Robust Global Control Strategies for Improvement of Angular Stability using FACTS and HVDC Devices

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Abstract—System-wide feedback signals made available by Wide Area Measurement Systems technology can be used in FACTS/HVDC based controllers for the improvement of angular stability. These global signals can facilitate the efficient use of controller effort to stabilize critical swing modes. This paper introduces a restricted global strategy which involves the use of specific global feedback signals which are available at the HVDC/FACTS locations. The strategy is expected to be robust to changes in the power grid as well as communication uncertainties. This paper presents a heuristic introduction to this strategy using a circuit analogy of a simplified model of a power system. Preliminary results on a small system are also presented.

Index Terms—HVDC, FACTS, WAMS, Angle Stability, Power Swing Damping

I. INTRODUCTION

Disturbances like load changes, faults and tripping of components trigger oscillations in a power system ("swings"), which are due to relative angular motion between the rotors of synchronous machines. Under certain operating conditions, the oscillations may be poorly damped or unstable. Large disturbances may also lead to a loss of synchronism between machines. Several methods have been proposed and implemented to improve angular stability. Fast acting excitation systems and a strong transmission system reduce relative angular deviations following a disturbance ("synchronizing effect"), thereby reducing the susceptibility to loss of synchronism. The damping of swings can be improved using auxiliary damping controllers, e.g., Power System Stabilizers in generator excitation systems.

The advent of Wide Area Measurement Systems (WAMS) [1] and the deployment of HVDC and FACTS devices have ushered in new possibilities in the design of robust and effective controls for improving angular stability. FACTS devices like Thyristor Controlled Series Compensator (TCSC) and Static Synchronous Series Compensator (SSSC) which are series connected devices, and Static VAR Compensator (SVC) and Static Synchronous Compensator (STATCOM), which are shunt connected devices, have been deployed in many large power grids. HVDC links (both Line Commutated Converter(LCC) and Voltage Source Converter-Voltage Sourced Converter(VSC) based) are also embedded in many synchronous grids to transfer bulk power over large distances. The fast response provided by the power electronic converters can be used to improve angular stability. However, swing mode controllability depends on their location in the grid and the quantity which is controlled (e.g., shunt-series real or reactive power).

Local feedback signals are an obvious choice for damping controllers, as communication reliability will not be a concern. However, local signals may have some constraints: a local feedback signal may contain, in addition to the critical modes of interest, several other non-critical modes. Therefore, control effort may be unnecessarily expended due to the presence of these non-critical modes. Moreover, controller design has to be constrained so as to prevent possible destabilization of these modes. Therefore, it is worthwhile to consider a wide choice of candidate feedback signals including global signals available from WAMS.

WAMS involve the use of time synchronized measurements obtained using Phasor Measurement Units deployed at several locations in a network. High accuracy system-wide relative angular information is made available by this technology, which can be made available to stabilizing controllers. These remote measurements can be used for the synthesis of signals with good observability properties (i.e., only the modes of interest are dominantly observable in them). However, if a controller is designed with remote measurements, it should be robust to a partial or complete loss of remote signals and communication latencies. The control strategy should also work well for different operating conditions including major changes in the network. This motivates us to look for special control strategies which can exploit the benefits of remote measurements, while maintaining a satisfactory level of robustness.

A strategy based on “collocated” sensor-actuator pairs, which was proposed for the damping of space structures [2], has generated interest [3] due to its inherent robustness. The strategy introduces damping in the system by mimicking the effect of viscous friction. Similar strategies for the damping of swings in a power system using certain local measurements and FACTS/HVDC devices (the actuators) have been described in [4]–[7].

