Abstract: This paper presents a technique for real-time prediction of stability/instability of a power system based on synchrophasor measurements obtained from phasor measurement units (PMUs) at generator buses. For stability assessment the technique makes use of system severity indices developed using bus voltage magnitude obtained from PMUs and generator electrical power. Generator power is computed using system information and PMU information like voltage and current phasors obtained from PMU. System stability/instability is predicted when the indices exceed a threshold value. A case study is carried out on New England 10-generator, 39-bus system to validate the performance of the technique.

Key words—Synchronized phasor; measurements; severity index; power system stability; prediction;

I. INTRODUCTION

Prediction of transient stability or instability of power systems upon occurrence of contingency can be an important defense plan for the operator in energy management system (EMS). Prediction of power system stability using conventional stability analysis methods like time domain and energy function analysis are far from satisfactory. PMUs, a widely accepted measurement system of choice are being utilized intelligently by utilities to capture the dynamics of an event [1], [2]. With the help of GPS system, PMU provides fundamental frequency, positive sequence component of voltage phasors of the installed bus and positive sequence components of current phasors of all incident lines to the installed bus. The advantage is that these information’s are time synchronized [3]. Though the technology related to synchrophasor measurements is well established but the implementation of this technique is yet to be properly explored in dynamic security assessment (DSA) of power systems.

When fault is cleared from a system the signature of final behavior of the system, stable or unstable, is reflected in the path of the dynamic variation. A careful observation can trace this signature early enough in the control centre. Use of severity indices [4]-[5] was a common practice for contingency screening and ranking for DSA in energy EMS. For judgment of stability/instability of transient event, a coherency based, integral square generator angle (ISGA) index is suggested in [6]. A stability threshold was used for the ISGA index. Value of the threshold was determined after large number of simulation. However, the performances of the indices were depending either on the time domain simulation method or some hybrid analysis method. With the availability of synchrophasor measurements, a huge scope is generated for improving real time transient stability analysis.

For stability analysis, dynamic states such as rotor angle and speed of synchronous machines are essential. These states were estimated using synchrophasor data from PMUs in the extra high voltage sides of substations [7], [8]. These works rely on offline trained artificial neural network. However, instead of estimation of these states, they can be computed directly considering PMUs at generator terminal buses. Based on generator rotor angles derived from PMU measurements and generator parameter, transient stability assessment (TSA) approach is reported in [9], considering similar prefault and post-fault network. In this method PMUs were considered at the terminals of the generator buses. Fuzzy rule-based classifiers using wide area response signals from PMUs are developed for rapid stability assessment [10]. Synchrophasor assisted adaptive under frequency load shedding scheme, by quantifying disturbance power is found in [11].

In this paper system severity indices (SSIs) are developed using synchrophasor measurements. Each SSI is evaluated at every instant immediately after fault clearing time when a new set of PMU data arrives. At any instant when the computed SSI violates its threshold value, stability or instability is declared immediately. The indices actually declare the post-disturbed system unstable or stable early enough before the system actually becomes unstable. Effectiveness of the indices are tested on 58000 cases, generated for New England 39-bus system.

II. PROPOSED METHOD

SSIs are based on synchronized measurements like voltage and current magnitude, angle etc. obtained from PMUs.

A. SSI using generator bus voltamagnitude is defined as

$$\eta |V(t)|=\sum_{i=1,\ldots,ng}|V_i(t)|/\sum_{i=1,\ldots,ng}|V_i(t=t_{cl}+)|$$

for $$t > t_{cl}$$
where $n_g$ is the number of generator in the system. Power system
desired operating voltages are usually lies within 0.95 p.u. to
1.05 p.u. The voltage index represents sum of the voltage
magnitude of buses where PMUs are installed with respect to the
sum immediately after fault clearing time. After fault clearing if
the system is stable the voltage profile successfully maintained
in the desired range with sinusoidal variation. If the system is
unstable voltage profile starts decreasing below the specified
range. If the voltage index starts growing after fault clearing, this
is an indication of stable case. On the other hand continuous
decrease of this index from the fault clearing point indicates an
unstable case. However, there are observed cases which show
stability even after decrement in few consecutive steps from
fault clearing point. An extensive simulation finds a threshold
value such that if the index value becomes less than the
threshold value, the system is unstable.

