Abstract—This paper presents the analysis of different techniques used for the detection of various power quality disturbances such as voltage sag, swell, interruption and harmonics. Fast Fourier transform technique applied to analyze the voltage signal can correctly identify the magnitudes and phase of harmonic components present in the signal. But it does not give the time information of these harmonics in the analyzed signal. Also, Fast Fourier Transform cannot detect disturbances such as voltage sag, swell and interruption. Measurement of RMS is applied to overcome this. But this method is affected by the instant at which the disturbances occur and hence introduce error in the calculated duration of disturbance. Wavelet Transform of a disturbance signal can detect these disturbances accurately—the magnitude and time information. S-Transform application to detect disturbances is also introduced in this paper, which gives phase information of the harmonics and has capability to detect sag, swell and interruption in the presence of noise. Simulation results of performance comparison of these techniques are presented with disturbance generated by MATLAB code and MATLAB/Simulink.

Index Terms— FFT, wavelet transform, power quality, S-transform, RMS measurement

I. INTRODUCTION

Most of the voltage sensitive critical loads are non-linear in nature, due to the application of fast acting semiconductor switches and their specific control strategy. These non-linear loads introduce current harmonics and voltage disturbances in the utility, which affect the performance of other non-linear and linear loads at the point of common coupling (PCC). Failure of sensitive electronic loads such as data processing, process control and telecommunications equipment connected to the system has become a great concern [1],[2]. One of the important issues in power quality (PQ) problems is to detect and classify disturbance waveforms automatically in an efficient manner. To detect, solve and mitigate the PQ problem, many utilities perform PQ monitoring for their industrial and key customers.

The rms magnitude of voltage supply is used in the power quality standards for detection and characterization of voltage events [1]. The standard method is simple and easy to implement but it does not give information about the phase angle of voltage supply during the event or the point on the wave where the event begins and, as can be seen in [2], presents important limitations in the detection and estimation of magnitude and duration of short-duration voltage events.

Detection of voltage dips is subject to a trade-off between speed and selectivity/accuracy. Fast detection might cause an unnecessary operation either because the detected disturbance is not a voltage dip or because it is a voltage dip but not severe enough to justify the operation of the mitigation equipment.

Wavelet transform (WT) has the capability of extracting information from the signal in both time and frequency domain simultaneously and has been applied in the detection and classification of power quality disturbances [3]-[5]. But it exhibits some disadvantages that it is sensitive to noise, requires proper selection of mother wavelet and the level of decomposition are to be chosen based on the disturbance. S-Transform (ST), considered as the extension of Fast Fourier Transform (FFT) or WT, provides magnitude and phase information of the harmonics [6] and detects the sag/swell in the presence of noise or transient. This paper provides the simulation analysis of voltage events such as sag, swell, interruption and harmonics and detection using RMS measurement, FFT, WT and ST techniques. Comparison of these techniques is provided.

II. TECHNIQUES FOR POWER QUALITY EVENT DETECTION

This section introduces several transformation techniques that are used for the detection of power quality disturbances.

A. Discrete Fourier Transform (DFT)

The frequency content of a periodic discrete time signal with period N samples can be determined using the discrete Fourier transform (DFT). The DFT of the sequence x(n) is expressed as

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)e^{-j\Omega n/k}$$

where \(\Omega = 2\pi/N\) and k is the frequency index. Considering frequency index that varies from 1 to \((N/2) - 1\), the time domain sequence x(n) can be written as

$$x(n) = \sum_{k=1}^{(N/2)-1} \left[ X_R(k) \cos \left( \frac{2\pi kn}{N} \right) - X_I(k) \sin \left( \frac{2\pi kn}{N} \right) \right]$$

where \(X_R\) and \(X_I\) are the in-phase and quadrature-phase components of X(k), respectively. The phasor quantity at the analog frequency \(\omega_0\) is determined as