In this paper, we present an extension of this concept to multiple devices and non-local feedback signals. The proposed strategy is a restricted global strategy in the sense that it uses specific local and remote feedback signals - the remote feedback signals used are those obtained from the other
FACTS/HVDC device locations only. Although the use of collocated control with multiple FACTS devices and global measurements has been explored by other authors recently [3], the signals are synthesized using generator speeds. However, the feedback signals which lend themselves to this strategy are, in fact, available at the FACTS/HVDC locations itself [7]. Recognition of this leads us to the control strategy presented in this paper, which is inherently robust to loss of communication and changes in the network. This paper presents an intuitive introduction to the concept using a circuit analogy and also presents preliminary numerical results by applying it to a three machine system.

### List of Main Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta)</td>
<td>Generator rotor angle</td>
</tr>
<tr>
<td>(\omega)</td>
<td>Generator rotor speed</td>
</tr>
<tr>
<td>(\omega_B)</td>
<td>Machine base speed</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Bus voltage phase angle</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Line current phase angle</td>
</tr>
<tr>
<td>(H)</td>
<td>Generator inertia constant</td>
</tr>
<tr>
<td>(I)</td>
<td>Line current magnitude</td>
</tr>
<tr>
<td>(I_{sh})</td>
<td>Shunt injected reactive current</td>
</tr>
<tr>
<td>(P_{sh})</td>
<td>Shunt injected real power</td>
</tr>
<tr>
<td>(P_{ser})</td>
<td>Series injected real power</td>
</tr>
<tr>
<td>(P_{dc})</td>
<td>Two terminal link DC link Power Flow</td>
</tr>
<tr>
<td>(P_e)</td>
<td>Generator Electrical Power</td>
</tr>
<tr>
<td>(P_{ij})</td>
<td>Transmission line power flow</td>
</tr>
<tr>
<td>(V)</td>
<td>Bus voltage magnitude</td>
</tr>
<tr>
<td>(V_{sh})</td>
<td>Series injected reactive voltage (voltage in quadrature with current)</td>
</tr>
<tr>
<td>(X_{ij})</td>
<td>Transmission line reactance</td>
</tr>
</tbody>
</table>

II. UNDERSTANDING POWER SWINGS: A CIRCUIT ANALOGY

The control concept of this paper can be easily understood if we consider a circuit analogy [8] of the electro-mechanical model of a power system, where the following is assumed:

1. Generators are represented by the classical model (voltage source of constant magnitude behind a transient reactance).
2. Generator mechanical power inputs and load active powers are constant.
3. Transmission losses are neglected.
4. Bus voltage magnitudes are constant(1.0 pu).

Further, let us consider the following analogous quantities:

- Power flow through a branch \(\leftrightarrow\) “Current”
- Time derivative of bus voltage phase angle \(\leftrightarrow\) “Voltage”

Note that the sum of power flows incident at a node is zero (Kirchoff ‘Current’ Law), and the sum of the time derivatives of bus voltage phase angle differences in a loop is zero (Kirchoff ‘Voltage’ Law). By this analogy, the small-signal behaviour of the simplified model of a power system can be represented by an L-C circuit: From the swing equation, the time derivative of the “voltage” across a generator is proportional to the “current” and therefore is analogous to a capacitor \(C\) (see Fig. 1a). Similarly, a lossless ac transmission line between buses \(i\) and \(j\) is analogous to an inductance \(L_{ij}\) because time derivative of the “current” is proportional to the “voltage” across it (see Fig. 1b).

![Fig. 1. Circuit Analogy of (a) Generator (b) AC Transmission Line](image)

The small signal analog circuit of a three machine system [9] is shown in Fig. 2. The oscillatory modes of the L-C circuit are the swing modes of the original power system.

III. CONTROL OF POWER SWINGS

A. Local Control Strategy using a DC Link

The main utility of the circuit analogy is that it suggests a control strategy which can improve angular stability with controllable network elements, say, a DC link - see Fig.3.