The system stabilityinstability is declared according to
satisfaction of the following criterion.

The system is stable if \( \eta V(t_i) - \eta V(t) > 0 \) \( t_i > t > t_{cl} \)
The system is unstable if \( \eta V(t) < 0.855 \) \( t > t_{cl} \)

Let us consider a fault at line number 4 between bus 2 and 25 in
New England 39-bus system for demonstration. Synchronized
measurements are simulated using transient stability program
with integration time of one cycle such that only one set of
measurements are available in that cycle. The fault is considered
near bus 2 and is cleared after 0.1s from the inception of the
fault. Fig 1(a) shows variation of \( \eta V \) with respect to time (after
fault clearing), the system is declared stable at \( t = 0.705s \). Fig.
1(b) shows the variation of the maximum angle deviation of any
generator with respect to time for the same fault and duration.
Maximum angle deviation is defined as

Maximum angle deviation = \( \max (\abs{\delta_i - \delta_{COI}}) \), for \( i = 1 \ldots n_g \)

Where \( \delta_{COI} = \frac{1}{M_T} \sum_{i=1}^{n_g} \delta_i \cdot M_i \) and \( M_T = \sum_{i=1}^{n_g} M_i \)
\( \delta_i, M_i \) are generator angle and moment of inertia.

Fig. 1(b) justifies that the decision taken from observation of
Fig. 1(a) is correct.

Let us consider the same fault at the same location sustained for
0.3s after which the fault is cleared by tripping the line between
bus 2 and 25. Fig. 2(a) shows variation of \( \eta V \) with respect to
time (after fault clearing). The proposed index detects the case as
unstable at \( t = 0.6s \). Fig. 2(b) shows the maximum angle deviation with time for this situation, it is clearly understood that
the system actually becomes unstable at \( t = 0.72s \) when
maximum angle deviation becomes more than 180°. Considering
the computation time negligible, the unstable case is detected
120ms early before the system becomes actually unstable.

B. SSI using generator electrical output power

PMU does not provide any power measurement. However,
when a PMU is connected to a generator bus, electrical output
power (real) of the connected generator can be calculated using

the information of the PMUs along with the generator
information. The induced e.m.f \( (E) \) is computed as

\[
E = V + I(r_a + jx_a)
\]  

where \( V \) and \( I \) are voltage and current phasor available from the
PMU at generator bus and \( r_a, x_a \) are armature resistance and
reactance of generator, considering classical model. The (real)
electrical output power of a generator is obtained as

\[
PE = \frac{(E_r^2 - E_r V_r + V_r^2 - E_r V_i) r_a - (E_r V_1 - E_r V_2) x_a}{(r_a^2 + x_a^2)}
\]  

where \( r \) and \( i \) refers to real and imaginary component of \( E \) and
\( V \).

\[
\eta PE(t) = \frac{\sum_{i=1}^{n_g} PE_i(t)}{\sum_{i=1}^{n_g} PE_i(t)}
\]

for \( t > t_{cl} \)

The electrical power index actually represents the sum of the
electrical power of the generators with respect to the sum
immediately after fault clearing time. If the system is stable sum of electrical power starts growing with sinusoidal variation as soon as the fault is cleared. If the system is unstable electrical power decreases. If the index starts growing after fault clearing time, it indicates that this is a stable case. But if the index starts decreasing and become less than a threshold value that is an unstable case. Threshold is used to reduce misclassification at the cost of available time for control action. Value of threshold is decided after extensive simulation.

The system stability/instability is declared according to satisfaction of the following criterion.

