111) of the PMU measurement to the conventional measurement, the condition number of the gain matrix is found to be $4.7237\text{x}10^4$ and 536.5598 for cases A and B, respectively. From Fig. 6, it is apparent that the condition number of the gain matrix increases rapidly with the increase in $w$ for case A, whereas, in case B, the condition number is nearly stable. Also, for a given weight ratio, the condition number increases with the system size. One can, therefore, infer that for a large practical power system, the condition number will be very high in the presence of hybrid measurements consisting of PMU voltage and current measurement.

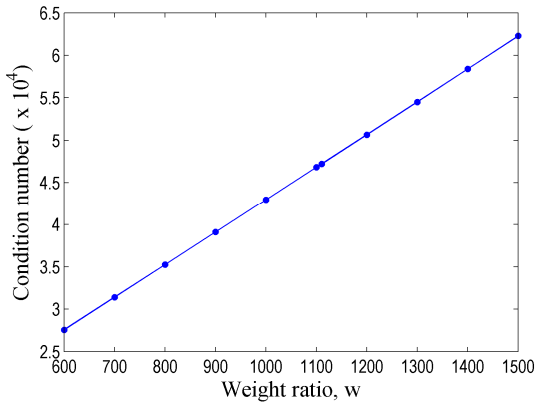


Fig. 6. Effect of weight ratio '$w$' on the condition number of the gain matrix for IEEE 14-bus system (**Case A**)

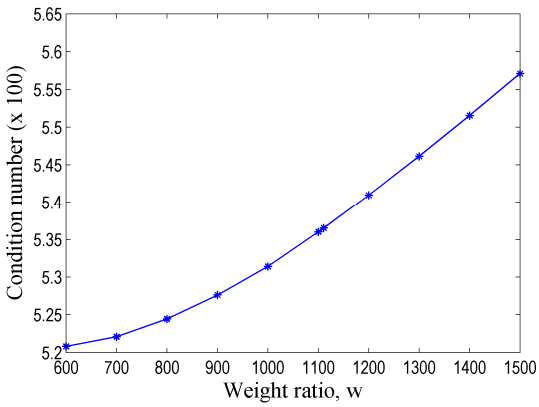


Fig. 7. Effect of weight ratio '$w$' on the condition number of the gain matrix for IEEE 14-bus system (**Case B**)

To deal with the problems emanated from the ill-conditioned gain matrix due to PMUs, two solutions are suggested. First solution lies in the conversion of the PMU current measurements into corresponding voltage measurements as reported in [11]. The other way to resolve the poor conditioning is to evolve new state estimator techniques. A number of SE methods are evolved by the researchers to overcome ill-conditioning of the gain matrix [4]-[8]. Reference [8] proposes an L-curve based regularization method to solve the ill-conditioned state estimation problems, which may not be solved by the conventional WLS algorithms.

## VI. Conclusion

The iterative solution at each step of WLS SE depends on the inversion of the gain matrix. Due to high condition number of the gain matrix, the convergence as well as accuracy of the

state estimator (SE) may get affected. This paper tries to find the cause of such numerical ill-conditioning in the presence of both PMU and conventional measurements. The condition number of the gain matrix of the hybrid state estimator is found to increase with the increase in the ratio of the weights assigned to the PMU measurements and the conventional measurements. It is, therefore, advisable to use proper numerical methods to address this ill-conditioning problem while including PMU measurements in a conventional SCADA based estimator.

## REFERENCES


[1] A. Abur and A. Gómez Expósito, *Power system state estimation: theory and implementation*. New York: Marcel Dekker, 2004.

[2] A. Monticelli and A. Garcia, "Modeling zero impedance branches in power system state estimation," *IEEE Trans. Power Systems*, Vol. 6, No. 4, pp. 1561-1570, Nov 1991.

[3] Antonio de la Jaen and A. Gomez Exposito, "Including ampere measurements in generalized state estimators," *IEEE Trans. Power Systems*, vol 20, no 2, pp. 603-610, May 2005.

[4] J. W. Gu, K. A. Clements, G. R. Krumpholz, and P. W. Davis, "The solution of ill-conditioned power system state estimation problems via the method of Peters and Wilkinson," *IEEE Trans. Power Apparatus and Systems*, Vol. PAS-102, No. 10, pp. 3473-3480, Oct 1983.

[5] A. Simoes-Costa and V. H. Quintana, "A robust numerical technique for power system state estimation," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-100, No. 2, pp. 691-698, February 1981.

[6] A. Simoes-Costa and V. H. Quintana, "An orthogonal row processing algorithm for power system sequential state estimation," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-100, No. 8, pp. 3791-3800, August 1981.

[7] N. D. Rao and S. C. Tripathy, "Power system static state estimation by the Levenberg-Marquardt algorithm," *IEEE Trans. Power Apparatus and Systems*, Vol. PAS-99, No. 2, pp 695-702, March 1980.

[8] S. K. Mallik, S. Chakrabarti, and S.N. Singh, "A robust regularized hybrid state estimator for power systems," *Electric Power Components and Systems*, 42(7), pp 671-681, 2014.

[9] R. Ebrahimian and R. Baldick, "State estimator condition number analysis," *IEEE Trans. Power Systems*, vol. 16, No. 2, pp. 273–279, May 2001.

[10] J. De La Ree, V. Centeno, J.S. Thorp, and A. G. Phadke, "Synchronized phasor measurement applications in power systems," *IEEE Trans. Smart Grid*, vol. 1, No. 1, pp. 20–27, June 2010.

[11] S. Chakrabarti, E. Kyriakides, G. Ledwich, and A. Ghosh, "Inclusion of PMU current phasor measurements in a power system state estimator," *IET Generation Transmission and Distribution*, Vol. 4, issue 10, pp. 1104-1115, 2010.

[12] Ming Zhou, V. A. Centeno, J. S. Thorp and A. G. Phadke, "An alternative for including phasor measurements in state estimators," *IEEE Trans. Power Systems*, vol. 21, No. 4, pp. 1930-1937, November 2006.

[13] N. Logic, E. Kyriakides and G. T. Heydt, "Lp state estimators for power systems," *Electric Power Components and Systems*, Vol. 33(7), pp. 699-712, 2005.

[14] Model 1133A, "GPS-Synchronized Power Quality/Revenue Standard: Operation manual", Arbiter System Inc.