Abstract: The Luenberger Observer based sensorless control of a five-phase synchronous reluctance machine with current control in the stationary reference frame is presented in this paper. The extended Luenberger Observer based sensorless operation of a three-phase induction machine is well established and the same principle is extended in this paper for a five-phase synchronous reluctance machine. Performance, obtainable with hysteresis current control, is presented for a number of operating conditions on the basis of simulation results. Dynamics, achievable with a five-phase vector controlled synchronous reluctance machine, are shown to be essentially identical to those obtainable with a three-phase induction machine.

Index Terms: Multi-phase machines, Extended Luenberger Observer, Synchronous reluctance motor, Sensorless speed control.

I. INTRODUCTION

On multi-phase a number of interesting research results have been published over the years and detailed reviews are available in [1,2]. Major advantages of using a multi-phase machine instead of a three-phase machine are detailed in [1-3] and are higher torque density, greater efficiency, reduced torque pulsations, greater fault tolerance, and reduction in the required rating per inverter leg (and therefore simpler and more reliable power conditioning equipment). Additionally, noise characteristics of the drive improve as well [4].

Higher torque density in a multi-phase machine is possible since, apart from the fundamental spatial field harmonic, space harmonic fields can be used to contribute to the total torque production [3,5,6]. This advantage stems from the fact that vector control of the machine’s flux and torque, produced by the interaction of the fundamental field component and the fundamental stator current component, requires only two stator currents (d-q current components). In a multi-phase machine, with at least five phases or more, there are therefore additional degrees of freedom, which can be utilised to enhance the torque production through injection of higher order current harmonics. In a five-phase machine third harmonic current injection can be used [5-8] to enhance the overall torque production.

Multi-phase synchronous reluctance (Syn-Rel) motor has the same stator structure as that of a multi-phase induction motor; however, the rotor can have a simple structure obtained by removing cut out from a round rotor. Syn-Rel offers the inherent ruggedness, simplicity and ease of maintenance of squirrel cage motors. It has gained popularity as an alternative solution to induction motor drive due to low cost and simpler structure. The vector control approach to a five-phase Syn-Rel motor with concentrated winding on the stator is reported in [9,10] with current control in rotating reference frame. The space vector pulse width modulation (SVPWM) is used to control five-phase voltage source inverter. Only the outer large vectors are used to realise the SVPWM. However, this method produces unwanted low-order harmonics [11] and has complicated control structure. In contrast this paper deals with Syn-Rel with distributed winding on the stator and simpler hysteresis current control with current control in the stationary reference frame.

Sensorless vector control of three-phase induction machine has attracted wide attention in recent years [12-13]. Different, more sophisticated techniques are required for high performance applications in vector controlled drives [14]. In a sensorless drive, speed information and control should be provided with an accuracy of 0.5% or better, from zero to the highest speed, for all operating conditions and independent of saturation levels and parameter variations. In order to achieve good performance of sensorless vector control, different speed estimation schemes have been proposed and a variety of speed estimators exist nowadays.

Sensorless operation of a vector controlled three-phase induction machine drive is extensively discussed in the literature [15], but the same is not true for multi-phase induction machine. Only few application specific sensorless operation of multi-phase machine is elaborated in the literature. The problem of using the position sensor in 'more-electric' aircraft fuel pump fault tolerant drive is highlighted in [16]. The drive utilises a 16 kW, 13000 rpm six-phase permanent magnet motor with six independent single-phase inverters supplying each of the six-phases. The authors proposed an alternative sensorless drive scheme. The proposed technique makes use of flux linkage-current-angle model to estimate the rotor position.

Although several schemes are available for sensorless operation of a vector controlled drive, but the most popular is the Luenberger Observer because of ease of their practical implementation [17-20]. An attempt is made in this paper to extend the Luenberger Observer based technique of a three-phase machine to an indirect field oriented five-phase synchronous motor drive.

In this paper, only the fundamental spatial field harmonic is used to produce the machine’s torque under vector control conditions and a five-phase system is analysed. The well known rotor flux oriented control (RFOC) method, used to
achieve high performance control of three-phase machines, can be easily extended to five-phase machines under this condition. It has been shown in [8] that multi-phase machine models can be transformed into a system of decoupled equations in orthogonal reference frames. The $d$-$q$ axis reference frame currents contribute towards torque and flux production, whereas the remaining $x$-$y$ components plus the zero sequence component(s) do not. This allows a simple extension of the RFOC principle in that the rotor flux linkage is maintained entirely in the $d$-axis, resulting in the $q$-axis component of rotor flux being maintained at zero. This reduces the electromagnetic torque equation to the same form as that of a dc machine or a rotor flux oriented three-phase machine. Thus the electromagnetic torque and the rotor flux can be controlled independently, by controlling the $d$ and $q$ components of stator current independently. The decoupled control of torque and flux using rotor flux oriented control for a five-phase induction machine is illustrated in [8].