$$X(\omega_0) = X_R(k) + jX_I(k)$$

The calculation of all the phasor components requires

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\[ \text{N}^2/2 \text{ operations. The fast Fourier Transform (FFT) was} \]

\[ \text{developed to reduce the computational burden of the DFT. When the period} \]

\[ \text{N for a waveform is an integer power of two, the determination of the Fourier coefficients can be} \]

\[ \text{stream lined. The number of operations is reduced from N}^2/2 \]

\[ \text{to N} \log_2(N). \text{However, misapplication of the FFT algorithm} \]

\[ \text{would lead to incorrect results. For example, if the number of} \]

\[ \text{s samples taken includes a fraction of a cycle, the FFT results} \]

\[ \text{would show incorrect frequency spectrum due to leakage [7].} \]

\[ \text{Sampling rates of data acquisition systems are usually set} \]

\[ \text{at fixed values, multiples of KHz. If the sampled waveforms} \]

\[ \text{do not contain an integer number of samples per integer} \]

\[ \text{number of cycles, the results of the DFT algorithm will} \]

\[ \text{include errors. The resulting error is known as spectral} \]

\[ \text{“leakage”. The DFT or FFT of such a sampled waveform will} \]

\[ \text{incorrectly indicate nonzero values for all of the harmonic} \]

\[ \text{frequencies. Furthermore, for the FFT algorithm to be} \]

\[ \text{applied, the number of samples needs to be an integer power} \]

\[ \text{of two.} \]

\[ \text{B. RMS Measurement} \]

\[ \text{Voltage signals are recorded as sampled points in time} \]

\[ \text{and the RMS value of a sampled time-domain signal is} \]

\[ \text{calculated using (4).} \]

\[ V_{\text{rms}} = \frac{1}{N} \sum_{i=1}^{N} v_i^2 \]  

\[ (4) \]

\[ \text{where} \ N \ \text{is the number of samples per cycle and} \ v_i \ \text{are the} \]

\[ \text{magnitudes of sampled signal. RMS calculation can also be} \]

\[ \text{performed by considering samples for one half-cycle [2]. The} \]

\[ \text{window length has to be an integer multiple of one-half} \]

\[ \text{cycle. Any other window length will produce an oscillation in} \]

\[ \text{the rms plot with a frequency twice the fundamental frequency.} \]

\[ \text{C. Wavelet Transform (WT)} \]

\[ \text{Wavelet transformation has the ability to analyze different} \]

\[ \text{power quality problems simultaneously in both time and} \]

\[ \text{frequency domains. Wavelet analysis expands functions in} \]

\[ \text{terms of wavelets, which are generated in the form of} \]

\[ \text{oscillates, has finite energy and zero mean value. Compared} \]

\[ \text{with Fourier transform, wavelet can obtain both time and} \]

\[ \text{frequency information of signal, while only frequency} \]

\[ \text{information can be obtained by Fourier transform. The signal} \]

\[ \text{can be represented in terms of both the scaling and wavelet} \]

\[ \text{function [5] as follows:} \]

\[ f(t) = \sum_{j=0}^{J-1} c_j \Phi(t-n) + \sum_{j=0}^{J-2} d_{j+1} (n) 2^{j/2} \Psi(2^{j/2}t-n) \]  

\[ (5) \]

\[ \text{where} \ c_j \ \text{is the J level scaling coefficient} \]

\[ \text{d}_j \ \text{is the} \ j \ \text{level wavelet coefficient} \]

\[ \Phi(t) \ \text{is scaling function} \]

\[ \Psi(t) \ \text{is wavelet function} \]

\[ J \ \text{is the highest level of wavelet transform} \]

\[ t \ \text{is time} \]

\[ \text{Each wavelet is created by scaling and translation operations of mother wavelet. Wavelet theory is expressed by} \]

\[ \text{continuous wavelet transformation as} \]

\[ CWT_{\Psi}(a,b) = W_{\Psi}(a,b) = \int_{-\infty}^{\infty} x(t) \Psi^*_{a,b}(t) \, dt \]  

