

# A New Approach of Harmonic Load Flow for Radial Distribution Networks

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**Abstract**—This paper presents a novel volt-ampere balance equation for non-linear load characteristic. The application of this balance equation converts non-linear load into equivalent constant power harmonic source. The coupling between the proposed volt-ampere balance equation with Newton-Raphson based power flow equations enables to separate calculations for every harmonic order. This will greatly reduce the complexity level and computation burden attributed with harmonic power flow. The proposed volt-ampere balance equation combines with Newton-Raphson based algorithm and tested on distorted IEEE 33-bus radial distribution system. The obtained results are compared with existing method available in literature, and are found promising.

**Keywords**—harmonic load flow; non-linear load; distribution systems

## I. INTRODUCTION

IN the last few decades, harmonic load flow analysis has become an important tool of power system analysis and design [1, 2]. Nonlinear loads are continuously increasing. Harmonics have become an important power quality problem because of the increased distortion levels in the power system as a result of an increasing number of harmonic producing loads [3, 4]. The main effects of harmonics in power systems are heating, overloading, aging of equipment, increased losses and system unbalancing. Harmonics may lead to malfunctioning of power system components and electronic devices. Relays and measurement equipment may function erroneously under non-sinusoidal voltage and currents. Harmonic analysis is then applied to the study of resonant conditions in network, harmonic filter designs and also to investigate other effects of harmonics on the power system, i.e., notching and ringing, neutral currents, saturation of transformers, and overloading of system components. Identification and measurement of harmonic producing load has become an important issue in electric power systems since increased use of power electronic devices and equipment sensitive to harmonics has increased the number of adverse harmonic related events.

Many methods have been proposed to solve the harmonic pollution produced by nonlinear loads. The commonly used harmonic analysis methods can be divided into two categories. The first category is based on the transient-state analysis techniques, such as the time domain analysis and Wavelet analysis, etc. [7]–[9]. The second category is the steady-state analysis [10]–[12]. Frequency scan is the most commonly used non-iterative method in many commercial software

packages. In this method, a harmonic current source of specified magnitude is injected into a linear network to determine the voltage distortion level. One of the limitations of this method is that in the presence of multiple harmonic sources, a fundamental frequency load flow is required to determine the magnitudes and phase angles of the injected currents accurately. Another drawback of frequency scan is its reliance on typical harmonic spectra to represent nonlinear devices. This results in inaccurate results under conditions of voltage-dependency, unbalanced operation, and the generation of non-characteristic harmonics. The steady-state algorithms are developed based on load-flow programs and employed frequency-based component models. The steady state-based algorithms are more efficient than the transient state-based algorithms. Steady state-based algorithms are better choice for large-scale power system analysis due to the computational economy. The conventional harmonic analysis methods used the Newton-Raphson power flow method, the admittance matrix, or the impedance matrix to obtain harmonic penetration in distribution systems [13]–[15]. These methods achieve separation of the nonlinear elements from the linear part of the network and obtain solution iteratively using the Newton-Raphson load flow. References [16, 17] present a review of the modeling and analysis of harmonic propagation in electric power systems, along with practical considerations and sample case studies. In [16], the theoretical aspects of harmonics modeling and simulation are focused. Concepts and characteristics of power system harmonics, modeling of harmonic sources and network components, and techniques for network-wide harmonic analysis are discussed. In [17], the review on the nature and modeling of harmonic sources in electrical power systems and the analysis of harmonics propagation is presented.

In this paper, a new volt-ampere balance equation is developed for non-linear load characteristic. In this proposed formulation the non-linear load is converted into constant power harmonic source. The proposed volt-ampere balance equation combines with Newton-Raphson power flow algorithm for harmonic load flow analysis and tested on distorted IEEE 33-bus radial distribution system. This is based on the balance of active power and reactive volt-amperes, irrespective of fundamental or harmonic frequency. The active and reactive power balance is forced to zero by the bus voltage iterations. The system solution, with linear and nonlinear loads under balanced non-sinusoidal three-phase conditions, is achieved by forcing total (fundamental and harmonic)

mismatch active and reactive powers to zero using the Newton-Raphson method.

## II. PROBLEM DESCRIPTION

### A. Objectives

The purpose of harmonic analysis is to ascertain the distribution of harmonic voltages and harmonic distortion indices in a power system.

- To find out voltage  $THD$  levels ( $V_{THD}$ ) in radial distribution system.
- To find out total real power losses in the system with the presence of harmonics.

