Locating Mechanical Deformations in Electromagnetic Model of Transformer Winding

Shah Krupa Rajendra and Ragavan K
Indian Institute of Technology Gandhinagar, India

Abstract—This paper presents a diagnostic method for locating and assessing the severity of mechanical deformations in an electromagnetic model of transformer winding. The proposed method involves performing frequency response analysis on the model and capturing its high frequency behavior through terminal measurement. The terminal quantities are natural frequencies, magnitude and phase of impedance, neutral current, dc resistance, equivalent inductance and capacitance. Utilizing these data and performing constrained-optimization, a physically realizable, electrically and magnetically coupled ladder network pertaining to healthy model is synthesized. Then, mechanical deformations in model are introduced by producing discrete changes in capacitances. Once again synthesizing ladder network is achieved corresponding to faulty state of the electromagnetic model. Comparing the elements of those synthesized circuits will identify the fault location together with its severity. The electromagnetic model is capable of producing high frequency behaviour of winding. Hence, applicability of the proposed method to the transformer winding is feasible.

Index Terms—Circuit synthesis, condition monitoring and diagnostics, electromagnetic model, frequency response analysis, transformer winding

I. MOTIVATION

Stability and reliability of power system is highly affected by the state of apparatus connected to it. Transformer is one of the crucial elements associated with it and hence its uninterrupted functioning is very important. Transformer is prone to short circuit forces, lightning impulses and switching surges. Among all, electromagnetic forces developed by short circuit currents are enormously high. Cumulative effect of exposure to such abnormality can cause permanent mechanical deformation in transformer [1]-[3]. As a result, different parts of windings and insulation system would possibly be subjected to stresses of higher value than the recommended values. All such events consequently would lead to catastrophic failure. It is therefore necessary to identify mechanical deformations during its incipient stage.

There exist various condition monitoring and diagnostic techniques for assessing the status of transformer. Frequency Response Analysis (FRA) is one of the widely accepted techniques for detecting the mechanical deformations in transformer windings [4]-[9]. This method involves acquiring steady-state response to sinusoidal excitation and the generation of sinusoidal signals of certain frequency is not difficult. Thus, the measurement is repeatable making the judgement as very reliable. Further it has higher sensitivity in detecting winding deformation [4] and offers higher signal to noise ratio [5]. However, for FRA to be effective diagnostic tool, the location of the fault is to be identified and its seriousness to be understood for the remedial measures. This research work is aimed at using FRA data for identifying location of the mechanical deformation and assessing its severity in transformer winding.

Mechanical deformations are reflected in high frequency region and hence, it is essential to characterize high frequency behaviour of the winding. It is reported in [10] that the transient behaviour, internal and external performance, natural frequencies of the winding can be reproduced (with more damping) by constructing its electromagnetic model. It is scale-down model of the winding with provision for connecting capacitors externally. By selecting appropriate values of capacitance, it is possible to reproduce high frequency behaviour of transformer winding.

With electromagnetic model, external capacitances can be changed to emulate fault. Such flexibility does not exist with the winding. The damage to the winding would lead to permanent deformations and attaining healthy state is not possible afterwards. To this end, the objective is formulated as locating and assessing severity of mechanical deformations in electromagnetic model.

II. THEORY OF AN ELECTROMAGNETIC MODEL

In the past, for the purpose of predicting transient voltage distribution in power transformer windings, the corresponding geometrically scaled-down models were developed. However, there exist certain bottlenecks with such models:

- The geometrical model would have low internal capacitance, the value of which depends on the scaling factor. Hence, the transient behaviour of actual winding cannot be emulated directly.
- To correlate the transient responses, the scaling in physical dimensions should be compensated by the scaling in time. Suppose the actual winding is tested by a wave then its geometrical model scaled by ‘l’ is to be tested by l-times faster wave. However, there exist some issues in studying the effect of impulses with shorter wave front duration.

To overcome these limitations, electromagnetic models were developed which satisfy the equation (1).

\[ \mu \ell^2 = l^2 \]  

Here, \( l \), t, \( \epsilon \), and \( \mu \) are the scale factors (ratio of the respective values of the model to the original) of length, time, permittiv-
ity, and permeability respectively. It was concluded in [10] that electromagnetic model is successful in reproducing the performance of transformer winding by accounting both magnetic and electric fields. By connecting appropriate capacitances externally, the time-scale could be made as independent of length-scale. This becomes the chief feature of electromagnetic models.

To develop an electromagnetic model corresponding to transformer winding of single layer, 101 kVA, 22 kV and 128 turns, table in [11] can be used. With scale factors $N_t$, $l$ and $t$ as 0.5, 0.25 and 1, various quantities of model can be decided. Few of these quantities are presented in Table I.