The system is stable if \[ \eta PE(t_2) - \eta PE(t_1) > 0 \quad t_2 > t_1 > t_{cl} \]
The system is unstable if \[ \eta(t) < 0.70 \quad t > t_{cl} \]

To see the performance of the proposed index \(\eta PE\) compared to the voltage index \(\eta V\), let us considering the same fault at same line between bus 2 and bus 25 in the NE 39-bus system. The fault occurred near bus 2 and is cleared after 0.1s from the inception of the fault. Fig. 3 shows variation of \(\eta PE\) with respect to time, the system is declared stable at \(t=0.705s\). It is understood from the variation of the maximum angle deviation with respect to time of Fig. 1(b) that the case is actually stable. Thus the stable case is successfully detected by the electrical power based index.

Fig. 4, shows variation of \(\eta PE\) with respect to time (after fault clearing) for fault sustained period of 0.3s, when the fault is cleared by tripping the line. The system is declared as unstable at \(t=0.57s\). According to Fig. 2(b), the case truly becomes unstable at \(t=0.72s\). This implies that the case is detected as unstable before it actually becomes unstable. Thus the unstable case is detected 150ms early.
For further illustration of the proposed indices, let us consider another fault occurred at line no. 23 between bus 16 and bus 21 in the NE 39-bus system. The fault occurred near bus 16 and is cleared after 0.1s from the inception of the fault. Fig. 5(a) shows variation of $\eta V$ with respect to time, the system is declared stable by this index at $t=0.585$s and Fig. 5(b) shows variation of $\eta PE$ with respect to time, the system is also declared stable at the same time i.e. at $t=0.585$s. Fig. 5(c) shows the variation of the maximum angle deviation with respect to time for the same fault and duration. Fig. 5(c) justifies that the decision taken from observation of fig. 5(a) and 5(b) is correct. If the fault at the same location persists for 0.2s and cleared by tripping the line,

![Graph](image1)

![Graph](image2)

![Graph](image3)

![Graph](image4)

Fig. 5. Time variation of (a) voltage based index ($\eta V$), (b) electrical power index ($\eta PE$) and (c) maximum angle deviation for fault at line no. 23 which is cleared at 0.1s by tripping the line.

Fig. 6. Time variation of (a) voltage based index ($\eta V$), (b) electrical power index ($\eta PE$) and (c) maximum angle deviation for fault at line no. 23 which is cleared at 0.2s by tripping the line.
case. It is understood from Fig. 6(c) that the system actually becomes unstable at t=1.09s, when maximum angle deviation becomes greater than 180°. Thus, the case is detected properly by both the indices. The unstable case is detected 765ms and 585ms early, respectively by voltage index and electrical power index before the system becomes actually unstable.

III. CASE STUDIES

This paper investigates prediction of stability or instability when n-1 contingency occurs in the transmission lines. The case studies are carried out on New England 39-bus system [12]. The cases are generated by varying load from 90% to 110% at an interval of 1%, which gives 20 operating condition. For each operating condition 29 numbers of n-1 contingency is considered starting from 1 to 30th line. Since the outage of 22nd line leads to islanding, synchrophasor assisted parallel computation technique is necessary for its analysis and thus does not considered in this paper. For each line outage, fault clearing time varies from 0.1s to 0.3s at an interval of 0.002s which give 100 fault clearing time. Total 58000 cases are generated and studied. For each case the stability /instability is detected using the voltage index and electrical power index.

IV. SIMULATION RESULTS AND DISCUSSIONS

Quantitative measurement of performance of the proposed indices are obtained through the following criterion [10].

1. Mis-classification: Total number of cases converted to stable from actually unstable cases/ (Total number of cases - number of undetected cases)
2. False Alarm: Total number of cases converted to unstable from actually stable cases/ (Total number of cases - number of undetected cases)
3. Reliability: (Total number of unstable cases – total number of cases converted to stable cases)/total number of unstable cases
4. Efficiency: (Total number of stable cases – total number of cases converted to unstable cases)/total number of stable cases
5. Accuracy: (Total number of cases - number of mis-classification)/ total number of cases
6. Undetection: Number of undetected cases/ total number of cases

The complete data base is summarized in table I. Each index performance is evaluated using these criterions. Performance measurement of the voltage index is given in table II. Among the total cases the voltage index shows 1.65% mis-classifications, 1.01% false alarms and 0.05% undetected cases. Performance measurement of the electrical power based index is given in table III. Among the total cases this index shows 0.98% mis-classifications, 1.92% false alarms and 0.72% undetected cases.

