Abstract—Worldwide stress on low carbon economy and reducing aggregate CO₂ emission levels exposed fossil fuel GenCos to carbon risk associated with prices of carbon permits needed to be environmentally compliant. This risk is substantial for fossil fuel GenCos aiming to maximize profit. This paper addresses a power portfolio optimization approach for a fossil fuel GenCo to maximize profit and minimize risk of volatile input costs and output revenue. Carbon price risk along with fuel and electricity price risk with correlation between all revenue and cost side markets has been considered for portfolio selection using Markowitz mean variance theory. A realistic case study on Nordpool market illustrate that carbon price uncertainty considerations after trading decision of GenCo in electricity market for efficient risk management. Correlated revenue and cost side markets provides better tradeoff in terms of profit and risk for same risk averse level for GenCo. Under strong correlation of electricity and carbon market a higher allocation in spot market provides risk protection. The carbon price risk impact is prominent for high emission GenCos.

Index Terms—Electricity price risk; carbon price risk; fuel price risk; mean variance portfolio theory.

I. INTRODUCTION

Deregulation of electric power industry has introduced competitive electricity markets, where suppliers and consumers trade electricity to meet their electricity supply and demand. The market prices are volatile and players face uncertainty of securing profits thus with growing competition, it becomes imperative for the players to minimize their exposure to economic risks, considering all involved uncertainties that may affect their expected profit [1].

Power producers generally consider the market price risks associated with electricity trading side while optimizing their portfolio, and tend to overlook the cost side risks, associated with price uncertainty of production resources [2]-[5]. Fuel is a major production resource and procured by generators from the markets. Its volatile prices impact electricity trading planning of generating companies (GenCos) [6].

Fossil fuel GenCos are the largest contributor to CO₂ emissions which need to procure carbon permits, to emit in the environment, as per existing Emissions Trading Schemes (ETS) [7]. ETS is worldwide accepted market mechanism for providing solutions to emission reduction. This scheme establishes caps on emission and allows trading of carbon permits to fulfill emission targets [8].

In upcoming scenario of power industry, starting from year 2013, allocation of allowances would be based on auction mechanisms, instead of the prevailing free allocation. Power sector would have to buy all its allowances in open competitive markets, i.e. Carbon Exchanges [9]. Since power sector has 50% weightage in ETS, total auction levels would increase by up to 50%, which would significantly boost the demand and prices for such credits [10]. This would result in considerable increase in overall production, bason cost and volatility of price of such credits [11]. Hence, there is strong motivation for fossil fuel GenCos to consider emission costs and its uncertainty, along with fuel and electricity price uncertainties for making their electricity trading plans.

Investigations have been carried out to understand the impact of emission trading scheme, on decision making of generation companies for self scheduling [12, 13], unit commitment [14, 15] and generation technologies selection [19], etc. Empirical researches suggest that electricity, carbon and fuel markets (coal, natural gas, etc.) are usually interdependent, i.e. price dynamics of one affects the other, because of the direct link between them arising due to overlapping goals [16]. The three markets are considered together for profit maximization but their market price uncertainties have been ignored [17, 18]. Though, the correlation of carbon market prices with coal, natural gas and electricity prices has been considered for selection of fuel mix for generation technologies of the power sector [19]. Still, researches are lacking to investigate the impact of carbon price uncertainty on medium term trading decision making of GenCos.

This paper aims to discover the impact of carbon price uncertainty on electricity trading portfolio optimization of a fossil fuel GenCo. For this medium term planning overall risk of revenue and cost sides i.e. electricity, fuel and carbon market prices, is considered. To handle the analogy between price uncertainties of revenue and cost sides a mean variance optimization approach has been used. Electricity trading portfolio of a price-taker GenCo, involves pool and bilateral contracts markets. This work highlights the significance of co-movement of different market prices, on optimal portfolio selection. Simulations based on realistic case study of Nordpool market illustrate the effectiveness of the proposed approach by providing efficient and improved risk management for fossil fuel GenCos.
II. PROBLEM DESCRIPTION AND FORMULATION

For a price taker fossil-fuel based GenCo a trading portfolio optimization problem is modelled for its medium-term trading decision making. For a medium term (month to year) planning and hedging decision making take precedence over operational and scheduling issues. For presumed generation GenCo plans its future electricity trading in pool and bilateral contracts markets. To meet its electricity production requirements, it procures fuel and carbon permits from their respective markets at competitive price. Long term contracts for fuel and carbon markets are ignored in this work [28].