\[ (6) \]

\[ \text{where} \ \Psi_{a,b}(t) = \frac{1}{\sqrt{a}} \Psi \left( \frac{t-b}{a} \right), \]

\[ a \ \text{(scale) and} \ b \ \text{(translation) are real numbers.} \]

\[ \text{For discrete-time systems, the discretization process leads} \]

\[ \text{to the time discrete wavelet series as} \]

\[ DWT_{\Psi}(m,n) = \sum_{k=-\infty}^{\infty} x(k) \Psi^*_{m,n}(t) \]  

\[ (7) \]

\[ \text{where} \ \Psi_{m,n}^*(t) = a_{m,n}^{-m/2} \Psi \left( \frac{t-nb_{m,n}}{a_{m,n}} \right), \]

\[ a = a_{m,n}^m \ \text{and} \ b = nb_{m,n}^m \]

\[ \text{Using the discrete wavelet transformation power quality} \]

\[ \text{problems can be classified easily.} \]

\[ \text{D. S-Transform (ST)} \]

\[ \text{ST can be seen as the “phase correction” of continuous} \]

\[ \text{wavelet transform (CWT). The ST of function} \ h(t) \ \text{is defined} \]

\[ \text{as a CWT with a specific mother wavelet multiplied by the} \]

\[ \text{phase factor} \]

\[ S(\tau, f) = e^{-j 2 \pi f \tau} W(d, \tau) \]  

\[ (8) \]

\[ \text{where} \ h(t) \ \text{is defined as} \]

\[ w(t, f) = \frac{1}{\sqrt{2 \pi}} e^{-j \frac{f}{2} \tau^2} e^{-j 2 \pi ft} \]  

\[ (9) \]

\[ \text{The scale parameter} \ d \ \text{is the inverse of the frequency} \]

\[ \text{f. The wavelet in (9) does not satisfy the condition of zero} \]

\[ \text{mean for an admissible wavelet; therefore, (8) is not strictly a} \]

\[ \text{CWT. Written out explicitly, the ST is defined as} \]

\[ S(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{1}{\sqrt{2 \pi}} e^{-j \frac{f}{2} (t-\tau)^2} e^{-j 2 \pi ft} \]  

\[ (10) \]

\[ \text{The ST can also be written as operations on the Fourier} \]

\[ \text{spectrum} \ H(f) \ \text{of} \ h(t) \]

\[ S(\tau, f) = \int_{-\infty}^{\infty} H'(\alpha + f) e^{-j \frac{f}{2} \tau^2} e^{-j 2 \pi ft} d\alpha, \ \ \alpha \neq 0 \]  

\[ (11) \]

\[ \text{The power system disturbance signal} \ h(t) \ \text{can be} \]

\[ \text{expressed in a discrete form as} \ h(kT), \ \text{k = 0, 1, \ldots,} \ N-1, \ \text{where} \]

\[ \text{T is the sampling time interval and} \ N \ \text{is the total sampling} \]

\[ \text{number.} \]

\[ \text{The discrete Fourier Transform of} \ h(kT) \ \text{is obtained as} \]

\[ H \left[ \frac{n}{N T} \right] = \frac{1}{N} \sum_{k=1}^{N-1} h(kT) e^{-j 2 \pi \frac{k n}{N}}, \]  

\[ (12) \]

\[ \text{where} \ n = 0, 1, \ldots, N-1. \]

\[ \text{Using (12), the ST of a discrete time series is given by} \]

\[ \text{(let} \ \tau \rightarrow kT \ \text{and} \ f \rightarrow n / NT) \]
The discrete ST can be computed quickly by taking advantage of the efficiency of the FFT and the convolution theorem. The ST localizes the phase spectrum as well as the amplitude spectrum. ST analysis results in a complex valued matrix (STC matrix). Each row of the STC matrix gives the frequency components of the signal analyzed at various sampling times. Each column of the STC matrix represents the harmonic magnitude and phase of the disturbance signal at a given time or sample.

