Improved Fault Location Computation using Prony Analysis for Short Duration Fault

Faraz Zafar Khan, R. Nagaraja and H. P. Khincha
Power Research and Development Consultants Pvt. Ltd., Bangalore

Abstract—Estimating accurate phasor calculations and selection of best fault data window are the basic requirements of any fault location algorithm. This paper highlights the issues with conventional Fourier transform based filtering techniques to compute fundamental components. It also discusses the effect of RDFT on fault location especially in case of short duration faults. The paper introduces an improved fault location scheme based on Prony analysis. The comparison of RDFT filter and Prony analysis for the computation of fault location is also presented. Performance of the proposed scheme is showcased with the help of case studies and practical data.

Keywords—Fault location; Prony analysis; Short duration fault

I. INTRODUCTION

Digital filters are used to compute fundamental components of voltage and currents, which can be further used as inputs for various applications. A number of digital filters and their modifications are reported in literature [1, 2]. Some widely used filtering algorithms are Discrete Fourier Transform (DFT), Cosine filter, least square and wavelet transform. Comparison of these methods shows their suitability for ideal conditions. However the performance is usually affected in practical conditions as the waveform contains harmonic content, system transients, decaying dc offset under varying instant of fault and fault resistance [3]. Extracting frequency component using Prony estimation is well known. It gained more popularity with the introduction of Phasor Measurement Unit technology. However the application is restricted to monitor low frequency oscillations during steady state, though it has advantage over other signal processing techniques under system transients [4-6].

The Discrete Fourier transform (DFT) is a popular filtering technique having two variants, Non recursive (NDFT) and Recursive (RDFT). The recursive form is commonly used, since RDFT require less number of mathematical operations compared with NDFT. Implementing RDFT in numerical relay pose challenges in the form of accumulated error in the estimated magnitude and phase during real time operation due to input signal dynamics and frequency drifts [7]. In general, the response time of RDFT is around one cycle, which is bound to increase if the input is non sinusoidal in nature. This indicates that output of the filter may fluctuate for definite period of time and settles to final value depending on the system conditions. If these values are further used for any power system computation, possibility of errors due to variation in filter output cannot be ignored.

Fault location is one such application which requires fundamental components of voltage and current as inputs. Determining actual fault location and its cause can help the power system operators. This information reduces the system down time and increases the system reliability, which has imposed a requirement for fast and accurate fault clearance. Faults must be cleared fast and selectively to avoid widespread damage to the power system [8]. This role is assigned to relay and the associated circuit breaker. Relay senses the abnormal conditions and gives signal to the circuit breaker for its operation. Thus fault clearing time is summation of relay response time and time of circuit breaker to isolate the fault. As the relay and breaker technology is advancing, the fault clearing time is reducing consequently. However information about the short duration fault may give limited data for processing and it may pose issues in accurate computations of fault location [9]. Thus precise phasor estimation technique can improve computation of fault location especially for short duration faults.

II. SHORT DURATION FAULTS

Short duration faults are defined as the faults that are cleared within two power frequency cycles [9]. If the faults are cleared fast, the current may not reach its faulted steady state value and the voltage may not drop to its faulted steady-state value. Such information collected may give an incomplete picture about the faulted event and if this data is further utilized for analysis such as fault location, the results obtained may be misleading. Following a fault, the voltage and current waveforms possess the fundamental frequency signal along with the harmonics and the DC component. The presence of noise results in errors when estimating the fundamental components. Most fault location algorithms perform best when accurately measured fundamental voltage and current quantities are available. The accuracy of algorithms requires efficient signal filtering technique and signals of long enough duration to measure.

III. ISSUES WITH CONVENTIONAL FILTERING

The Discrete Fourier transform (DFT) extracts fundamental components of voltage and current from the signal. For an ideal sinusoidal input the response time of DFT is around one cycle, which is bound to increase if the input is non-sinusoidal in nature. The output of DFT is also found affected, if the system works on off-nominal frequency [1]. Typical plot for current
with instantaneous and r.m.s values computed using RDFT is shown in Fig. 1.

![Fig. 1. Fault current with instantaneous and corresponding r.m.s value](image1)

It is observed that initially the r.m.s values of current are almost constant and after the occurrence of fault, the current magnitude tries to rise gradually in the first cycle. This variation is highly dynamic and if the same window is utilized for any application, it may give uncertain results. The current value is maintained almost constant in the period of second and third cycle but differ in magnitude. This variation in magnitude is significant and it may also affect the application results. Beyond second cycle, the fault current is almost constant and can be considered as the best value for further computations. In practical cases, filtering techniques require more than one cycle time for stabilizing the output and if the fault is cleared before settling of the value, possibility of error cannot be ignored.