Consider a system with  $n+1$  buses, bus 1 is a slack bus, buses 2 through  $m$  are conventional load buses, and buses  $m$  to  $n$  have non-linear loads. It is assumed that the active power and the reactive volt-ampere balance are known at each bus and that the nonlinearity is known. The power balance equations are constructed so that the active power  $P$  and the reactive power  $Q$  at all non-slack buses are zero for all harmonics. The form of  $P$  and  $Q$  as a function of bus voltage and phase angle is the same as in conventional load flow, except that admittance matrix  $Y_{bus}$  is modified for harmonics.

$$\Delta M = J \Delta U \quad (1)$$

where  $\Delta M$  is mismatch active and reactive volt-amperes,  $\Delta U$  is voltage correction and  $J$  is Jacobian matrix for the harmonic power flow analysis, the power flow equations are defined according to Xia *et al.* [10, 11].

### B. Constraints

**Bus Voltage Limits:** The bus voltage magnitudes are to be kept within acceptable operating limits throughout the optimization process

$$V_{min} \leq |V_i| \leq V_{max} \quad (2)$$

where  $V_{min}$  and  $V_{max}$  lower and upper bound of bus voltage limits;  $|V_i|$  is the root mean square (rms) value of the  $i$ th bus voltage and defined by

$$|V_i| = \sqrt{|V_i^1|^2 + \sum_{h=h_0}^{h_{max}} |V_i^{(h)}|^2} \quad i=2, 3, \dots, n \quad (3)$$

where  $n$  is the number of buses,  $h$  is the harmonic order,  $h_0$  and  $h_{max}$  represents fundamental and highest harmonic order of interest, respectively.

The rms value of the  $n$ th bus voltage involves only the fundamental component, the first term of (3), when harmonics are not of interest.

**Total Harmonic Distortion Limits:** The total harmonic distortion at each bus is to be kept less or equal to the maximum allowable harmonic distortion level as shown.

$$THD_i (\%) \leq THD_{max} \quad (4)$$

where  $THD_{max}$  is the maximum allowable harmonic distortion level at each bus.

### C. General Harmonic Indices

The general harmonic indices may be defined as

$$THD = \frac{\sqrt{\text{sum of squares of amplitudes of all harmonics}}}{\sqrt{\text{squares of amplitudes of fundamental component}}} \times 100\% \quad (5)$$

$$V_H = \sqrt{\sum_{h=2}^{h_{max}} V_h^2}; \quad V_{THD} = \sqrt{\frac{\sum_{h=2}^{h_{max}} V_h^2}{V_1^2}} \times 100\% \quad (6)$$

where  $V_H$  is equivalent totalized harmonic component of the voltage,  $V_h$  is harmonic component of voltage of the order  $h$ .

## III. MATHEMATICAL MODELLING

Before starting with the development of the mathematical framework, the main assumptions to be considered are as follows.

- Three-phase balanced system is considered.
- Skin effect is neglected in the system.
- In distribution systems, the phase shift between the system bus voltage angles is very small.

### 1) Power system component modelling

#### a) Line parameter's modeling at harmonic frequencies:

At harmonic frequencies, accurate models for distribution lines and shunt capacitors are available. However, harmonic modeling of the power system supplying the distribution feeder, substation transformer, linear and nonlinear loads are not well established. If skin and proximity effects are ignored, a feeder segment and shunt capacitor may be represented by the following admittances at the  $h$ th harmonic frequency [18, 19]:

$$Y_{i,i+1}^h = \frac{1}{(R_{i,i+1} + jhX_{i,i+1})} \quad (7)$$

where  $R_{i,i+1}$  and  $X_{i,i+1}$  represent the resistance and reactance of the line segment between buses  $i$  and  $i+1$ .

#### b) Shunt capacitor modeling at harmonic frequencies:

In a harmonic-rich environment, shunt capacitors are important elements to be modeled for harmonic studies as they may amplify harmonic distortion levels [2]. Thus, modeling of such components is very important. Generally, shunt capacitors are specified by their kVAR ratings and nominal voltages. Single-phase shunt capacitors are represented by single-phase shunt capacitances, while three-phase capacitors are described by three-phase shunt capacitances [18, 19]. Shunt capacitor banks are represented as shunt connected elements

$$Y_{capacitor,i}^h = hY_{capacitor,i} \quad (8)$$

where  $Y_{capacitor,i}$  is the admittance of shunt capacitor  $C$  at bus  $i$ .