1. If power flow in the DC link is changed in proportion to the voltage phase angular difference between the buses to which it is connected (i.e. a “collocated” measurement), then the DC line behaves like an AC line, i.e., the effect is the addition of another inductive link \(L\) in the analogous circuit. This reduces angular deviations following disturbances (synchronizing effect).
2. If power flow is changed in proportion to the derivative of the voltage phase angular difference between the buses (i.e., the bus frequency difference), the DC line behaves like a resistance \(R\) in the analogous circuit. This creates a damping effect.

Both these effects are beneficial. There are certain advantages which are obvious if such a strategy is used:

1. The control strategy is simple and uses local measurements.
2. The introduction of “resistive” and “inductive” links will always enhance damping and synchronizing effects respectively, for all swing modes in the system (except for those modes which are not observable in the phase angular difference at the DC link location).
3. When multiple DC links are present, the same strategy may be applied for each of them (more resistors/inductors get added); their independent actions are not adversarial.

The use of bus frequency difference signal for power swing damping with DC link power control is not new, and has been derived earlier for a two machine system [5]. However, the circuit analogy shows that it is an appropriate strategy even for larger multi-machine systems and multiple links.

1. The figure shows a VSC-based DC link, but a Line-Commutated Converter(LCC) based link may also be used.
B. Multiple DC links: A Restricted Global Strategy

As mentioned in the previous section, the local strategy which emulates the effect of resistance and inductance in the analogous circuit may be used in every DC link present in the system. However, the circuit analogy suggests that with multiple links and global measurements, one may also be able to emulate “mutual inductance” or “mutual resistance”. This is depicted in Fig. 4 for a three machine system with two DC links. This strategy gives us additional degrees of freedom in the form of the parameters $\mathcal{M}$ and $\mathcal{R}_m$. These may be used to selectively increase the leverage on a particular swing mode.

The matrices shown in Fig. 4 are symmetric positive definite, which also implies that $L_1, L_2, R_1$ and $R_2$ are positive. A mutual loss of communication $^2$, is equivalent to setting $\mathcal{M}$ and $\mathcal{R}_m$ to zero. Therefore, we essentially revert to the local strategy in case of communication problems. While there will be a quantitative change in the damping and synchronizing effects due to this, it will not cause any harm since the local strategy to which we revert also contributes to positive damping and synchronizing effects.

The strategy for two DC links can be generalized to a larger number of links - the inductance and resistance matrices will have correspondingly larger sizes, but should be symmetric positive definite. If communication from/to a DC link is prob-

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$^2$Both Transmit and Receive: the loss of reception at location A from location B for a preset time will automatically stop transmission from location A to location B
lematic, then that DC link should use only local measurements and all other DC links should not use measurements from that link. Equivalently, all off-diagonal elements of the inductance and resistance matrices corresponding to the link from/to which communication is problematic are set to zero. Even if the values of other matrix elements is unchanged, this will preserve the symmetry and positive definitiveness of the matrices, and therefore the positive damping and synchronizing effects (see Appendix A).

C. Multi-terminal DC links

With a multi-terminal DC link, we can emulate multiple inductive and resistive links in the analogous circuit as shown in Fig. 5. If partial loss of angular information due to communication failure, say between buses 8 and 6 is detected, then \( R_2 \) and \( L_2 \) should be set to infinity, i.e., the inductor and resistor, \( L_2 \) and \( R_2 \), are removed. In such a case, the control law will continue to provide a positive damping and synchronizing effect.

D. Extension to FACTS Devices

The utility of the circuit analogy presented in the previous section is limited to suggesting control laws related to real power flows and phase angular differences. Actually, a larger set of actuator-sensor pairs and similar control laws can be identified which introduce damping and synchronizing effects
in the system. These can be inferred from a mathematical formulation which also uses the classical generator model as before, but replaces the assumption of constant voltage magnitude at buses [7]. Series and shunt reactive power devices can therefore be made eligible for similar control strategies using the corresponding actuator-sensor pairs - see Table 1.

