Price Based Demand Response of Aggregated Thermostatically Controlled Loads For Load Frequency Control

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Abstract—Demand Response has the potential to participate in grid operations and management systems with the advent of Information Communication Technology. In this paper, a demand response scheme for Thermostatically Controlled Loads that respond to real time price is proposed. The proposed control scheme is economical to the consumers as the objective of the scheme is to minimize the increase in amount paid due to increase in price, which is supported by simulation results. The power gained from the proposed price based demand response of aggregated TCL is then modeled using coupled Fokker-Planck equations. The model is then applied to a single area Load Frequency Control scheme, to investigate the contribution of price based demand response. Simulation results show that the load frequency control scheme with demand response improves system stability and leads to economic operation of the system at both consumer and generation level.

Keywords—Coupled Fokker-Planck Equations, Load Frequency Control, Price Based Demand Response, Thermostatically Controlled Loads.

I. INTRODUCTION

Demand Response (DR) is an emerging feature in Smart grid which is now gaining much importance in Home Energy Management System. DR broadly classified into price based and incentive based, has been extensively studied and analyzed in various aspects. The focus was mainly on peak clipping, load shifting and direct load control schemes. For residential load control, thermostatically controlled loads (TCLs) like Heating, Ventilating and Air Conditioner (HVAC) loads are considered as they cause minimum inconvenience to the users.

The feasibility of such TCL control was studied in [1]. Aggregation of these loads and the impact of DR were studied in [2]. An ON/OFF scheme for HVAC units was discussed in [3]. A control scheme that manipulates the thermostat set point of such loads to balance fluctuations from intermittent renewable generators was developed in [4]. A change in thermostat set point results in change in state of the unit and this has been modeled and studied in [5]. Thermostat set point was considered to be varying with market price based on a predefined linear relation. But a control scheme that can find the optimal thermostat set point corresponding to market price that varies in real time was not studied from the consumer/user level. In this paper the authors attempt to provide a mathematical formulation and control scheme required to determine the optimal thermostat set point corresponding to the market price from a consumer perspective.

Frequency Adaptive Power Energy Scheduler (FAPER) with a ‘load follows supply’ concept was introduced in [7]. Later in [5] contribution from demand side to primary frequency control was discussed by proposing a DR algorithm for primary frequency control using load controllers. An active role for intelligent loads in frequency control was proposed in [8-9] by hypothesizing that modulation of load real power based on frequency error would result in appropriate modulation in output power of generator. Thus Demand Response can participate in Load Frequency Control (LFC). In this paper a price based demand response model for aggregated thermostatically controller loads is developed to investigate the contribution of demand response to Load Frequency Control.

The paper is organized as follows. Section II provides the mathematical formulation for price based demand response in thermostatically controlled loads (TCLs). The price based demand control scheme is then proposed in Section III. Section IV discusses the model for aggregated TCLs participating in the proposed demand response scheme that is required for LFC. Section V provides the simulation results. Conclusions are then drawn in Section VI.

II. PRICE BASED DEMAND RESPONSE IN TCL

When there is an increase in electric price signal, Demand Response (DR) is expected from the customers who have contracted for price based Demand Response. Adjustment of thermostat set point results in change in power consumption, which can be derived quantitatively as shown below.

HVAC Loads/ TCLs are rated by their Coefficient of Performance (COP) given by [6]

\[ \text{COP} = \frac{\text{Work Done}}{\text{Electric Power Input}} \]  

\[ \text{Work Done} (Q) \text{ by the HVAC unit is given by} \]

\[ Q = m \cdot C_p \cdot (T_{\text{out}} - T_{\text{in}}) \]
where

\[ m = \text{mass of the coolant} \]
\[ C_p = \text{specific heat capacity of the coolant} \]
\[ T_{out} = \text{outside temperature} \]
\[ T_{in} = \text{inside temperature} \]

Assuming Thermostat set point to be same as inside temperature, change in work done (\( \Delta Q \)) for a change in thermostat set point (\( \Delta T_{st} \)) is

\[ \Delta Q = -m * C_p * (\Delta T_{st}) \] (3)

A change in work done (\( \Delta Q \)) results in a change in electric power consumed (\( \Delta P_{input} \)), given by equation 1. Thus

\[ \Delta P_{input} = \frac{\Delta Q}{\text{COP}} = \frac{-m * C_p * (\Delta T_{st})}{\text{COP}} \] (4)

Equation 4 establishes a linear relation between change in thermostat set point and change in power consumed by the TCL unit. An increase in electricity price motivates consumers to initiate DR as they aim at minimizing the increase in total amount paid. Equation 4 shows that Demand Response-contracted customers can adjust their thermostat set point so as to reduce the power consumption. Thus an increase in price can be responded with an increase in thermostat set point so as to reduce the power consumption. This in turn minimizes the increase in amount paid.