III. DETECTION OF VOLTAGE EVENTS

This section shows the application of FFT, RMS measurement, WT and ST for detection of voltage events such as voltage sag, swell and interruption. Also, detection of harmonics is discussed.

A. Voltage Sag/Swell

Voltage signals with different levels of sag are generated using MATLAB code. Voltage amplitude of 1 pu is considered for normal condition without any disturbance. The sampling frequency is 6.4 kHz with 128 samples per cycle for supply frequency of 50 Hz. Fig. 1 shows the voltage sag waveform of two cycle duration. RMS measurement technique is applied to detect voltage sag. Fig. 1 also shows the performance of 1-cycle and half-cycle RMS measurement. Voltage sag of two cycle duration (256 samples) starting at the end of first cycle (from sample 129) is considered for analysis.

Half-cycle RMS measurement identifies the instant of sag at sample 159 and 1-cycle RMS measurement identifies the sag at sample 172. Estimation time is taken as the time taken to detect the voltage amplitude to go below 0.9 pu (IEEE std. 1159) from normal value. Fig. 2 shows the time of estimation of voltage amplitude for 1-cycle and half-cycle RMS methods. It is seen that the half-cycle RMS method is suitable for voltage sag measurement. Same is true for the case of voltage swell and interruption.

WT is applied to detect the voltage disturbances such as voltage sag, swell and interruption as shown in Fig. 3. The voltage events are shown in Figs. 3(a) to (c). Figs. 3(d) to (f) shows the detailed coefficients of WT at level 1 obtained with db6 mother wavelet. There is increase in the coefficient at the instant of disturbance start and end. Disturbance or change in the magnitude of signal is identified instantaneously by the WT. These instances may be used to calculate the duration of the disturbance. Performing multi-resolution analysis of the disturbance signal for 10 levels and calculating delta-STD at each level, delta-STD curves are drawn for sag, swell and interruption conditions as shown in Figs. 3(g) to (i) respectively. The frequency bands of WT coefficients are shown in Table I. The magnitude of delta-STD at level 6 which contains the fundamental component is used calculate the percentage of disturbance. Plot of delta-STDs for voltage sag of 10% to 50% are shown in Fig. 4. The peak value of delta-STD at level 6 determines the magnitude of voltage sag. Hence WT analysis identifies magnitude and duration of the voltage events such as sag, swell or interruption.

\[
S \left[ kT + \frac{n}{NT} \right] = \sum_{m=0}^{N-1} H \left[ \frac{n + m}{NT} \right] e^{-\frac{2\pi^2 m^2}{n^2}} e^{j2\pi \frac{mk}{N}} = 0
\]  

where \(k, m = 0, 1, \ldots, N-1\), and \(n = 0, 1, \ldots, N-1\).

Fig. 1. Detection of voltage sag by RMS analysis

Fig. 2. Amplitude estimation by RMS measurements.

Fig. 3. Wavelet analysis of the voltage signal. (a) Voltage sag waveform, (b) Voltage swell waveform, (c) Voltage interruption waveform, (d),(e),(f). Detailed coefficient at level 1 for (a), (b) and (c) respectively and (g),(h),(i). Delta STDs at different levels for (a), (b) and (c) respectively.

Detection of voltage sag is also performed by ST analysis of the signal in Fig. 3(a). ST of the signal results in STC matrix from which the magnitude of the signal can be extracted. Fig. 5(a) shows the magnitude plot (in per unit) of the signal versus samples. The rms values are obtained from

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STC matrix by identifying the maximum amplitudes of the signal at every sample using (14).

\[ | S \{ J T, n / N T \} | \]

where \( T \) is the sampling time interval, \( N \) is the total sampling number and \( n = 0, 1, \ldots , N-1 \).