One may also extend the restricted global strategy to combine dissimilar devices like a SSSC and a DC link, as shown in Fig. 6. For the scheme shown in the figure, the signals $\Delta f$ and $\Delta \phi$ and the time derivatives are used to modulate $\Delta P_d$ and $\Delta V_f$ using gain matrices. A formal state space analysis using these input-output pairs is needed to prove this - see Appendix A.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>COLLOCATED ACTUATOR-SENSOR PAIRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator Variable</td>
<td>Sensed Signal</td>
</tr>
<tr>
<td>$\Delta P_{sh}$</td>
<td>$\Delta f$</td>
</tr>
<tr>
<td>$\Delta I_f$</td>
<td>$\Delta V$</td>
</tr>
<tr>
<td>$\Delta P_{ser}$</td>
<td>$\Delta V$</td>
</tr>
<tr>
<td>$\Delta V_f$</td>
<td>$\Delta f$</td>
</tr>
</tbody>
</table>

![Fig. 6. The Control Strategy with Dissimilar Devices: DC Link and SSSC](image)

IV. DISCUSSION

A. Local Strategies, The Restricted Global Strategy and other Global Strategies

The restricted global strategy uses the set of signals available at all the HVDC/FACTS locations to arrive at the control law at each of those locations. The main advantage of using this strategy is that one gains extra degrees of freedom (as compared to a local strategy) due to the off-diagonal elements in the gain matrices used in the control law. Therefore one may be able to selectively use the control effort to affect a critical swing mode. However, the extent to which this can be achieved depends on the actual locations of the HVDC/FACTS devices.

An alternative means to achieve the same end is to synthesize a modal signal which exclusively contains the critical swing mode, using a larger set of global signals. However, this synthesis is not robust to loss of communication or a change of system conditions. The restricted global strategy merely emulates self/mutual resistive and inductive effects and is therefore robust to change in system conditions. Since the strategy reverts to an acceptable local one if communication is lost, it retains a level of robustness to loss of communication. The control laws are also very simple and the effects of the parameters are more easily understood as compared to those obtained from a more mechanistic control design procedure.

B. Detailed Models

The introduction of synchronizing and damping effects by the use of the control laws described in the previous section is strictly valid only for a power system for which generators are modelled by the classical model, transmission losses are neglected and loads are of constant real power type [7]. Their efficacy needs to be verified using detailed models. Studies in [7] indicate that the strategies described in this paper are likely to work well with shunt real power devices (HVDC converters, STATCOMs with energy sources) and series real power devices (TCSC, SSSC).

Since the control laws have been derived using simple models, the gains will require to be “tuned” by studying the eigenvalue movement (for the detailed model) when the controller is introduced.

C. Implementation of the Control Laws

The control laws described in the previous section assume that the actuator plant time constants are small. The assumption is not unreasonable because HVDC/FACTS are very fast acting relative to the time scales associated with swing modes.

Direct use of phase angular differences for enhancing synchronizing effects will cause a steady state change in power flows if there is a change in the network, generation or load. If this is not desired, then a washout block may be provided in cascade with the gain block of the controller. The control laws improving damping using real power devices require the derivative of phase angle (i.e., bus frequency), while other devices - see Table 1 - require derivatives of bus voltage magnitude, line current/angle magnitude.

The bus frequency computation can be modelled by a first order transfer function $\frac{s}{1+Ts} \Delta \phi(s)$. Typically $T$ will be small - about 10 to 40 ms - and is unlikely to affect the control law significantly. Incidentally, the phase delay due to this transfer function creates a synchronizing effect. This is because, at the swing mode frequency $\omega$, $\frac{s}{1+j\omega} \Delta \phi(j\omega)$ has a component in phase with $\Delta \phi(j\omega)$. This is not entirely undesirable.

In practice, instead of directly using proportional and approximate derivative functions, one may use phase-lead transfer functions which mimic the frequency response of the desired control law in the frequency range of the swing modes. The design outside this frequency band should be done to prevent destabilization of other modes.