Let \( \rho_1 \text{Rs/W} \) be the current rate at which consumers are charged. Thus amount paid (\( P_1 \)) for an input power, \( D_1 \), of HVAC units is

\[ P_1 = \rho_1 * D_1 \] (5)

Let \( \rho_2 \text{Rs/W} \) be the new rate, such that \( \rho_2 > \rho_1 \). Then the amount to be paid (\( P_2 \)) for the same input power is

\[ P_2 = \rho_2 * D_1 \] (6)

Customers aim to minimize the increase in amount paid. This can be achieved by reducing the demand from to \( D_2 = D_1 - \Delta D \). Thus

\[ P_2 = \rho_2 * D_2 \] (7)

The increase in amount paid (\( \Delta P \)) is

\[ \Delta P = P_2 - P_1 = \rho_2 * D_2 - \rho_1 * D_1 = (\rho_2 - \rho_1) * D_1 - \rho_2 * \Delta D \] (8)

Substituting equation 4,

\[ \Delta P = (\rho_2 - \rho_1) * D_1 + \rho_2 * \frac{m * C_p * (\Delta T_{st})}{\text{COP}} \] (9)

The relation is as shown in Fig 1. Thus the objective of the customer is to minimize equation 8.

### III. CONTROL SCHEME FOR PRICE BASED DEMAND RESPONSE IN TCLS

The previous section derived the relation between amount paid by the consumer and the thermostat set point of the TCL. The objective of the consumer was also defined. This section proposes a control scheme for the TCL units so as to achieve the consumer objective. Fig 2 shows the relation between the increase in amount paid and the change thermostat set point/demand.

The difference in amount paid is \( \Delta P_1 \) and shown as operating point A. Customer objective is to minimize the difference in amount paid. The optimal difference in amount paid should be zero. The operating point corresponding to optimal difference in amount paid is B. This optimal point can be achieved by shifting \( \Delta T_{st} \), as given in Fig 3.
IV. PRICE BASED DEMAND RESPONSE MODEL OF AGGREGATED TCL FOR LFC

The price based demand response scheme discussed in the previous section determines the optimal thermostat set point change required. In this section modeling of aggregated TCLs that are contracted for a price based demand response is discussed. The model is based on the Coupled Fokker-Planck equations (CFPE) [7] and its exact solution [4] that provide the dynamics of aggregated population of TCL due to change in thermostat set point. Aggregate power response model in a homogeneous population of TCLs was derived in [8] based on the exact solutions of CFPE. In this section the exact solution of CFPE is utilized to model the change in power consumption in aggregated TCL due to price based demand response.

Consider N homogeneous TCLs in steady state. Let \( \theta_0 \) be the initial thermostat set point, with dead band of \( \Delta \). Let \( P \) be the power drawn by the TCL. \( \theta_0 - \) from \( \theta_0 + \) is the heating period and \( \theta_0 + \) to \( \theta_0 - \) is the cooling period. The expressions for time taken to reach temperature \( \theta_n \) during cooling period \( t_c(\theta_n) \) is [8]

\[
t_c(\theta_n) = C * R * \ln \left( \frac{PR + \theta_n^+ - \theta_{amb}}{PR + \theta_n^- - \theta_{amb}} \right) \tag{10}
\]

and during heating period \( t_h(\theta_n) \) is

\[
t_h(\theta_n) = C * R * \ln \left( \frac{-\theta_n^- + \theta_{amb}}{-\theta_n^+ + \theta_{amb}} \right) \tag{11}
\]

where,

- \( C \) is thermal capacitance
- \( R \) is thermal resistance
- \( \theta_{amb} \) is ambient temperature

In steady state cooling time constant \( T_C \) and heating time constant \( T_H \) are defined as [8]

\[
T_C = C * R * \ln \left( \frac{PR + \theta_0^+ - \theta_{amb}}{PR + \theta_0^- - \theta_{amb}} \right) \tag{12}
\]

\[
T_H = C * R * \ln \left( \frac{-\theta_0^- + \theta_{amb}}{-\theta_0^+ + \theta_{amb}} \right) \tag{13}
\]

The ON probability density function \( f_1(\theta) \) and OFF probability density function \( f_0(\theta) \) for N TCLs loads are are obtained by solving the CFPE equations [4] and the solution is given in the below equations [8].

\[
f_1(\theta) = \frac{C * R}{(PR + \theta - \theta_{amb}) * (T_C + T_H)} \tag{14}
\]

\[
f_0(\theta) = \frac{C * R}{(-\theta + \theta_{amb}) * (T_C + T_H)} \tag{15}
\]

A. Application of Price based demand response of TCL in LFC

A change in thermostat set point results in a shift in the temperature band of operation (\( \theta_- \) to \( \theta_+ \)) Four different TCL conditions (a,b,c,d) on the probability density curve defined by the equations 14, 15 are considered as shown in Fig 4.

The power waveforms for the four operating points at the initial thermostat set point and the new thermostat set point are denoted in Fig 5. Initial waveform is shown using dotted lines and new power waveforms are shown with thick lines.

From Fig 5 the difference in power consumed can be calculated and it is shown in Fig 6.