Few of these quantities are presented in Table I.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Scaling</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>$\frac{P}{P_0}$</td>
<td>1578 VA</td>
</tr>
<tr>
<td>Voltage</td>
<td>$\frac{V}{V_0}$</td>
<td>688 V</td>
</tr>
<tr>
<td>Current</td>
<td>$\frac{I}{I_0}$</td>
<td>2.3 A</td>
</tr>
<tr>
<td>Turns</td>
<td>$\frac{N_t}{N_0}$</td>
<td>64</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$\frac{C}{C_0}$</td>
<td>16</td>
</tr>
</tbody>
</table>

The schematic arrangement of winding and its model is shown in Figure 1. Further, corresponding dimensions are given in Table II. Apart from this, it is necessary to calculate value of external capacitances pertaining to model. As per Table I, these capacitances are 16 times that of winding capacitances. Thus, the prime step is to calculate winding capacitances. To this end, following equations can be used [2].

- Calculating Effective Series Capacitance, $C_{s,eff}$:
  \[
  C_{s,eff} = \frac{\epsilon_0 \epsilon_i \pi 2ac}{\tau_i F}
  \]  
  (2)
  Where, absolute permittivity, $\epsilon_0$ is 8.854 x $10^{-12}$ and relative permittivity of insulation, $\epsilon_i$ is 7.

- Calculating Effective Shunt Capacitance, $C_{g,eff}$:
  \[
  C_{g,eff} = \frac{\epsilon_0 \epsilon_{air} \pi (D_l+GD_l)}{\tau} H
  \]  
  (3)
  With this, effective series- and shunt-capacitance of winding becomes 0.02 nF and 0.52 nF respectively. Thus, effective series- and shunt-capacitance of model becomes 0.32 nF and 8.32 nF respectively.

III. High Frequency Circuit Representation

The electromagnetic model can equivalently be represented by a linear circuit as shown in Fig. 2. This is because, for the high frequency excitation, most of the flux passes through air and core does not play much role [12].

The ladder circuit consists of $N$-sections that are coupled electrically and magnetically. Each section has series- and shunt- capacitances ($C_s, C_g$). The coil pertaining to any section could be represented as series connection of resistor ($r$) and inductor (of self inductance $l_s$). All these coils are magnetically coupled. The dielectric losses could be represented by resistors in parallel with capacitors. However, they can be safely neglected in almost all the cases.

For the purpose of building such ladder circuits, terminal data can be used. To this end, relevant literature was explored. In [13], a physically realizable ladder network was generated for identifying the discrete changes introduced in an air-core coil through terminal measurement. For this purpose, information on poles and zeros of its Driving-point impedance (DPI) function were utilized. However, the methodology was based on sequential iterations and hence the size of the circuit model was restricted. The non-gradient optimization approaches such as genetic algorithm and bacterial swarming algorithm were deployed in [14] and [15] respectively. Usually, these algorithms are computationally very expensive [16].

In this paper, gradient-based algorithm is used for synthesizing a physically realizable circuit of an electromagnetic model through terminal measurement. Since, the algorithm use decision based iterations, solving large size networks and obtaining quick convergence is possible.

IV. Proposed Methodology for Detecting Faults

To obtain the equivalent ladder circuits, FRA is to be performed on the electromagnetic model initially. Then, the variation in DPI with the frequency is to be plotted. This can be termed as FRA plot. The peaks and troughs in the plot can be termed as open circuit natural frequencies (ocnf, $f_o$) and short circuit natural frequencies (scnf, $f_s$) respectively [17]. The other parameters of the model namely dc resistance ($R_{dc}$), equivalent inductance ($L_{eq}$), equivalent capacitance ($C_{eq}$),

\[
\text{Fig. 1. Schematic representation of winding and its model}
\]
Algorithm 1 Constrained optimization for obtaining Reference circuit

1: The number of sections, $N$, in the circuit to be synthesized should be at least equal to the number of observable ocns [19].
2: The resistance $r$ in series with inductor can be given by,

$$ r = \frac{R_{dc}}{N} $$

3: flag ← 0
4: while (flag == 0) do
5: Variables:
   Initially, the model considered is free from faults. Hence, it is more appropriate to assume that it has symmetry and uniformity. As a result, its equivalent representation can be considered to have the parameters as mentioned below.
   - series capacitances are identical
   - shunt capacitances except the terminal sections are identical
   - self inductances are identical
   - mutual inductances between any two coils separated by same physical distance are equal
   The variables, $X$, of this optimization problem are,

$$ X = \left\{ \begin{array}{ll}
C_{si} = C_{s1}, & \forall i = 1, \ldots, N \\
C_{gi} = C_{g1}, & \forall i = 1, N + 1 \\
C_{gi} = 2C_{g1}, & \forall i = 2, \ldots, N \\
l_{si} = l_s, & \forall i = 1, \ldots, N \\
m_i, & \forall i = 1, \ldots, N - 1
\end{array} \right. $$