Another issue is communication latency. This can be variable, but for this application it is reasonable to expect that
practically all signal packets will be made available with latencies less than 100 ms. A fixed latency 3 can be compensated by a phase lead transfer function.

V. ILLUSTRATIVE EXAMPLE

We now present an illustrative example based on the three machine system of Fig.3. A detailed generator model is considered. The generators are equipped with static excitation systems with AVRs which are modelled by a single time constant transfer function and gain, 250 pu/pu and 0.05s respectively. The loads are of constant impedance type. The two swing modes of the system and their characteristics are described in Table II. Note that the modes are not well damped. The eigenvalues are obtained using a MATLAB [10] based small signal analysis program.

<table>
<thead>
<tr>
<th>Mode Description</th>
<th>Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Swing Mode I</strong>: Generators 2 and 3 swinging against Generator 1</td>
<td>-0.0556 + j 8.6550</td>
</tr>
<tr>
<td><strong>Swing Mode II</strong>: Generator 3 swings against Generator 2, with low participation of Generator 1</td>
<td>-0.1353 + j 13.2844</td>
</tr>
</tbody>
</table>

Two VSC based DC links are introduced in the system as shown in Fig. 4 in order to study the effects of the control strategy. The quiescent power flow in both links is set to zero, so that the study can be done for the same equilibrium condition as before. The small-signal analysis program models the dc links as controllable power injections at the buses to which they are connected. The reactive power injections by the DC links are assumed to be regulated to zero by the VSCs. We consider three sets of gain matrices (corresponding to the control law shown in Fig. 4) to illustrate the utility of the degrees of freedom. The primary concern in this system is the damping of the swing modes. Therefore, we consider only the “resistance” matrix - the “inductance” matrix is set to zero, i.e., \( L_2 = L_3 = M = 0 \). Also, the time constant in the approximate implementation of the derivative of phase angle (see the previous section) is assumed to be small: \( T = 10 \) ms.

The two eigenvalues of the “resistance” matrices in Cases I and II given above are the same (\( \mu_1 = 1.0, \mu_2 = 0.1 \)), and are positive. However, the eigenvectors are different in each case. Case III denotes the case where communication is lost (the mutual term becomes equal to zero) resulting in a local strategy - the eigenvalues of the gain matrix are 0.28 and 0.82. Case IV is also a local strategy but the eigenvalues of the gain matrix are the same as in Case I and II.

Case I: \( R_1 = 0.28, R_2 = 0.82, R_m = -0.36 \)
Case II: \( R_1 = 0.82, R_2 = 0.36, R_m = 0.36 \)
Case III: \( R_1 = 0.28, R_2 = 0.82, R_m = 0 \)
Case IV: \( R_1 = 0.1, R_2 = 1.0, R_m = 0 \)

The gains are in pu/(rad/s) on a 100 MVA base.

In Case I, we align the eigenvectors of the gain matrix such that the effect on Swing Mode I is preferentially higher. In Case II, we preferentially try to increase the effect on Swing Mode II. This effect of this is shown in Table III. In both cases, damping is improved although to different extents, as expected.

In Case III, where remote signals are lost, an improvement in damping from the no-damping-controller case of Table II is still observed, although the damping of mode I is not as good as in Case I. Case IV shows that inspire of using a local strategy with a gain matrix with the same eigenvalues as in Case I, it does not achieve the same damping as Case I.