![Fig. 2. Two bus system](image2)

In order to better understand the phenomena, a simple two bus system of 400 kV and line length 100 km is considered as shown in Fig. 2 and the details of transmission line parameters are given in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive sequence resistance</td>
<td>0.03</td>
<td>ohm/km</td>
</tr>
<tr>
<td>Positive sequence reactance</td>
<td>0.3</td>
<td>ohm/km</td>
</tr>
<tr>
<td>Zero sequence resistance</td>
<td>0.3</td>
<td>ohm/km</td>
</tr>
<tr>
<td>Zero sequence reactance</td>
<td>0.75</td>
<td>ohm/km</td>
</tr>
</tbody>
</table>

A single line to ground fault in R phase is created on Bus2 at 0.1s and the corresponding voltage and current samples are collected by the relay to determine the fault location. Initially the data is passed through fault detection and classification algorithm to identify the presence of fault and classify its type [10]. Then fault data is passed to fault location algorithm for further processing. A simple reactance based fault location algorithm is considered for analysis [9].

The per unit distance to the fault ‘m’ is given as:

$$m = \text{Im} \left( \frac{V_x}{I_x} \right) / \text{Im}(Z_1)$$  \hspace{1cm} (1)

For single line-to-ground fault, fault location is computed as:

$$m = \text{Im} \left( \frac{V_x}{I_x + 3k_0I_0} \right) / \text{Im}(Z_1)$$  \hspace{1cm} (2)

Where,

- \(I_0\) is the zero sequence current
- \(Z_0\) and \(Z_1\) are zero and positive sequence impedance of line.

Zero sequence compensation factor: \(k_0 = \frac{Z_0 - Z_x}{3Z_1}\)

Fault location value computed by using (2) is shown in Fig. 3. It is observed that the fault location computed in the pre-fault region is very high which indicates healthy system condition. However after the occurrence of fault the location settles to a much lower value as seen from the post-fault region.

![Fig. 3. Fault location value computed](image3)

In order to have insight of the variation in fault location, zoomed portion of the faulted region is shown in Figure 4. The variation of fault location will be very high in the first cycle due to response time of filter as explained earlier. Fault location value is observed to be damping for the consecutive cycles and oscillate around the value of 95 km. The final value obtained at the end of simulation at 1s is 94.965 km. The difference in fault location of around 5km from actual value may be due to the inherent error of the simple reactance based algorithm in computing fault location.

![Fig. 4. Zoomed portion of faulted region](image4)
The focus here is not to improve the existing algorithm by itself, but analyze the variation in final value possible due to filtering at different fault clearing time. Now assume if the same fault is cleared in two cycles i.e. at 0.14s, the fault location value at 0.14s is 93.61km. The minimum and maximum value in the cycle just before the fault clearing is 86.05km and 103.44km. Reported fault location values for different fault clearing time along with minimum and maximum value in the window is furnished in Table II.

### TABLE II. FAULT LOCATION CONSIDERING RDFT

<table>
<thead>
<tr>
<th>Fault clearing duration (Time)</th>
<th>Reported Fault Location (km)</th>
<th>Min. Fault Location (km)</th>
<th>Max. Fault Location (km)</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Cycle (0.02s)</td>
<td>93.61</td>
<td>86.05</td>
<td>103.44</td>
<td>17.39</td>
</tr>
<tr>
<td>2 Cycle (0.04s)</td>
<td>92.41</td>
<td>-1427.22</td>
<td>4358.57</td>
<td>5785.79</td>
</tr>
<tr>
<td>3 Cycle (0.06s)</td>
<td>94.22</td>
<td>89.16</td>
<td>96.93</td>
<td>10.96</td>
</tr>
<tr>
<td>4 Cycle (0.08s)</td>
<td>95.45</td>
<td>90.23</td>
<td>98.14</td>
<td>6.91</td>
</tr>
<tr>
<td>5 Cycle (0.10s)</td>
<td>94.71</td>
<td>92.58</td>
<td>96.93</td>
<td>4.35</td>
</tr>
<tr>
<td>6 Cycle (0.12s)</td>
<td>94.81</td>
<td>93.45</td>
<td>96.19</td>
<td>2.74</td>
</tr>
<tr>
<td>50 Cycle (1.00s)</td>
<td>94.96</td>
<td>94.96</td>
<td>94.96</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Thus effect of fault duration on computation of fault location is summarized as follows:
1) Fault location value is dependent on fault clearing time.
2) Higher the fault clearing time, more accurate is the fault location value.