Thus, $X$ consists of $N + 2$ independent variables.
6: Initial guess and Bounds:
   There exists no constraint on choosing initial guess. The lower boundary can be set as zero. Hence, variables will not take negative values. The upper boundary can be kept open.
7: Constraints:
   The mutual inductance ($m_i, \ i \leq N - 2$) between the coils decreases with the increase in the physical separation and hence the following constraint could be formulated.

$$ \psi_{\text{inductance}} : \left\{ \begin{array}{l}
0.4 \ l_s < m_1 < 0.8 \ l_s \\
0.4m_i < m_{i+1} < 0.8m_i
\end{array} \right. $$

Now, equivalent inductance, equivalent capacitance, effective shunt capacitance will be estimated from the synthesized circuit as indicated below (superscripted with *),

$$ L_{eq}^* = Nl_s + 2\sum_{i=1}^{N-1}(N - i)m_i $$

$$ C_{eq}^* = C_{g1} + \frac{1}{C_{s1}} + \frac{1}{C_{g2}} + \frac{1}{C_{s2}} + \cdots $$

$$ C_{g,e,f}^* = C_{g1} + \ldots + C_{g(N+1)} $$

These estimated values are expected to match the corresponding measured values within certain tolerance $T$. Hence the constraints could be formulated as,

$$ \psi_{\text{terminal quantities}} : \left\{ \begin{array}{l}
\left| \frac{L_{eq}^* - L_{eq}}{L_{eq}} \right| - \text{tolerance} \\
\left| \frac{C_{g,e,f}^* - C_{g,e,f}}{C_{g,e,f}} \right| - \text{tolerance} \\
\left| \frac{C_{eq}^* - C_{eq}}{C_{eq}} \right| - \text{tolerance}
\end{array} \right. $$

8: Objective function:
   With node voltages and inductor currents as state variables, develop state space model [20]. Then, estimate ocnf and scnf ($f_o, f_s$). Using the measured and estimated natural frequencies, an objective function is formulated as,

$$ \eta(X) : \left( \frac{f_o - f_o^*}{f_o} \right)^2 + \left( \frac{f_s - f_s^*}{f_s} \right)^2 $$

9: if ($\eta(X) \leq \eta_{\text{min}}$) then
10: flag = 1
11: else
12: $N = N + 1$
13: end if
14: end while

Once the values of $X$ for which the algorithm converges are identified, then the Reference-circuit could be built.

B. Synthesizing Deformed-circuit

For characterizing the deformed model by an equivalent circuit, same number of sections as in Reference-circuit can be used. However, the model cannot be considered uniform.
Hence, the number of variables to be estimated is more. This would require identifying few more constraints and are explained in Algorithm 2.

**Algorithm 2 Constrained optimization for obtaining Deformed circuit**

1: Variables, \(X\):
   The variables of this algorithm are the series capacitances (\(N\)), shunt capacitances (\(N + 1\)) and self- and mutual-inductances (0.5\(N(N + 1)\)).

2: Initial guess and Bounds:
   Elements of the Reference-circuit are chosen as the initial guess. The Algorithm can be run by assigning numerous values to upper and lower bounds. Boundaries that result into minimum value of objective function and constraints can be considered as the most appropriate limits.

3: Constraints:
   - The constraints on equivalent inductance, equivalent capacitance and the effective shunt capacitance are formulated as given in (8).
   - The difference between the measured and estimated values of DPI can be construed as constraint. From the FRA plot, the magnitude of DPI at any desired frequency can be directly obtained and is referred as \(|Z|\).

The DPI at the same frequency can also be estimated by following the state variable analysis [20]. The driving-point admittance on the node-basis can be expressed as,

\[
Y = [G] + [K]s + [T] \left( [L]s + [R] \right)^{-1} [T]^t \tag{10}
\]

where,
- \(s\): complex frequency
- \([G]\): conductance matrix on the node-basis
- \([K]\): capacitance matrix on the node-basis
- \([L]\): inductance matrix with its diagonal elements as self-inductances and non-diagonal elements as mutual-inductances
- \([R]\): diagonal matrix with resistances in series with inductors
- \([T]\): Transformation matrix with all of its main-diagonal and sub-diagonal elements as 1 and -1 respectively.