The analysis is here restricted to Extended Luenberger Observer algorithm for speed estimation in an indirect rotor flux oriented synchronous reluctance machine, with current control in the stationary reference frame. Phase currents are controlled using hysteresis current control method. A simulation study is performed for speed mode of operation, for a number of transients, and the results are reported in the paper.

II. D-Q MODELLING OF A FIVE-PHASE SYNCHRONOUS RELUCTANCE MACHINE

Standard assumptions of general theory of electrical machines apply to the modelling described in what follows, including the one related to the sinusoidal spatial distribution of the field in the machine. Spatial displacement between any two consecutive phases in a five-phase machine is $\alpha = 2 \pi / 5$.

The stator has five-phase distributed winding while there is no winding on the rotor. The cross sectional view of a five-phase Syn-Rel with two pole pair on the rotor is shown in Fig. 1. The modelling is performed by analogy with corresponding three-phase synchronous machine modelling and multi-phase induction machine modelling. Hence only $d$-$q$ models are elaborated here. The synchronous machine has to be modelled in the rotating reference frame firmly fixed to the rotor. Correlation between original phase variables and new variables in the rotating reference frame is given with

$$
\begin{align*}
    v_{dq}' &= A_s v_{abcde}' \\
    i_{dq}' &= A_s i_{abcde}' \\
    \psi_{dq}' &= A_s \psi_{abcde}'
\end{align*}
$$

where the transformation matrix $A_s$ is given by equation (2).

The superscript $s$ stands for stator.

$$
A_s^{-1} = A_s
$$

The angles of transformation for stator quantities are related to the speed of the selected common reference frame $\omega$ through:

$$
\theta_s = \int \omega dt = \theta
$$

where $\omega$ and $\theta$ are rotor angular (electrical) speed and instantaneous rotor position, respectively. Application of (3) in conjunction with (1) and the phase-domain model of a five-phase synchronous reluctance machine leads to the following set of equations in the rotational common reference frame (symbol $p$ stands for $p = d/dt$):

$$
\begin{align*}
    v_{ds}' &= R_i i_{ds} - \omega \psi_{qs} + p \psi_{ds} \\
    v_{qs}' &= R_i i_{qs} + \omega \psi_{ds} + p \psi_{qs} \\
    \psi_{ds} &= \left( L_{ds} + L_{md} \right) i_{ds} = L_{ds} i_{ds} \\
    \psi_{qs} &= \left( L_{qs} + L_{mq} \right) i_{qs} = L_{qs} i_{qs} \\
    \psi_{as} &= L_{a} i_{as} \\
    \psi_{bs} &= L_{b} i_{bs}
\end{align*}
$$

The torque equation is

$$
T_c = P \left( L_T - L_q \right) i_a i_b
$$

where $P$ is the pole pair, $L_T, L_q$ are the direct axis (along e Fig. 1) and quadrature axis (along d+e, Fig. 1) inductances. Equations (4)-(6) represent the complete $d$-$q$ models for a five-phase Syn-Rel machine. The only difference between the
five-phase machine model, given with (4)-(6), and the corresponding three-phase machine model is the presence of x-y component equations in (4) and (5). Zero sequence component equations can be omitted from further consideration due to short-circuited star connection of the stator winding. Finally, since stator x-y components are fully decoupled from d-q components and one from the other and vector control is applied (i.e. only d-q axis current components are generated), the equations for x-y components can be omitted from further consideration as well. This means that the model of the five-phase synchronous reluctance machine in rotational reference frame becomes identical to the model of a three-phase Syn-Rel machine. Hence the same principles of RFOC can be utilised, as for a three-phase Syn-Rel machine.

III. VECTOR CONTROL OF A FIVE-PHASE SYNCHRONOUS RELUCTANCE MACHINE

The indirect rotor flux oriented controller used for five-phase Syn-Rel is shown in Fig. 2. The simplest possible method of vector control, called constant d-axis current control is utilised here. Operation in the base speed region only is assumed, so that the rotor flux reference (and consequently stator d-axis current reference) is constant at all times. The five-phase voltage source inverter is controlled using hysteresis current control and closed-loop speed control is investigated. The relationship between the star connected machine phase-to-neutral voltages and inverter leg voltages is given with

\[ v_a = (4/5)v_A - (1/5)(v_B + v_C + v_D + v_E) \]
\[ v_b = (4/5)v_B - (1/5)(v_A + v_C + v_D + v_E) \]
\[ v_c = (4/5)v_C - (1/5)(v_A + v_B + v_D + v_E) \]
\[ v_d = (4/5)v_D - (1/5)(v_A + v_B + v_C + v_E) \]
\[ v_e = (4/5)v_E - (1/5)(v_A + v_B + v_C + v_D) \]

(7)

where capital letters denote inverter leg voltages (which take the value of either dc link voltage or zero) and lower case indices denote phase-to-neutral voltages of the machine.