<table>
<thead>
<tr>
<th>Table I</th>
<th>SUMMARY OF THE DATA BASE USED IN THE PROPOSED METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td># total cases</td>
</tr>
<tr>
<td>NE 39-bus</td>
<td>58000</td>
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</table>

<table>
<thead>
<tr>
<th>Table II</th>
<th>QUANTITATIVE MEASUREMENT OF PERFORMANCE OF THE PROPOSED VOLTAGE INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ind-</td>
<td>Mis-classification (%)</td>
</tr>
<tr>
<td>ex</td>
<td>(%)</td>
</tr>
<tr>
<td>$\eta V$</td>
<td>1.65</td>
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<tr>
<th>Table III</th>
<th>QUANTITATIVE MEASUREMENT OF PERFORMANCE OF THE PROPOSED ELECTRICAL POWER INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ind-</td>
<td>Mis-classification (%)</td>
</tr>
<tr>
<td>ex</td>
<td>(%)</td>
</tr>
<tr>
<td>$\eta E$</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Performance comparison of the proposed voltage based index and electrical power index, with two classifier [10] namely system wise fuzzy rule classifier and heuristic fuzzy logic classifier for 2s _Global observation window, are given in Table IV.
TABLE IV
COMPARISON OF THE PROPOSED INDEX WITH THE FUZZYCLASSIFIERS [10]

<table>
<thead>
<tr>
<th>Classifier</th>
<th>Misclassification (%)</th>
<th>False Alarm (%)</th>
<th>Reliability (%)</th>
<th>Efficiency (%)</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed ($\eta V$)</td>
<td>1.65</td>
<td>1.01</td>
<td>94.39</td>
<td>98.55</td>
<td>98.34</td>
</tr>
<tr>
<td>Proposed ($\eta PE$)</td>
<td>0.98</td>
<td>1.92</td>
<td>96.67</td>
<td>97.29</td>
<td>99.01</td>
</tr>
<tr>
<td>System wise fuzzy-rule classifier[2]</td>
<td>3.37</td>
<td>3.04</td>
<td>85.38</td>
<td>96.04</td>
<td>93.58</td>
</tr>
<tr>
<td>System wise heuristics fuzzy-rule classifier[2]</td>
<td>0.019</td>
<td>27.52</td>
<td>99.91</td>
<td>64.21</td>
<td>72.45</td>
</tr>
</tbody>
</table>

Discussion

Computational advantage of the proposed prediction method is that these indices detect the characteristics (stable/unstable) of the studied cases well advance before the actual happenings. As already observed from the Fig. 2, 4 and 6, that significant computational advantage is achieved using this synchrophasor assisted system severity indices. In most cases the indices provides computational advantage around 100ms. It is observed cases where greater than 500ms time advantage is obtained. In few cases the time advantage is around 50ms or less than that. It is observed that to increase reliability of the approach, computational time advantage becomes insignificant to take any control action. Reliability and accuracy of the proposed index based study can be further increases at the cost of reduction of computational time advantage by varying the selected thresholds. Considering all the criterion such as misclassification, false alarm, reliability, efficiency and accuracy, the proposed indices shows superiority over the fuzzy classifier [10]. An index which shows minimum false alarms, minimum misclassification and minimum undetection cases is a good index. Considering these issues voltage based index is comparatively better than electrical power based index.

V. CONCLUSION

In this paper, synchronized phasor measurements based prediction technique of stability/instability of power system is proposed. Synchronized measurements like voltage and current both magnitude and angle obtained from PMUs are used to develop system severity indices. Each index is applied for prediction of stability/instability of power system. Performance of the SSI’s studied on a large size database consisting of 58000 cases (70.40% stable and 29.60% unstable) in NE 39-bus system and found to be efficient.

REFERENCES