In all trading markets, electricity, fuel and carbon prices are considered uncertain. Uncertainty in fuel and carbon permit prices adds risk to cost side, while uncertainty in electricity prices makes revenue risky. For efficient risk management in this situation, a GenCo should consider all these risks in an integrated framework, while developing its electricity trading portfolio. It has to allocate its produced output optimally, in multiple trading approaches, considering the interdependencies of these three markets. Power traded in pool and bilateral contract are decision variables in this multi objective optimization problem of maximizing profit and minimizing risk. In this work, the markets are assumed to be completely liquid and the GenCo is assumed to have sufficient emission caps. It is also assumed that generation is already scheduled.

A. Cost of Electricity Generation

The generation cost is considered to best represent operation behavior. Quadratic heat rate cost equation has been considered to calculate fuel and emission cost for required fuel and emission permits. Under ETS, emission cost calculations are based on the consideration that electricity production with emission requires direct purchase of emission allowances from the market for the quantum of produced emission [14]. CO₂ emissions are related to the quantum of fuel consumed, and can be calculated using emission factor ef [22]. So, for the total cost calculation of electricity generation, considered fuel cost $C^F$ and emission cost $C^E$ can be expressed in terms of heat rate function $\phi(P_i^g)$ as:

$$\phi(P_i^g) = a (P_i^g)^3 + b P_i^g + c$$ (1)

$$C^F = \sum_{i=1}^{I} \phi(P_i^g) \lambda_i^F$$ (2)

$$C^E = \sum_{i=1}^{I} \phi(P_i^g) e_i \lambda_i^E$$ (3)

Where $i$ is the index of trading interval for the planning period $I$; $a$, $b$, $c$ are heat rate constants for a generating unit and $e_i$ represents emission factor to calculate required emission permits for a particular amount of electricity generation, $t$ is time in each trading interval (hours); $\lambda_i^F$ is the fuel price and $\lambda_i^E$ is the carbon price.

B. Revenue Generated from Sale

Fossil fuel GenCo aims to fix its future trading plan for the planning period $I$, for an optimal allocation of its scheduled generation $P_i^o$ between spot market and bilateral contracts. Revenue generated from the spot market $R^S$ and bilateral contract market $R^B$ are calculated as

$$R^S = \sum_{i=1}^{I} \lambda_i^S P_i^o$$ (4)

$$R^B = \sum_{i=1}^{I} \lambda_i^B P_i^o$$ (5)

Where $\lambda_i^S$ is spot market price and $\lambda_i^B$ is bilateral contract price, while $P_i^o$ and $P_i^e$ are power traded in spot market and bilateral contract respectively, each for $i^{th}$ trading interval.

C. Total Profit

Total profit of the GenCo $\pi_c$ can be calculated as the difference of revenue generated by selling electricity in different contracts and involved generation cost, as

$$\pi_c = (Revenue - Cost)$$

$$\pi_c = R^S + R^B - C^F - C^E$$ (6)

The expected value of future profit can be obtained by considering expected future prices of different markets, for each trading interval. Bilateral contract prices are deterministic, i.e. known at the time of planning, so expected values are not relevant in their case. So, expected profit is:

$$\pi_c^{Exp} = \text{Exp}_{x_i^S, x_i^B, x_i^E} \{ R^S + R^B - C^F - C^E \}$$ (7)

$$\pi_c^{Exp} = \text{Exp}_{x_i^S, x_i^B, x_i^E} \{ R^S - C^F - C^E \} + R^B$$ (8)

$$\pi_c^{Exp} = \text{Exp}_{x_i^S, x_i^B, x_i^E} \{ t \sum_{i=1}^{I} \lambda_i^S P_i^o - t \sum_{i=1}^{I} \phi(P_i^o) \lambda_i^F \}

$$

$$+ t \sum_{i=1}^{I} \lambda_i^B P_i^o$$

The expected future prices of different markets can be calculated from various forecasting techniques. A precise forecast may help a efficient portfolio selection, but not the focus of this paper, thus an average of historical price vector has been considered as expected value.

D. Uncertainty Model

In Markowitz mean variance theory, variance is considered as a measure of risk. This theory seeks to reduce the variance of profit function [20]. This also considers dependencies of risk factors affecting the profit function and highlights the importance of correlation between trades.