IV. LUENBERGER OBSERVER BASED SENSORLESS CONTROL

A state observer is a model-based state estimator which can be used for the state and/or parameter estimation of a non-linear dynamic system in real time. In the calculations, the states are predicted by using a mathematical model, but the predicted states are continuously corrected by using a feed back correction scheme. The actual measured states are denoted by \( \hat{x} \) and the estimated states by \( \hat{\hat{x}} \). The correction term contains the weighted difference of some of the measured and estimated outputs signals (the difference is multiplied by the observer feedback gain, \( G \)). The accuracy of the state observer also depends on the model parameters used. The state observer is simpler than the Kalman observer, since no attempt is made to minimize a stochastic cost criterion.

To obtain the full-order non-linear speed observer, first the model of the induction machine is considered in the stationary reference frame, which can be described as follows:

\[ \dot{x} = Ax + Bu \]
and the output vector is

\[ \hat{i}_s = Cx \]

(9)

By using the derived mathematical model of the induction machine, e.g. if the component form of the equation (8), is used, since this is required in an actual implementation and adding the correction term, which contains the difference of actual and estimated states, a full-order state observer, which estimates the stator currents and rotor flux linkages, can be described as follows:

\[ \dot{x} = A\hat{x} + Bu + G(i_s - \hat{i}_s) \]

(10)

and the output vector is

\[ \hat{i}_s = C\hat{x} \]

(11)

where \( A \) is a state matrix , \( B \) is the input matrix, \( G \) is the observer gain matrix, \( C \) is the output matrix, \( x \) is the state vector, \( u \) is the input vector, \( i_s \) stator current vector.

Also the state matrix of the observer ( \( \hat{A} \) ) is a function of the rotor speed, and in a speed –sensorless drive, the rotor speed must be estimated. The estimated rotor speed is denoted by \( \hat{\omega}_r \), and in general \( \hat{\dot{\omega}}_r \) is a function of \( \hat{\dot{\omega}}_r \). The estimated speed is considered as a parameter in \( \hat{A} \), however in extended Kalman filter considered as a state variable. In equations (8) and (9) the different terms are explained as follows:

\[ \hat{A} = \begin{bmatrix} -\frac{1}{T_s}(1 - \sigma)/T_r & \frac{1}{T_m(T_s L_r)} \frac{I_2}{T_r} & -1/T_r & \hat{\omega}_r \end{bmatrix} \]
\[ B = \frac{I_2}{L_s}O_2 \]
\[ C = \left[I_2, O_2 \right]^T \]
\[ u = u_s = \left[u_{ds}, u_{qs} \right]^T \]
\[ \hat{x} = \left[\hat{i}_s, \hat{\dot{i}}_s \right]^T \]

(12)
\[ i_s = \begin{bmatrix} i_{ds} \ i_{qs} \end{bmatrix}^T, \quad \hat{i}_s = \begin{bmatrix} \hat{i}_{ds} \ \hat{i}_{qs} \end{bmatrix}^T \]

\[ J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \]

\( I_2 \) is a second order identity matrix. \( O_2 \) is a 2x2 zero matrix.

In state matrix \( \hat{A} \), the different terms are as follows: \( L_m \) and \( L_r \) are the magnetizing inductance and rotor self-inductance respectively. \( L_s \) is the stator transient inductance. \( T_s = L_s / R_s \) and \( T_r = L_r / R_r \) are the stator and rotor transient time constants respectively, and \( \sigma = 1 - L_m^2 / (L_s L_r) \) is the leakage factor.

The observer gain matrix is defined as

\[ G = \begin{bmatrix} g_1 I_2 + g_2 J \\ g_3 I_2 + g_4 J \end{bmatrix} \]

which yields a 2x4 matrix. The four gains in \( G \) can be obtained from the eigen-values of the induction motor as follows:

\[ g_1 = -(k - 1) \left( \frac{1}{T_s} + \frac{1}{T_r} \right) \]

\[ g_2 = (k - 1) \hat{\omega}_r \]

\[ g_3 = (k^2 - 1) \left( \frac{1}{T_s} \right) \left( \frac{1}{T_r^2} \right) \left( \frac{L_s L_m}{L_r} + \frac{L_m}{T_r} \right) \]

\[ + \frac{L_s L_m}{L_r} (k - 1) \left( \frac{1}{T_s} + \frac{1}{T_r} \right) \]

\[ g_4 = -(k - 1) \hat{\omega}_r \frac{L_s L_m}{L_r} \]

It follows that the four gains depend on the estimated speed, \( \hat{\omega}_r \). By using equations (8) and (9) it is possible to implement a speed estimator which estimates the rotor speed of an induction machine by using the adaptive state observer shown in Fig. 3. In this observer only d and q components of voltage are utilized since x and y components do not offer any significant contribution in the estimation.