To mitigate the effect of fault duration on computation of fault location, a scheme based on prony analysis is proposed and discussed in the following section.

### IV. PROPOSED SCHEME

In the proposed scheme, Prony analysis is used to determine the fundamental component of the voltage and current signals during the fault period. The values obtained are then passed through the fault location algorithm to compute the desired output.

#### A. Prony Analysis

Prony analysis extracts valuable information from a uniformly sampled signal and builds a series of damped complex exponentials or sinusoids [4]. The power system is highly dynamic in nature and output of a dynamic system can be expressed as sum of exponential functions corresponding to its eigen values is given in (3)

\[
y(t) = \sum_{i=1}^{n} R_i e^{\lambda_i t}
\]

Where,
- \(R_i\) is the output residual
- \(\lambda_i\) is the eigen value
- \(n\) is the order of system
- \(j\) is the number of outputs

In discrete form, output can be represented as:

\[
y(k) = \sum_{i=1}^{n} R_i z_i^k
\]

Where, \(z_i = e^{\lambda_i \Delta t}\) and \(\lambda_i = \sigma_i + j\omega_i\)

Equation (4) can be written in matrix form as:

\[
\begin{bmatrix}
y(0) \\
y(1) \\
\vdots \\
y(N-1)
\end{bmatrix} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
z_1 & z_2 & \cdots & z_n \\
\vdots & \vdots & \ddots & \vdots \\
z_1^{N-1} & z_2^{N-1} & \cdots & z_n^{N-1}
\end{bmatrix} \begin{bmatrix}
R_1 \\
R_2 \\
\vdots \\
R_n
\end{bmatrix}
\]

(5)

The \(Z_i\) are necessarily the roots of an nth order polynomial with unknown coefficients \(a_n\), and thus satisfies equation.

\[
z^n - (a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n z^0) = 0
\]

If we multiply both sides of (5) by (7), then we get eq. (8)

\[
\begin{bmatrix}
-a_n & -a_{n-1} & \cdots & -a_1 & 1 & 0 & \cdots & 0
\end{bmatrix}
\]

(7)

\[
y(n) = a_1 y(n-1) + a_2 y(n-2) + \cdots + a_n y(0)
\]

(8)

Same can be extended in matrix form as follows

\[
\begin{bmatrix}
y(n) \\
y(n+1) \\
\vdots \\
y(N-1)
\end{bmatrix} = \begin{bmatrix}
y(n-1) & y(n-2) & \cdots & y(0) \\
y(n) & y(n-1) & \cdots & y(1) \\
\vdots & \vdots & \ddots & \vdots \\
y(n-N+1) & y(n-N+2) & \cdots & y(N-1)
\end{bmatrix} \begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n
\end{bmatrix}
\]

(9)

The procedure of Prony method can be explained in the following three steps.
1. Solve (9) to get the coefficients \(a_n\).
2. Calculate the roots of (6) to get \(z_i\).
3. Solve (5) for complex residues \(R_i\).

The output parameters calculated are as follows:
- Amplitude: \(A_i = 2|R_i|\)
- Phase angle: \(\phi_i = \tan^{-1}(Im(R_i)/Re(R_i))\)
- Damping factor: \(\sigma_i = \frac{1}{T} \ln(|z_i|)\)
- Frequency: \(f_i = \frac{1}{2\pi T} \ln(arg(z_i))\)

#### B. Flowchart

Flowchart of the proposed scheme to determine the fault location is shown in Fig. 5. Initially consider the instantaneous values of voltage and current data during fault period along with the system frequency. Fault period is the time interval between fault initiation and fault clearing time. The faulted data window is analyzed using Prony analysis to compute various modes with frequency, amplitude and phase angle as the output parameters.
The parameters computed using Prony analysis for 2 cycle fault is shown in Table III. All the modes are scanned to determine the frequency mode nearest to the nominal value i.e. 50 Hz. Since actual system frequency will not be constant, Prony analysis will determine the components at actual system frequency unlike DFT which always computes at fixed nominal frequency. This is an added advantage of using Prony analysis as a filtering technique as compared with DFT. Another advantage of using Prony analysis is that it supplies only one output for the total fault period. Once the dominant mode is identified, corresponding amplitude and phase angle obtained for voltage and current signal is passed to the simple reactance based fault location algorithm. Thus fault location algorithm will also provide a single value for the whole fault period. The proposed method completely avoids the variation in fault location value which improves the analysis results.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Frequency (Hz)</th>
<th>Magnitude (kV/A)</th>
<th>Angle (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>49.99</td>
<td>53.30</td>
<td>-65.58</td>
</tr>
<tr>
<td>Y</td>
<td>50.00</td>
<td>230.01</td>
<td>-174.77</td>
</tr>
<tr>
<td>B</td>
<td>49.99</td>
<td>231.10</td>
<td>64.84</td>
</tr>
</tbody>
</table>

The sample two bus system discussed earlier is considered and fault location computed using prony analysis as the filtering technique are furnished in Table IV. It is observed that fault location value computed is almost constant for the different fault clearing time. The values obtained are very near to 94.96 km as computed by DFT for 50 cycle fault.