### 2) Linear load modeling at harmonic frequencies:

Linear loads are basically those loads that can be described as passive loads in terms of harmonics. In this proposed formulation the generalized model of linear load, composed of a resistance in parallel with an inductance, is considered. The load admittance of the  $i$ th bus is expressed by

$$Y_i^1 = \frac{P_i - jQ_i}{|V_i|^2}; Y_i^h = f_i(Y_i^h, h) \quad (9)$$

$$Y_i = \frac{1}{R_i} + \frac{1}{jhX}; Y_i^h = f_i(Y_i^h, h) \quad (10)$$

where the voltage  $V_i$  is determined from the fundamental load flow. This expression depends on the fundamental voltages only because of the hypotheses of no harmonic voltage influence on power consumption and non-linear load behavior [20].

Linear passive loads that do not produce harmonics have a significant effect on system frequency response primarily near resonant frequencies. With respect to linear loads, it is suggested [12, 18] to use a generalized model which is composed of a resistance in parallel with an inductance selected to account for the respective active and reactive powers at fundamental a shunt capacitor and feeder segment may be represented by the following admittances at the  $h$ th harmonic frequency [13]:

$$Y_i^h = \frac{P_i - jQ_i}{|V_i^1|^2} \text{ for } i = 2, 3, \dots, m-1 \quad (11)$$

where  $P_i$  is real power of load (kW),  $Q_i$  is reactive power of load (kVAr),  $V_i$  is fundamental  $i$ th bus voltage (p.u.) and  $Y_i^h$  is shunt admittance at  $h$ th harmonic order.

The equations (9)–(11) depend on the fundamental voltages only because of the hypotheses of no harmonic voltage influence on power consumption and non-linear load behavior [20].

### 3) Non-linear load modeling at harmonic frequencies:

The type of nonlinear load is assumed to be of three phase, six-pulse converter. Nonlinear loads inject harmonics into distribution systems, and are commonly modeled as harmonic current sources. In this formulation non-linear load is modeled as constant current source. If the Fourier analysis is made on the output wave of this converter, it would contain all the odd harmonic current expect triplets [1]–[4].

Therefore, the following observations can be made

(a) The absence of triple harmonics. (b) The presence of harmonics of orders  $6k \pm 1$  for integer values of  $k$ . (c) Those harmonics of orders  $6k + 1$  are of positive sequence and those harmonics of orders  $6k - 1$  are of negative sequence. (d) The r.m.s. magnitude of the  $h$ th harmonic is

$$I_h = \frac{I_{fund}}{h}; h = kq + 1 \quad (12)$$

where  $h$  is the harmonic order,  $q$  is the pulse number of the circuit (6 in case of 6-pulse converter),  $k$  is an integer (1, 2, 3, ..., etc.),  $I_h$  is the amplitude of the harmonic currents of order  $h$  and  $I_{fund}$  is the amplitude of the rms value of the fundamental current.

### 4) Line Power Losses

At the  $h$ th harmonic frequency, line power loss in the line section between busses  $i$  and  $i + 1$  is expressed below:

$$P_{loss(i,i+1)}^{(h)} = R_{i,i+1} \left( |V_{i,i+1}^{(h)} - V_i^{(h)}| |Y_{i,i+1}^{(h)}| \right)^2 \quad (13)$$

The total power loss, including losses at harmonic frequencies, for an  $n$  bus system is

$$P_{loss} = \sum_{i=1}^H \left( \sum_{i=1}^n P_{loss(i,i+1)}^{(h)} \right) \quad (14)$$

where  $P_{loss}$  is total real power losses,  $V_i^h$  is  $h$ th harmonic order bus voltage,  $R_{i,i+1}$  is resistance between branches  $i$  and  $i+1$  and  $Y_{i,i+1}^h$  shunt admittance at  $h$ th harmonic order [18, 19].

## IV. PROPOSED HARMONIC LOAD FLOW METHOD

Conventionally, fundamental frequency load flow solution which gives the base operation point for system that is fundamental bus voltages and then fundamental powers for non-linear loads and fundamental bus voltages are used to obtain the network power flow. The current drawn by the non-linear loads can be evaluated as a function, which depends on the fundamental bus voltage and the non-linear load parameters. Unlike this, the proposed method formulates a new equation by which the power consumed by non-linear load for every harmonic order can be directly calculated as discussed below.