In Fig. 3 the estimated rotor flux-linkage components and the stator current error components are used to obtain the error speed tuning signal and given by equations:

\[ \dot{\psi}_r = \psi_{dr} + j \psi_{qr} \]

\[ \dot{\epsilon} = e_{ds} + j e_{qs} \]

The estimated speed is obtained from the speed tuning signal by using a PI controller thus,

\[ \hat{\omega}_r = K_p (\psi_{dr} e_{ds} - \psi_{dr} e_{qs}) + K_i \int (\psi_{qr} e_{ds} - \psi_{dr} e_{qs}) dt \]

where \( K_p \) and \( K_i \) are proportional and integral gain constants respectively, \( e_{ds} = i_{ds} - \hat{i}_{ds} \) and \( e_{qs} = i_{qs} - \hat{i}_{qs} \) are the direct and quadrature axis stator current errors respectively. The adaptation mechanism is similar to that as used in the MRAS-based speed estimators, where the speed adaptation has been obtained by using the state-error equations of the system considered.

V. SIMULATION RESULTS

Per-phase equivalent circuit parameters obtained from [10] of the 50 Hz, five-phase synchronous reluctance machine, used in the work, are \( R_s = 0.6 \Omega, L_m = 82.95 mH, L_s = 9.8 mH, L_a = 4.17 mH \).

Inertia and the number of pole pairs are equal to 0.23kgm\(^2\) and 2, respectively. Rated phase current, phase-to-neutral voltage and per-phase torque are 14.3 A, 220 V and 12.333 Nm, respectively. Rated per phase power is 1.9333 kW. Parameters of the speed PI controller are \( K_p = 15, K_i = 0.06 \).

Hysteresis band is set to ±2.5% of the rated peak phase current (i.e. ± 0.5056 A). Torque limit is at all times equal to one half the rated motor torque (i.e. 92.5 Nm). Dc link voltage equals 587 V (\( \sqrt{2} \times 415 \) V) and provides approximately 10% voltage reserve at rated frequency.

Synchronous reluctance machine is excited at zero time instant by applying the constant (rated) value of the stator d-axis current reference (17.687 A RMS) and is kept constant throughout the simulation period. Speed command of 1200 rpm (252.72 rad/s electrical) is applied at t=0.3 s in a ramp wise manner from \( t = 0.3 \) to \( t = 0.35 \) s and further kept unchanged. Operation takes place under no-load conditions.

Figure 4 to 7 displays the different simulation results. The rotor flux, motor torque, motor speed and stator phase ‘a’ actual current are shown. Acceleration takes place with the maximum allowed value of the motor torque. Actual motor phase current tracks the reference very well. Consequently, torque response closely follows torque reference.

Disturbance rejection properties of the drive are also investigated. The steady state is the one (no-load operation at 1200 rpm) and a load torque equal to the motor rated torque is applied in a step-wise manner at \( t = 1 \) sec. Application of the load torque causes an inevitable dip in speed. Motor torque quickly follows the torque reference and enables rapid compensation of the speed dip. The motor torque settles at the value equal to the load torque and the motor current becomes rated at the end of the transient.
Finally, reversing transient is also examined as well. The steady state is the one (rated load torque operation at 1200 rpm) and the command for speed reversal is given at \( t = 1.2 \) s. Once more, actual torque closely follows the reference, leading to the speed reversal, with torque in the limit, in the shortest possible time interval.

Fig.4. Rotor flux characteristic for indirect RFOC of a five-phase Syn-Rel machine with Luenberger Observer estimator.

Fig.5. Motor speed characteristic for indirect RFOC of a five-phase Syn-Rel machine with Luenberger Observer estimator.

Fig.6. Motor torque characteristic for indirect RFOC of a five-phase Syn-Rel machine with Luenberger Observer estimator.

VI. CONCLUSION

The paper deals with Extended Luenberger Observer based sensorless vector control of a five-phase synchronous reluctance machine, utilising an indirect rotor flux oriented controller and current control in the stationary reference frame. Modelling of a five-phase synchronous reluctance machine is at first reviewed and it is shown that the resulting model is the same as for a three-phase machine. Hence the same vector control principles and speed estimation technique are applicable. Operation in the speed mode is further studied, utilising the hysteresis current control. The speed feedback signal is the estimated one obtained from Luenberger Observer based speed estimator. The attainable performance is examined by simulation. It is shown that the dynamic behaviour, obtainable with the indirect vector control, is the same as it would have been had a three-phase machine been used. Rotor flux and torque control are fully decoupled, enabling the fastest possible accelerations and decelerations with the given torque limit.

REFERENCES