This theory stipulates that for portfolio optimization, selection of any trade is not solely dependent on the characteristics that were unique to it. Rather, its co-movement with others is also important. Reflection of these co-movements provides an efficient portfolio with better risk management, than a portfolio constructed by ignoring the interactions between securities [21].
Total risk of expected profit function, considering the inter-dependencies, can be evaluated as:

\[ \pi_{\text{var}} = \text{Var}(\lambda^{s}, \lambda^{e}) \ni \{ R^s - C^F - C^E \} \]  
\[ (10) \]

\[ \pi_{\text{var}} = \text{Var}(R^s) + \text{Var}(C^F) + \text{Var}(C^E) - 2\text{Cov}(R^s, C^F) - 2\text{Cov}(R^s, C^E) + 2\text{Cov}(C^F, C^E) \]  
\[ (11) \]

\[ \pi_{\text{var}} = \sum_{i=1}^{I} \left( \text{Var}(P^s) \right)^2 \text{Var}(\lambda^{s}) + \sum_{i=1}^{I} \left( \text{Var}(P^e) \right)^2 \text{Var}(\lambda^{e}) \]  
\[ + \sum_{i=1}^{I} \sum_{j=1}^{I} \phi(P^s) \phi(P^e) \text{Cov}(\lambda^{s}, \lambda^{e}) \]  
\[ (12) \]

Where (12) considers variance of expected profit function of (9). Variance of market prices, \( \text{Var}(\lambda^{s}) \), \( \text{Var}(\lambda^{e}) \), \( \text{Var}(\lambda^{s}) \) and covariance between price vectors of different markets \( \text{Cov}(\lambda^{s}, \lambda^{s}) \), \( \text{Cov}(\lambda^{s}, \lambda^{e}) \), \( \text{Cov}(\lambda^{e}, \lambda^{e}) \) for each trading interval \( i \), can be statistically calculated [23]. Covariance represents the correlation between two prices, i.e. how the two prices are mutually co-related, over each time interval.

To manage the risk defined in (12), a GenCo selects a tradeoff between profit and risk, to optimize its portfolio, appropriate to its risk bearing capacity. This is represented by risk weighing factor \( \beta \), which reflects the risk taking desire of a GenCo, to maximize profit and minimize the involved risk. Higher values of \( \beta \) represents a strong risk averse nature of GenCo which selects portfolio with less risk in expected profit. There exists a trade-off between profit and risk. As the GenCo seeks higher profit it has to bear higher risk while if it seeks to reduce risk of expected portfolio profit it has to compromise with profit.

To maximize profit and minimize the involved risk, overall objective function \( Z \) is:

\[ \max_{P^s, P^e} Z = \pi_{\text{exp}} - \beta \pi_{\text{var}} \]  
\[ (13) \]

\[ P^G = P^s + P^e \forall i \]  
\[ (14) \]

Final portfolio selection depends upon the scores of objective function \( Z \) obtained for each portfolio, varying with the risk bearing desire of GenCo. Higher values of \( Z \) are assigned to portfolios with more attractive tradeoff between profit and risk, i.e. portfolio with high \( Z \) scores have higher expected profit but lower volatility, and vice versa.

The following simplified analytical calculation validates the impact of correlation between various market prices, on optimal energy allocation. The objective function \( Z \) would be maximized for optimal allocation in risky spot market \( P^s \).

Where, value of \( P^s \) is considered from (14) as \( P^s = P^G - P^s \).

Considering a fixed total generation, (13) is differentiated to obtain the optimum allocation in risky spot market, as

\[ \frac{\partial Z}{\partial P^s} = t \exp(\lambda^{s}) - t\lambda_{b} \]  
\[ -2\beta t^{2} \left[ \text{Var}(P^s) \text{Var}(\lambda^{s}) \right] \]  
\[ -2\beta t^{2} \sum_{i=1}^{I} \phi(P^s) \text{Cov}(\lambda^{s}, \lambda^{e}) \]  
\[ (15) \]

\[ P^s = \frac{\exp(\lambda^{s}) - \lambda_{b}}{2\beta t^{2} \text{Var}(\lambda^{s}) + \text{Var}(\lambda^{e}) + \text{Cov}(\lambda^{s}, \lambda^{e})} \]  
\[ + \text{Var}(\lambda^{s}) \]  
\[ (16) \]

The optimal risky allocation shown in (16) represents GenCos' electricity allocation in risky spot market. It depends upon the correlation of electricity market prices with fuel and carbon market prices.

A positive correlation of electricity prices, with other market prices, would enhance allocation in risky spot market. This represents the fact that a strong correlation between revenue and cost would enhance the investment in that trade, even if it is risky. This happens because their combined risk may be reduced due to positively correlated prices. Conversely, negative correlation would reduce the allocation in that trade, for better risk management.