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Frequency Band (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1</td>
<td>1600 – 3200 Hz</td>
</tr>
<tr>
<td>d2</td>
<td>800 – 1600 Hz</td>
</tr>
<tr>
<td>d3</td>
<td>400 – 800 Hz</td>
</tr>
<tr>
<td>d4</td>
<td>200 – 400 Hz</td>
</tr>
<tr>
<td>d5</td>
<td>100 – 200 Hz</td>
</tr>
<tr>
<td>d6</td>
<td>50 – 100 Hz</td>
</tr>
<tr>
<td>d7</td>
<td>25 – 50 Hz</td>
</tr>
</tbody>
</table>

Fig. 5(b) shows the magnitude versus frequency plot. There is peak only at fundamental frequency (50 Hz) and the magnitude is 0.7pu for sag of 30% in Fig. 3(a). ST contour shown in Fig. 5(c) gives the magnitudes of frequency components at each sample. Normalized frequency \( f_n \) in Fig. 5(c) is given by

\[ f_n = n f_o / f_s , \quad n = 1, 2, 3, \ldots \]

(15)

where \( f_o \) = Fundamental frequency (50 Hz)
\( f_s \) = Sampling frequency (6.4 KHz)

The fundamental component RMS is 0.49pu which is 0.7pu peak.

**B. Voltage Harmonics**

Analysis of voltage signals containing harmonic components are analyzed in this section. Two cases are considered to evaluate the performance of harmonic detection techniques with suitable percentage of harmonic signal added to the fundamental waveform. In first case, the synthetic signal of 3 cycles shown in Fig. 6(a) with magnitude and frequency content given Table II is considered.

FFT analysis of the harmonic signal can detect the magnitudes of the fundamental and harmonics accurately as shown in Fig. 6(b). But it does not provide the time information of these harmonics. Fig. 7 shows the WT analysis of signal in Fig. 6(a). The detailed coefficients at five levels (d1 to d5) and approximation coefficient at

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**Table I Frequency Bands of DWT Coefficients at Different Levels**

<table>
<thead>
<tr>
<th>Levels</th>
<th>Coefficients</th>
<th>Frequency Band (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d1</td>
<td>1600 – 3200 Hz</td>
</tr>
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<td></td>
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<td></td>
<td>d3</td>
<td>400 – 800 Hz</td>
</tr>
<tr>
<td></td>
<td>d4</td>
<td>200 – 400 Hz</td>
</tr>
<tr>
<td></td>
<td>d5</td>
<td>100 – 200 Hz</td>
</tr>
<tr>
<td></td>
<td>d6</td>
<td>50 – 100 Hz</td>
</tr>
<tr>
<td></td>
<td>d7</td>
<td>25 – 50 Hz</td>
</tr>
</tbody>
</table>

**Table II Harmonic Content of the Synthetic Signal (Case 1)**

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Harmonic No.</th>
<th>Percentage of Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000 – 0.020</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>0.020 – 0.060</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>0.060 – 0.100</td>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

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**Fig. 4. Delta-STD curves for different Percentages of sag.**

**Fig. 5. ST analysis of voltage sag. (a) RMS value v/s samples, (b) Frequency spectrum and (c) ST contour.**

**Fig. 6(a). Voltage signal with multiple frequencies (case1) and 6(b). Frequency spectrum obtained by FFT analysis.**

**Fig. 7. Detailed coefficients of the multiple frequency signal (case1).**
level 5 (a5) are shown in Fig. 7. With sampling frequency of 6400 Hz, the harmonics 150 Hz, 250 Hz and 500 Hz components are present in levels 5, 4 and 3 respectively as given in Table I. The fundamental frequency signal is present in the approximation coefficient plot (a5 plot).