<table>
<thead>
<tr>
<th>Case</th>
<th>Swing Mode I</th>
<th>Swing Mode II</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-0.1848 + j 8.7604</td>
<td>-0.2283 + j 13.3222</td>
</tr>
<tr>
<td>II</td>
<td>-0.0732 + j 8.6595</td>
<td>-0.4582 + j 13.8626</td>
</tr>
<tr>
<td>III</td>
<td>-0.1530 + j 8.7201</td>
<td>-0.3425 + j 13.5031</td>
</tr>
<tr>
<td>IV</td>
<td>-0.1534 + j 8.7550</td>
<td>-0.2471 + j 13.4536</td>
</tr>
</tbody>
</table>

The results of the study done at a different loading condition (a 50% increase in the load at the buses 5, 6 and 8) are shown in Table IV. The results for a different network topology - line 5-7 is removed are shown in Table V. The same set of gains as in Case I is used. Although the gains are not specifically designed for these cases, they work satisfactorily (i.e., damping is improved), attesting to the robustness of the control law.

<table>
<thead>
<tr>
<th>Case</th>
<th>Swing Mode I</th>
<th>Swing Mode II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Damping</td>
<td>-0.1665 + j 8.4615</td>
<td>-0.3497 + j 13.1670</td>
</tr>
<tr>
<td>Gains as in Case I</td>
<td>-0.3015 + j 8.5734</td>
<td>-0.4547 + j 13.1996</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Swing Mode I</th>
<th>Swing Mode II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Damping</td>
<td>0.1071 + j 6.5753</td>
<td>-0.2657 + j 13.2076</td>
</tr>
<tr>
<td>Gains as in Case I</td>
<td>-0.3534 + j 0.0270</td>
<td>-0.3712 + j 13.2634</td>
</tr>
</tbody>
</table>

We now present a detailed simulation result to verify the conclusions. The modeling is done on PSCAD [11]. The two DC links are rated at 100 MW each, have “inner” real and reactive current controls and use a PWM switching scheme. Reactive power injections by the links are regulated to zero. The control strategy discussed in the previous paragraphs is used to modulate the power flows of the DC links.

A disturbance is given in the form of a single phase to ground fault near bus 7 which is cleared in 60 ms by tripping line 5-7. The resulting swings without a damping controller are unstable - the growing oscillation of 1 Hz is clearly seen in Fig. 7. However, application of the damping controller is able to damp the oscillations (Fig. 8) as predicted by the eigenvalues of Table V.
strategy and the degrees of freedom that it provides, in terms of eigen-value movement.

The state space equations of a power system represented by the simple model described in Section II can be written as follows.

\[
\begin{bmatrix}
\Delta \delta \\
\Delta \dot{\delta} \\
\Delta \omega \\
\Delta \dot{\omega}
\end{bmatrix} =
A
\begin{bmatrix}
\Delta \delta \\
\Delta \dot{\delta} \\
\Delta \omega \\
\Delta \dot{\omega}
\end{bmatrix} +
B \begin{bmatrix} u \end{bmatrix}
\]
\[
y = C
\begin{bmatrix}
\Delta \delta \\
\Delta \dot{\delta} \\
\Delta \omega \\
\Delta \dot{\omega}
\end{bmatrix} +
D \begin{bmatrix} u \end{bmatrix}
\]

where,

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-MA & I & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B = -M^{-1}
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
C = -
\begin{bmatrix}
C_\delta & 0 & 0 & 0
\end{bmatrix}
\]

The matrices \( A \) and \( D \) are symmetric, while \( C_\delta = B_\delta^T \) [7]. The matrix \( A \) has \( n-1 \) pairs of complex eigenvalues (the swing modes) for a \( n \) machine system. The matrix \( M \) is a diagonal matrix with \( \frac{2H}{w_m} \) as its elements - \( i \) denotes the machine number.