III. RESULTS AND ANALYSIS

A fuel fired generation company has been considered for case study (specifications shown in Table I). Two types of fuel, coal and gas are, considered, each associated with certain carbon. Based on the fuel type, emission factors are estimated for CO2 emissions [22]. The planning period is one month and trading interval is one day. GenCo sells its total capacity as scheduled generation in day-ahead spot market and by bilateral contract. For procuring fuel and carbon permits, it directly trades in spot markets of fuel and carbon permits. Simulations are performed over several months, and one analysis as example is presented hence.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>GENERATING UNIT SPECIFICATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Type</td>
<td>Gas</td>
</tr>
<tr>
<td>Generation capacity</td>
<td>400 MW</td>
</tr>
<tr>
<td>Quadratic heat-rate coefficient</td>
<td>0.000115MBtu/MW</td>
</tr>
<tr>
<td>Linear heat-rate coefficient</td>
<td>4.515MBtu/MW</td>
</tr>
<tr>
<td>No-load heat-rate coefficient</td>
<td>185MBtu</td>
</tr>
<tr>
<td>Emission Factor</td>
<td>0.054 tCO2/MBtu</td>
</tr>
</tbody>
</table>

A. Data

The analysis is made by using historical data of Aug. month from 2008 to 2012, of electricity from Nordpool [25], of fuel
from Nordpool Gas [26] and emission as spot European Union Allowance (EUA) from Bluenext exchange [27]. Expected value of prices for each market is considered as the average price of each trading interval. Coal prices are assumed randomly due to unavailability of data. Bilateral contract prices are assumed fixed at 40 €/MWh for each considered scenario. Each EUA represents a right to emit 1 ton of CO$_2$ in the atmosphere.

### TABLE II  CORRELATION COEFFICIENTS WITH FUEL TYPE COAL

<table>
<thead>
<tr>
<th></th>
<th>Electricity</th>
<th>Coal</th>
<th>EUA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>1</td>
<td>0.6821</td>
<td>0.8871</td>
</tr>
<tr>
<td>Coal</td>
<td>0.6821</td>
<td>1</td>
<td>0.5975</td>
</tr>
<tr>
<td>EUA</td>
<td>0.8871</td>
<td>0.5975</td>
<td>1</td>
</tr>
</tbody>
</table>

### TABLE III  CORRELATION COEFFICIENTS WITH FUEL TYPE GAS

<table>
<thead>
<tr>
<th></th>
<th>Electricity</th>
<th>Gas</th>
<th>EUA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>1</td>
<td>0.3206</td>
<td>0.8871</td>
</tr>
<tr>
<td>Gas</td>
<td>0.3206</td>
<td>1</td>
<td>0.0914</td>
</tr>
<tr>
<td>EUA</td>
<td>0.8871</td>
<td>0.0914</td>
<td>1</td>
</tr>
</tbody>
</table>

Variance-covariance matrices between the price vectors of different markets, for each trading interval, have been calculated using appropriate functions in MATLAB® [23]. The matrices represent the correlation between these price vectors. All matrices could not be shown in paper. For understanding, average correlation matrices of different market prices, each with certain fuel type are represented in Tables II and III, respectively. Correlation varies from -1 to +1. A positive correlation represents co-movement of two data vectors while its negative value represents that the fluctuations between the two are opposite. The considered data has high correlation between electricity and carbon permits market prices, while a lesser correlation of electricity prices with fuel prices. Present analysis is made on the historical data corresponding to Phase II of EU-ETS, but carbon price risk would have a considerable impact on the upcoming Phase III. A complete auction based purchase mechanism for power sector would lead to a higher correlation between electricity and carbon prices.

### B. Scenario Consideration

For analysis, three scenarios are considered: in Scenario I carbon prices are deterministic, so no uncertainty or correlation with other markets is considered. In Scenario II, uncertainty of all markets and their correlation are considered. In Scenario III, carbon markets are considered independent from other markets, so carbon uncertainty is considered, but its correlation with other markets is neglected.

### C. Simulation and Result Discussion

Gas and coal fired GenCos are analyzed individually for portfolio selection, to trade their generated output. The generation cost and revenues corresponding to different contracts are calculated using (1)-(5) for each trading interval, based on specification shown in Table I and prices of different markets. Overall expected profit and involved risk has been calculated using (9) and (12), considering all trading alternatives. On the basis of total expected profit and involved risk, the objective function (13), subject to constraint (14), is optimized. This NLP optimization problem has been solved by commercially available software GAMS, on its solver CONOPT [24].