All the matrices of (10) are symmetric. For the circuit representation in Fig. (2), \([G]\) is null. With the use of estimated parameters in the equation (10) and substituting \(s = j\omega\), the value of driving-point admittance \(Y^*\) at that frequency can be determined. Its reciprocal would give the value of DPI at \(\omega\) (i.e. \(Z^*\)). Using the values of measured and estimated-DPI, the constraint could be formulated as,

\[
\psi_{\text{impedance}} : \frac{|Z| - |Z^*|}{|Z|} \text{ - tolerance} \tag{11}
\]

3: The reactive power at a particular frequency is linked with the input voltage, magnitude \(|Z|\) and phase \(\theta\) of the input impedance as,

\[
Q = \frac{V^2 \sin\theta}{2|Z|} \tag{12}
\]

If the reactive power determined using acquired FRA data is \(Q\) and the value of reactive power estimated in the synthesized circuit is \(Q^*\), then, the constraint could be formulated as,

\[
\psi_{\text{reactive power}} : \left| \frac{Q - Q^*}{Q} \right| \text{ - tolerance} \tag{13}
\]

3: Measurement of neutral current (\(I_n\)) can be done in frequency domain by exciting the model with sinusoidal signal. The same can also be estimated using below mentioned equation.

\[
I_n^* = I_N + sC_{sN}E_N \tag{14}
\]

Where,
- \(I_N\): current through the \(N^{th}\) inductor
- \(E_N\): voltage across the \(N^{th}\) capacitor (\(C_{sN}\))

From the measured and estimated values of neutral current, the constraint could be formulated as,

\[
\psi_{\text{neutral current}} : \left| \frac{I_n - I_n^*}{I_n} \right| \text{ - tolerance} \tag{15}
\]

4: Objective function:
   The objective function and the convergence criteria are considered to be the same as in Algorithm 1.

With the estimated values of \(X\), the Deformed-circuit can be built. A comparison between the Reference- and Deformed-circuits would reveal the extent of deformation of the electromagnetic model under inspection. *Note: The Algorithms are implemented in MATLAB with fmincon as solver.*

**V. EXPERIMENTAL INVESTIGATION**

For capturing the frequency behaviour of the winding, Precision Magnetics Analyzer (Model: 3260B, Make: WayneKerr Electronics) was used. With this instrument, it is possible to obtain the magnitude and phase of input impedance directly over wide range of frequencies from 20 Hz to 3 MHz. *Note: Any instrument that is capable of capturing the variation of impedance with frequencies can be used.*

The considered electromagnetic model has 9-equidistant taps with which external capacitances can be connected (series capacitance - connected between two consecutive taps; shunt capacitance - connected between tap and earth). Thus, the model is divided into 8-sections. Further, effective series- and shunt-capacitances are 0.32 nF and 8.32 nF respectively. Hence, value of series- and shunt-capacitance per section is considered as 2.5 nF and 1 nF respectively.

**A. Circuit realization of healthy model**

To fulfill the objective of synthesizing Reference-circuit, initially, FRA plot of the healthy model is obtained as shown in Fig. 3. Its natural frequencies are identified and listed in Table III. Then, terminal quantities of healthy model are obtained and presented in Table III.

In the FRA plot, only 5 peaks (that is, ocnsf) are well-pronounced and hence the initial value of \(N\) for the circuit to be synthesized is chosen as 5, for which the value of objective
function was found to be high (of the order of $6 \times 10^4$). Then, the Algorithm 1 converged to $N = 8$ with the value of objective function as low (of about 5). The corresponding synthesized circuit is shown in Fig. 4 and the values of mutual inductances between the coils are listed in Table IV. The natural frequencies of Fig. 4 are presented in Table V.

From the above table, it is clear that the winding resistance and inductance are not changed. Hence, only shunt- and series-capacitances are considered as variables while synthesizing Deformed-circuit. Once the features of the deformed winding are extracted, equivalent circuit is to be synthesized using Algorithm 2. For this purpose, the magnitude and phase of DPI are measured as indicated in Tables VII and VIII. The impedance information obtained at natural frequencies and outside the range of natural frequencies are used to formulate the impedance and reactive power based constraints respectively. Further, neutral current at different frequencies is measured and few of the readings are presented in Table IX. Those can be used to formulate few more constraints based on neutral current. The algorithm converged to the circuit shown in Fig. 6, the natural frequencies of which are presented in Table X.