The set of all controllable inputs are:

\[
u = \begin{bmatrix}
\Delta P_{sh}^T & \Delta I_R^T & \Delta P_{snc}^T & \Delta V_R^T
\end{bmatrix}^T,
\]

while the “collocated” observables are:

\[
y = \begin{bmatrix}
\Delta \phi_1^T & \Delta V_1^T & \Delta \phi_2^T & \Delta V_2^T
\end{bmatrix}^T.
\]

Consider a subset of controllable inputs \( u \) such that \( u = Su \). For example, if we have one two-terminal dc link connected at buses \( i \) and \( j \), then \( S \) is a column vector with its \( i^{th} \) entry \(-1\) and \( j^{th} \) entry \(+1\). This corresponds to shunt real power drawn at the two buses\(^4\). The collocated feedback signal corresponding to this input is \( \Delta \phi_1 - \Delta \phi_2 \), i.e., \( y = -S^T \). A scalar control law which involves the collocated input-output pair is \( u = \alpha y \).

If we have two devices, say one dc link and one SSSC, \( u \) has two rows, \( y \) has two columns, \( S \) has two columns, and the control law utilizes a \( 2 \times 2 \) gain matrix \( \alpha = \alpha K_\alpha \), where \( \alpha \) is a scalar. This can be generalized to any number of devices.

Following the analysis given in [7], we can infer that,

1) The sensitivity of an eigenvalue \( \lambda = j\Omega \) corresponding to a swing mode, to the parameter \( \alpha \) is given by

\[
\frac{d\lambda}{d\alpha} = \frac{j}{2\Omega} \left( e_\omega^T B_\omega^T K_\alpha S^T B_\alpha^T e_\omega - e_\omega^T B_\omega^T e_\omega \right),
\]

where \( e_\omega \) is a column vector which contains the right eigen-vector components corresponding to the rotor speed states for the swing mode under consideration. \( e_\omega \) can be chosen to be real because of the structure of \( A \) and symmetry of \( A \).

Therefore, if \( K_\alpha \) is positive definite, then the eigenvalue is a purely imaginary number - the control law increases the frequency of the swing mode (synchronizing effect).

\(^4\)For LCC-DC links, there is reactive power exchange concomitant with the shunt real power exchanged by a converter, but this is not considered here. For VSC based links reactive power exchange can be controlled independent of the real power drawn by the converter.
2) If instead, the control law, \( u = \mathcal{R} \frac{dy}{dt} \) is used, where \( \mathcal{R} = \alpha K_d \), then the sensitivity is given by

\[
\frac{d\lambda}{dx} = -\frac{1}{2} \left( \frac{e_u^T B_u S K_d S^T B_u^T e_u}{e_u^T \Sigma e_u} \right).
\]

If \( K_d \) is positive definite, then this is negative real, indicating that the control law introduces damping.

The eigenvalue sensitivities of all swing modes can be obtained using the above expressions. The actual values of the elements of \( K_s \) and \( K_d \) can be chosen so as to preferentially increase the eigenvalue sensitivities of the critical mode(s). Note that the eigenvalue movement for increasing overall gain \( \alpha \) is also determined by the zero locations \( [7] \) of the transfer function between \( u \) and \( y \). The zero locations are also affected by the choice of the gain matrices.

Positive definiteness is an important criteria of the gain matrices so as to obtain guaranteed damping and synchronizing effects. Additionally, if the gain matrices are also symmetric, it is advantageous because of the following reasons:

1) For a symmetric positive definite (SPD) matrix \( K \), if for a certain \( f \), the elements \( k_{ij} \) and \( k_{ji} \) are set to zero for all \( j \) except \( j = i \), then the matrix is still symmetric positive definite. This means that the control law will continue to provide damping/synchronizing effects even if mutual communication from/to the actuator \( f \) is lost.

2) Note that any gain matrix \( K \) can be decomposed into symmetric and skew symmetric components, i.e., \( K = \frac{(K + K^T)}{2} + \frac{(K - K^T)}{2} \). Therefore, for any \( x \),

\[
x^T K x = x^T \left[ \frac{(K + K^T)}{2} + \frac{(K - K^T)}{2} \right] x
\]

This means that the eigenvalue sensitivity for a gain matrix \( K_s \) is essentially affected by its "symmetric component" \( K_s + K_s^T \) only.

REFERENCES