The optimization is performed for various values of $\beta$, wherein each value of $\beta$ produces an efficient portfolio, in terms of profit and standard deviation. The contour of these portfolios is known as the efficient frontier. It represents risk averse nature of GenCo. Efficient frontier reflects the fact that with increasing risk averseness, both standard deviation and expected profit of portfolio decrease. It means that higher profit seeking GenCo has to bear higher risk and a lesser risk seeking GenCo has to compromise with profit. So, GenCo has to select an optimum trade-off between profit and risk, according to its risk bearing capacity, for desired value of profit.

1) **Impact on Coal-Fired GenCos**: For a fixed generation of 400MW, coal fired GenCo selects its optimum trading portfolio under each scenario, to opt the most profitable but secure position. For all three considered scenarios, optimum portfolios are obtained, and are shown in the form of efficient frontiers in Fig. 1. Each frontier has a different profit-risk profile for similar values of $\beta$. Scenario I considers deterministic carbon prices, i.e. when no carbon price uncertainty is considered, efficient frontier and optimal allocation in spot and bilateral market is as shown in Fig. 1 and 5. Scenario II considers uncertainty of carbon market and its co-movement with other markets. The efficient frontier in this case shows higher profits with lower risks, as compared to Scenario I, i.e. better trade-off in terms of profit and risk for similar values of $\beta$. This happens due to positive correlation between electricity and carbon prices. When the revenue of a particular trade commoves with cost, an investor would feel more secure, and would invest more in that trade. By this, generation cost variations can be compensated. In this case, due to strong correlation between prices of carbon and electricity markets, for a same risk aversion level, more energy can be allocated to the spot market for uncertain carbon prices as in (16). It means that price change in cost will be compensated by price change in revenue, so the risk of this situation is better controlled with positively correlated revenue and cost. Scenario III does not consider correlation of prices between carbon and electricity markets, and the energy allocation remains same as that for Scenario I. The efficient frontier offers highest risk, as compared to other scenarios.
This analysis shows that carbon price uncertainty enhances the risk of overall profit, but its correlation with electricity market prices may help to reduce risk, by higher allocation in the spot market. Hence, correlation considerations are important for the actual realization of trading strategy. If these prices were negatively correlated, then energy allocation in risk free bilateral contract would increase, and that in spot market would decrease. This situation would be more risky for a GenCo and dis-incentivize it to trade in that particular asset. In case of zero correlation between carbon and electricity markets, the allocation in different markets remains same as that for Scenario III. Here, the GenCo cannot compensate carbon risk by altering its trading decisions in the market.

2) Impact on Gas-Fired GenCos: In case of gas fired GenCos, emissions are comparatively low, so impact of emission cost and its uncertainty is also small as compared to that for coal fired GenCos. This difference can be visualized by a comparison of Figs. 1 and 3, where the latter shows a relatively small shift in the frontier of Scenario III from Scenario I. Simulation results shown in Fig. 3 indicates that positive correlation between carbon and electricity markets may work as risk protection here also. For the same risk aversion level, more electric energy can be allocated to the spot market, when carbon prices are considered uncertain, as compared to Scenario I. But here, the alteration in trading decisions is smaller, than in the case of coal fired GenCo, as visualised from Figs. 2 and 4.

Finally, it can be concluded that carbon price uncertainty significantly impact trading decisions of fuel fired GenCos. A string correlation of cost side and revenue side markets leads to risk hedging of portfolio by allocating more in spot electricity markets. This happens because price fluctuations of cost side markets are compensated by revenue side markets. This impact is prominent in the case of high-carbon coal fired GenCo, than a gas fired GenCo. Further, a better tradeoff between profit and risk is obtained for a fossil fuel GenCo.

IV. CONCLUSION

Carbon prices and their volatility consideration are important in the upcoming power sector scenario. A high emitting GenCo must consider these changes while selecting its electricity trading strategy. A fuel fired GenCo may efficiently manage the risk of its trading portfolio, considering the impacts of emission cost and its uncertainty. A mean variance portfolio theory offers optimal electricity trading portfolio for emitting GenCos, considering the analogy of cost and revenue side risks.
A realistic case of Nordpool electricity market illustrates that carbon prices and their correlation with electricity market prices, affect the optimum profit-risk trade-off and energy allocation of a GenCo. The results also indicate that carbon price uncertainty may lead to significant alteration of trading decisions, for better risk management of emitting GenCos. It is observed that the overall risk of total expected profit function can be reduced by investing more in the spot market, because its prices co-move with the emission prices. In such a situation, investing more in the spot market may also hedge risk, as the risk of price change in carbon market would be compensated by a price change in the spot market. A comparative analysis of coal and gas fired GenCo represents that carbon price uncertainty has a higher impact on trading of high emission GenCos. The presented work can be extended for trading portfolio optimization, involving multiple trading options, with their individual returns and securities.

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