By comparing the synthesized circuits (corresponding to healthy and deformed model), the major deviation in series- and shunt-capacitance was observed near section 3 and 6 respectively. Further, comparison of open and short circuit natural frequencies of the synthesized circuits with corresponding measured values shows that the deviation is below 2%. From the results presented, it is understood that the proposed algorithm is capable of characterizing the terminal behaviour of the

- $C_{s3}$ is decreased from 2.5 $nF$ to 2 $nF$
- $C_{g6}$ is increased from 1 $nF$ to 1.25 $nF$

This electromagnetic coil with the changes introduced will be referred as deformed model. Now, the objective is to realize this model coil by an equivalent ladder circuit with 8-sections. From the experimentally acquired FRA data (with the deformed model) in Fig. 5, the natural frequencies and terminal quantities are identified and presented in Table VI. The value of $R$ at different frequencies is approximated from dc resistance.
TABLE VII
IMPEDANCE MEASUREMENT AT CERTAIN NATURAL FREQUENCIES - DEFORMED MODEL

<table>
<thead>
<tr>
<th>f (kHz)</th>
<th>Z (Ω)</th>
<th>Z' (degrees)</th>
<th>Z&quot; (VAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>725</td>
<td>12.5k</td>
<td>46.80</td>
<td>156.18</td>
</tr>
<tr>
<td>342</td>
<td>3.3k</td>
<td>51.00</td>
<td>174.26</td>
</tr>
<tr>
<td>462</td>
<td>59</td>
<td>59.50</td>
<td>217.58</td>
</tr>
<tr>
<td>500</td>
<td>1.33k</td>
<td>63.50</td>
<td>242.50</td>
</tr>
<tr>
<td>615</td>
<td>145</td>
<td>67.50</td>
<td>267.45</td>
</tr>
<tr>
<td>635</td>
<td>281</td>
<td>71.75</td>
<td>301.40</td>
</tr>
<tr>
<td>700</td>
<td>130</td>
<td>76.00</td>
<td>335.30</td>
</tr>
<tr>
<td>725</td>
<td>210</td>
<td>80.25</td>
<td>381.80</td>
</tr>
</tbody>
</table>

TABLE VIII
REACTIVE POWER MEASUREMENT AT CERTAIN FREQUENCIES - DEFORMED MODEL

<table>
<thead>
<tr>
<th>f (kHz)</th>
<th>Z' (degrees)</th>
<th>Z&quot; (VAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-70.25</td>
<td>-115.68</td>
</tr>
<tr>
<td>2</td>
<td>-79.44</td>
<td>-123.38</td>
</tr>
<tr>
<td>10</td>
<td>-87.24</td>
<td>-103.65</td>
</tr>
<tr>
<td>15</td>
<td>-87.90</td>
<td>-104.87</td>
</tr>
<tr>
<td>90</td>
<td>-89.18</td>
<td>-30.97</td>
</tr>
<tr>
<td>95</td>
<td>-89.22</td>
<td>-29.35</td>
</tr>
</tbody>
</table>

TABLE IX
NEUTRAL CURRENT MEASUREMENT AT CERTAIN FREQUENCIES - DEFORMED WINDING

| f (kHz) | Phase - I_n | | I_n | (mAmp) |
|---------|-------------|--------|------|
| 1       | -70.25      | 115.68 |
| 2       | -79.44      | 123.38 |
| 10      | -87.24      | 103.65 |
| 15      | -87.90      | 104.87 |
| 90      | -89.18      | 30.97  |
| 95      | -89.22      | 29.35  |

Fig. 6. Synthesized circuit for representing deformed model (Resistances, inductances and capacitances are in ohm, mH and nF respectively)

TABLE X
NATURAL FREQUENCIES OF CIRCUIT - DEFORMED MODEL

<table>
<thead>
<tr>
<th>f_n (kHz)</th>
<th>f_p (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>126.2</td>
<td>263.2</td>
</tr>
<tr>
<td>339.7</td>
<td>458.3</td>
</tr>
<tr>
<td>498</td>
<td>614.4</td>
</tr>
<tr>
<td>627.6</td>
<td>711.5</td>
</tr>
<tr>
<td>723.1</td>
<td>711.5</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS

In this work, algorithms involving constrained optimization technique were utilized for obtaining ladder networks of electromagnetic model corresponding to its healthy and faulty states. A comparison of those two circuits would reveal the location and extent of deviation from that of reference value. As the outcome of the experimental investigation is promising, the proposed methodology could be considered reliable. Further, it is observed that the model is just replica of transformer winding. Hence, it would be possible to apply methodology directly to the actual transformer winding.

REFERENCES