The frequency spectrum obtained from ST analysis is shown in Fig. 8(a) from which, the magnitudes of the harmonics are extracted. The time information of these harmonic signals is obtained by plotting the ST contour. ST contour in Fig. 8(b) is obtained by plotting magnitudes of the STC matrix resulted from ST analysis. Hence ST has the feature of both FFT and WT and is considered as the extension of FFT and WT. Second case is considered with the signal in Fig. 9(a) simulated by shifting the harmonics as given in Table III.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Harmonic No.</th>
<th>Percentage of Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000 - 0.020</td>
<td>1</td>
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<td>20</td>
</tr>
<tr>
<td>0.020 - 0.060</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>0.020 - 0.060</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

The harmonic spectrum obtained from FFT analysis is given in Fig. 9(b) which is similar to that in Fig. 6(b). This shows that FFT can detect the magnitudes frequency components but does not give the time information. WT analysis the signal for case 2 resulted in the coefficient plots as shown in Fig. 10. Coefficient plot at level 3 (d3 plot) shows that the 500 Hz component is present from sample 129 to 384, and the 150 Hz and 250 Hz components are present from sample 1 to sample 128 which match with the time given in Table III. The frequency spectrum and ST contour plots are shown in Figs. 11(a) and (b) respectively. The results of ST match with signal definition given in Table III.

C. Voltage Sag/Swell with noise

Fig. 12 shows the case for consecutive voltage sag and swells with the noise signal present. Figs. 12(b) and (c) show the effectiveness of the ST in identifying the disturbances. When WT is performed the detailed coefficients are high for the instants when the signal contains noise or transients, and at the instants of start and end of sag/swell in the signal. Fig. 12(d) shows that when there are transients or noise in the signal WT fails to identify the sag/swell condition. Hence ST is having edge over the WT in detecting a disturbance under noisy condition. ST has the ability to detect the occurrence of disturbance correctly in the presence of noise.
IV. PERFORMANCE COMPARISON

Several cases of the PQ events are discussed in the previous sections and the comparison is made between the techniques employed for analysis and detection of PQ disturbances such as sag, swell, interruption and harmonics.

A. Feature Extraction

RMS measurement technique is suitable for detection sag, swell and interruption but cannot be used for harmonic measurement. FFT analysis provides features of harmonic magnitudes but does not provide time information. Detailed coefficient feature extracted by WT is used for detecting the start and end times of the disturbance. ST analysis results in STC matrix [6], [8], [9] from which the features of voltage start and end times of the disturbance. ST analysis results in coefficient feature extracted by WT is used for detecting the sag, swell or interruption without any time delay.

B. Effect of instant of disturbance initiation on the signal

The instant of sag or swell in the voltage signal affects the detection time calculated by using 1-cycle and half-cycle RMS techniques. The evaluation time of voltage magnitude for different instants of disturbance is shown in Fig. 2. Also, the detection of disturbance is fast by half-cycle RMS technique as compared to 1-cycle method. WT can detect the sag, swell or interruption without any time delay.

C. Duration of disturbance

FFT does not provide any time information of the frequency components in the signal and hence information of duration of disturbance cannot be found by FFT. WT gives better visualization of time-varying waveform distortions. However, this information is not sufficient for accurate analysis and classification of disturbance. Multi-resolution analysis and ANN techniques or fuzzy logic techniques [10] are further to be applied. By using magnitude plot obtained by ST, duration of sag, swell and interruption may be calculated.

D. Selection of mother wavelet

WT requires proper selection of mother wavelet used for analysis. Performance of WT varies with different mother selection. ST which is the phase extension of WT, on the other hand has a specific mother wavelet defined.

E. Effect of noise in the signal

When noise or transient is present in the voltage signal, WT cannot detect the sag and swell disturbances as it is very sensitive to noise. But ST can accurately detect the disturbance even in the presence of noise or transients.

V. CONCLUSIONS

This paper presented various techniques used for PQ event detection and their performances are compared. Voltage sag and swell detection is faster with half-cycle RMS technique as compared to 1-cycle RMS method. FFT analysis applied to voltage can detect the harmonics but does not give any time information of these harmonics. WT can detect the sag, swell, interruption. To detect the harmonics using WT, multiresolution technique is employed for different levels. Detection of disturbances by ST is accurate as it can detect sag and swell in the presence of noise.

VI. REFERENCES