It can be seen that for operating point a, difference in power i.e. power gain, after + occurs after cooling period which is in hours. Hence for applications like Load frequency Control power gain after + does not come into picture. Thus the Laplace transform of the waveform relevant to LFC time frame,i.e. waveform from a to + is computed and the total power gain is calculated by integrating over the distributions \( f_0 \) as the operating point a lies in the off period. Thus the total power gain from TCLs at operating point a, is

\[
\begin{equation}
Pa(s) = \int_{\theta_-}^{\theta_+} f_0(\theta_a) * G_a(s) d \theta_a \tag{16}
\end{equation}
\]
where $G_a(s)$ is the Laplace transform of the difference in power consumption of loads at a, given by

$$G_a(s) = \frac{P * e^{-st_a} * (e^{-st_a} - 1)}{s}$$

and $t_a = T_H - t_h(\theta_a)$

$\tau_d = T_d - t_c(\theta_d)$ which can also be defined as $\tau_d = T_H - t_h(\theta_d)$

For operating point b, difference in power i.e power gain, after $\theta_{-0}$ occurs after a time period equivalent to the initial heating period which is in hours. Thus the Laplace transform of the waveform from $\theta_b$ to $\theta_{-0}$ is computed and the total power gain is calculated by integrating over the distributions $f_i$. The total power gain from TCLs at operating point b, is

$$P_b(s) = \int_{\delta} f_b(\theta_b) * G_a(s) d \theta_b$$

where $G_a(s)$ is the Laplace transform of the difference in power consumption of loads at b, given by

$$G_a(s) = \frac{P * e^{-st_a} * (e^{-st_a} - 1)}{s}$$

and $t_a = T_H - t_h(\theta_a)$

$\tau_d = t_c(\theta_d) - t_c(\theta_b)$ with respect to initial temperature set point.

Similarly to point a, operating point c, difference in power i.e power gain, after $\theta_{0}$ occurs after coolingperiod. Thus the Laplace transform of the waveform from $\theta_c$ to $\theta_{0}$ is computed and the total power gain is calculated by integrating over the distributions $f_o$. The total power gain from TCLs at operating point c, is

$$P_c(s) = \int_{\delta} f_o(\theta_c) * G_c(s) d \theta_c$$

where

$$G_c(s) = \frac{P * e^{-st_c} * (e^{-st_c} - 1)}{s}$$

and $t_c = T_c - t_h(\theta_c)$

Similarly in operating point d, the Laplace transform of the waveform from $\theta_d$ to $\theta_{0}$ is computed and the total power gain is calculated by integrating over the distributions $f_i$. The total power gain from TCLs at operating point d, is

$$P_d(s) = \int_{\delta} f_i(\theta_d) * G_d(s) d \theta_d$$

where

$$G_d(s) = \frac{P * (e^{-sT_d} - 1)}{s}$$

and $t_d = T_c - t_h(\theta_d) + \tau_d$

Thus average change in power is given by

$$P(s) = P_a(s) + P_b(s) + P_c(s) + P_d(s)$$

$$= \frac{-P * (e^{sT_d} + 1)}{s(T_c + T_H)}$$

V. SIMULATION

The average change in power demand due to demand response can be calculated using the expression derived in the previous section. This aggregated TCL demand response model is then applied to a single area LFC, with real-time pricing scheme based on frequency deviation.

Simulation was done for a step load change of 0.01 p.u and 0.02 p.u. Aggregated TCL demand response model derived in section IV is then applied to a single area LFC scheme with real-time pricing scheme based on frequency deviation[9]. Real time price is proportional to the initial df/dt value[9], is assumed to be known at the system operator with the help of Phasor Measurement Units. System operator sends price information signals to the customers in demand response. 1000 TCLs were assumed to be available for demand response. The delay in communication is assumed to be negligible. The Load Frequency Control scheme with and without price based demand response are compared for a step load change of 0.01 p.u and 0.02 p.u and provided in Fig 7 and Fig 8.
Observation and inference

For an increase in price, the proposed control is sufficient for the consumers to find the optimal thermostat set point, so as to minimize the increase in amount paid to zero. Aggregation of such TCL loads that are contracted for the proposed price based demand response can play a significant role in Load Frequency Control scheme. The simulation results for a single area LFC show that the system which was unstable for 0.02p.u. becomes stable when demand response is included into the LFC scheme. The increase in generation required when there is an increase in load, can be reduced when demand response is included into LFC scheme. The results are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Step Change in Load</th>
<th>Change in Generation required</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0057</td>
</tr>
<tr>
<td>0.02</td>
<td>Unstable</td>
</tr>
<tr>
<td></td>
<td>0.0073</td>
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</tbody>
</table>

VI. CONCLUSION

A control scheme in which each TCL contracted for price based DR is equipped with an integral controller so as to adjust the thermostat set point is proposed in this paper. The advantage of this control is that customers are least affected by the increase in price with minimum inconvenience. Simulation carried out in Matlab/ Simulink environment supports the proposed scheme. Aggregated response of such TCLs is then studied and modeled. The power gain due to the proposed DR scheme is then utilized for ‘Load following Supply’ concept and applied to LFC scheme. The simulation results show that the single area system with proposed DR scheme has improved stability and more economical at customers level as they are less affected by increase in price, due to DR and also at the Generation level, as they are almost at fixed point operation.

REFERENCES