Comparing the results of Table II and Table IV highlights the better performance of the proposed scheme as compared with RDFT technique. The scheme is found effective to determine fault location, especially in the case of short duration faults.

V. CASE STUDY

An actual relay record collected from the field having phase to ground fault is considered, to analyze the performance of proposed scheme. Actual fault location reported by the relay is around 64 km and the fault period is equal to 0.0391s (approximately two cycles). Plot of three phase currents are shown in Fig. 6.

In order to compare the output of fault location algorithm employing RDFT and Prony analysis as filters, initially voltage and current samples are passed through RDFT to determine the fundamental component. Fault location computed by employing RDFT as a filtering technique is equal to 55.70 km with minimum value of 54.41km and maximum value of 2330.21km of fault location in the fault period. The variation in fault location value during the fault period is shown in Fig. 7.

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Table III. PARAMETERS USING PRONY ANALYSIS FOR 2 CYCLE FAULT

<table>
<thead>
<tr>
<th>Phase</th>
<th>Frequency (Hz)</th>
<th>Magnitude (kV/A)</th>
<th>Angle (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>49.99</td>
<td>53.30</td>
<td>-65.58</td>
</tr>
<tr>
<td>Y</td>
<td>50.00</td>
<td>230.01</td>
<td>-174.77</td>
</tr>
<tr>
<td>B</td>
<td>49.99</td>
<td>231.10</td>
<td>64.84</td>
</tr>
</tbody>
</table>

Table IV. FAULT LOCATION CONSIDERING PRONY ANALYSIS

<table>
<thead>
<tr>
<th>Fault clearing duration (Time)</th>
<th>Computed fault location (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Cycle (0.02s)</td>
<td>94.86</td>
</tr>
<tr>
<td>2 Cycle (0.04s)</td>
<td>94.94</td>
</tr>
<tr>
<td>3 Cycle (0.06s)</td>
<td>94.95</td>
</tr>
<tr>
<td>50 Cycle (1.00s)</td>
<td>94.95</td>
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Instantaneous values of voltages and current during the fault period are also passed through Prony analysis to determine the dominant mode around system frequency and the results obtained is furnished in Table V.

<table>
<thead>
<tr>
<th>Phase Voltage</th>
<th>Frequency (Hz)</th>
<th>Magnitude (kV/A)</th>
<th>Angle (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>50.0058</td>
<td>241.451</td>
<td>144.566</td>
</tr>
<tr>
<td>Y</td>
<td>50.5193</td>
<td>172.482</td>
<td>18.0044</td>
</tr>
<tr>
<td>B</td>
<td>49.9621</td>
<td>242.687</td>
<td>-97.554</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line Current</th>
<th>Frequency (Hz)</th>
<th>Magnitude (kV/A)</th>
<th>Angle (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>50.0819</td>
<td>526.137</td>
<td>-45.8305</td>
</tr>
<tr>
<td>Y</td>
<td>50.1701</td>
<td>5015.22</td>
<td>-60.1362</td>
</tr>
<tr>
<td>B</td>
<td>50.0959</td>
<td>380.882</td>
<td>85.0976</td>
</tr>
</tbody>
</table>

Prony analysis determines single value for each parameter over a fault period. By substituting these values in (2), the fault location is computed as 57.036 km. The fault location obtained using prony analysis is more nearer to the actual value by utilizing the same algorithm of fault location. This proves the superior performance of the proposed scheme for computation of fault location as compared with conventional RDFT filtering technique.

VI. CONCLUSIONS

An improved fault location scheme for short duration fault is presented in the paper. The proposed scheme is based on Prony analysis, which is used to extract the fundamental component. Issues with conventional filtering technique to analyze short duration faults are also highlighted. This paper presents a comparison between RDFT and Prony analysis to be used as filters for power systems applications. Results from simulation and practical data shows that the proposed scheme is effective in computing accurate fault location.

REFERENCES