The complex power flow at the  $i$ th node is

$$S_i = P_i + jQ_i = V_i I_i^* = |I_i|^2 Z_i \quad (15)$$

where  $Z_i$  is impedance that means  $Z_i$  is function of frequency. So  $Z_i$  can be written as:

$$Z_i^h = f(Z_i, h) \quad (16)$$

We can also write the above equation (15) and (16) as below:

$$P_i + jQ_i = h^2 \left| \frac{I_i}{h} \right|^2 Z_i^h = h^2 \left| \frac{I_{fund}}{h} \right|^2 Z_i^h \quad (17)$$

By equation (12) and (17)

$$P_i + jQ_i = h^2 |I_h|^2 Z_i^h \quad (18)$$

The above equation can also be written in the following form:

$$|I_h|^2 Z_i^h = \frac{P_i + jQ_i}{h^2} \quad (19)$$

$$V_h I_h^* = \frac{P_i + jQ_i}{h^2} \quad (20)$$

where  $P_i$  and  $Q_i$  are respective real and reactive power of non-linear load,  $V_i$  is fundamental  $i$ th bus voltage,  $Z_i^h$  is impedance at  $h$ th harmonic order,  $V_h$  is voltage at  $h$ th harmonic order and  $I_h$  current at  $h$ th harmonic order.

Therefore, using equation (20), the real and reactive power of the harmonic generating source can be obtained to get the corresponding harmonic bus voltages, branch currents. In the proposed formulation, the non-linear load is modeled as constant power harmonic source. System components are accurately modeled by above equation and the non-linear load in distribution system is modeled according to proposed

methodology and the Newton-Raphson method can be employed for harmonic load low analysis. The flowchart of the proposed harmonic load flow is shown in Fig. 1.

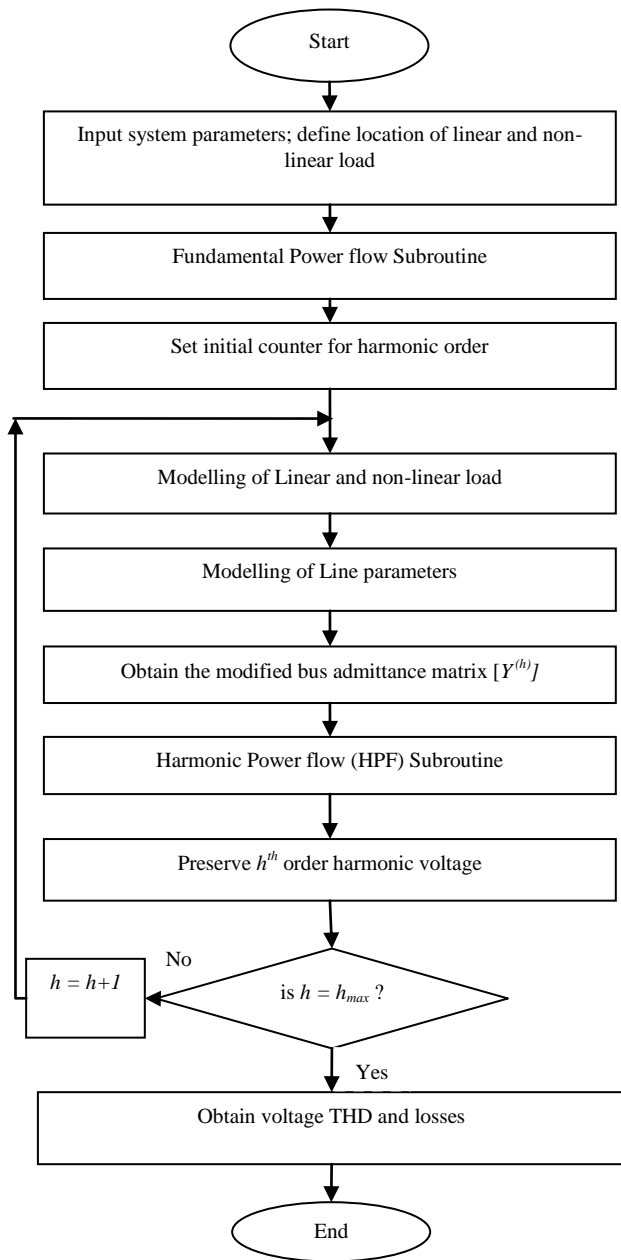


Figure 1. Flowchart for harmonic load flow algorithm

The proposed volt-ampere balance equation for non-linear load is coupled with the conventional Newton-Raphson method based algorithm for harmonic load low analysis. This is based on the balance of active power and reactive volt-amperes, whether at fundamental frequency or at harmonics. The active and reactive power balance is forced to zero by the bus voltage iterations. The system solution (with linear and nonlinear loads under balanced nonsinusoidal three-phase conditions) is achieved by forcing total (fundamental and

harmonic) mismatch active and reactive powers zero using the Newton-Raphson method.

## V. SIMULATION RESULTS

The proposed algorithm is tested on distorted IEEE 33-bus radial distribution system. The substation line voltage is 12.66 kV. The non-linear loads are assumed to be connected at bus 6 and 27 and the distribution generators are connected to bus 7, 24 and 25. The line and load data of the system is taken from [21]. The simulation is performed on the initial radial configuration with open lines 33, 34, 35, 36 and 37.

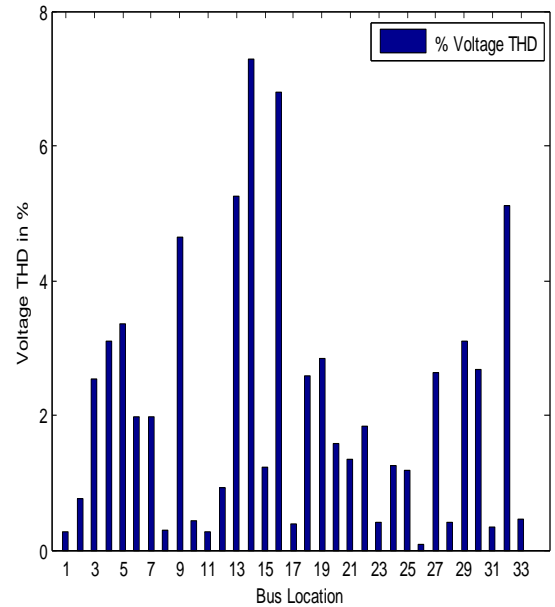


Figure 2. Voltage THD of the distorted IEEE 33-bus system with multiple nonlinear loads and DG's using proposed method

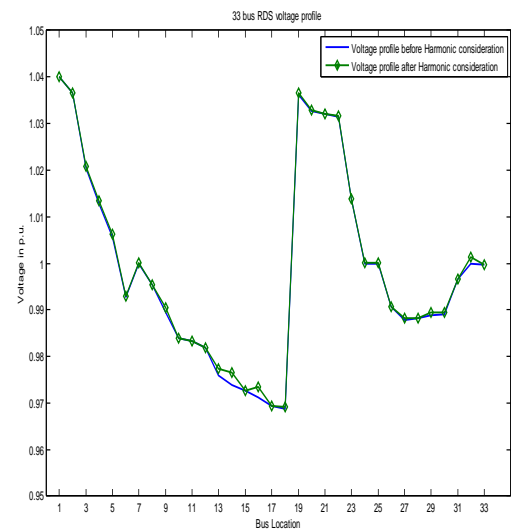


Figure 3. Voltage profile comparison of IEEE 33-bus system before and after harmonic consideration

The results of the proposed harmonic load flow method are shown in Fig. 2 and 3. Fig. 2 shows bus voltage THD levels at buses and it can be observed that the maximum voltage THD is found as 7.29 % at bus 14 using the proposed method. Fig. 3 shows the comparison of bus voltage profiles without and with the presence of non-linear loads using the proposed method. It can be observed from the figure that the maximum rms voltage distortion also occurs at bus 14. The results obtained using the proposed method is compared with the method of [21] in Table I. It can be depicted from the table that the maximum voltage THD using [21] is 7.41%, that is higher than using the proposed harmonic flow. Moreover, minimum voltage, maximum voltage, power loss as obtained using the proposed method are found to be better than that obtained using the method of [21]. This shows the superiority of the newly proposed harmonic flow algorithm.

TABLE I. COMPARISON RESULTS OF THE PROPOSED METHOD WITH [21]

Results	Method [21]	Proposed method
Max Voltage THD (%)	7.41	7.29
$V_{max}$ (p.u.)	1.05	1.04
$V_{min}$ (p.u.)	0.9573	0.9688
$P_{loss}$ (kW)	452	450.7

## VI. CONCLUSION

This paper presents a new harmonic flow method which can combine with the conventional Newton–Raphson load flow method to obtain network solutions in the presence of harmonics attributed due to the presence of non-linear loads in the distribution systems. The proposed volt-ampere balance equation converts non-linear loads into constant power harmonic source. The developed volt-ampere balance equation coupled with Newton–Raphson power flow algorithm and tested on distorted IEEE 33-bus radial distribution system. The results obtained are found better than the existing method on comparison. The key feature of the proposed algorithm is that it enables separate calculations for every harmonic order. The main objective of this work is to find out harmonic r.m.s. voltages and harmonic distortion indices in power system in the presence of harmonics. The proposed algorithm can also be used to solve reconfiguration, optimal reactive compensation, fault restoration, short-circuit and similar kind of other problems of distribution networks in the presence of harmonics.

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