Abstract—This paper examines the problem of optimally deploying distributed generation (DG) in a microgrid and proposes an heuristic approach to reduce the computational effort involved in determining the optimal solution. Much research has already been conducted in the area of optimal resource deployment in microgrids, and in the bulk of the work the problem has been posed as one of non-linear optimization, of considerable complexity. The proposed approach consists of planning for the load growth in a given distribution system, and systematically developing it into an optimal microgrid, where DG is deployed in the system and appropriate network additions are made to meet system-wide reliability guarantees in the presence of component unavailabilities. The paper presents a formulation of the optimization problem, its implementation using a method based on particle swarm optimization, and its demonstration on a small distribution system.

I. INTRODUCTION

The increasing penetration of distributed generation (DG) worldwide motivates the development of strategies for optimally augmenting distribution systems to form microgrids. Such problems have been addressed in various forms in prior research [1]–[5]. As an example, the “CERTS Microgrid Concept” [4], [5] focuses on the utilization of combined heat and power (CHP) mode of DG to benefit a compact cluster of loads. Many of the approaches have focused on the optimal placement of DG in the distribution network [1]–[3]. Most of the work tried to preserve the radial structure of the distribution system; on the other hand, we recognize that in order to exploit the reliability and economic advantages offered by DG, microgrids will evolve into weakly meshed networks, and develop a network augmentation strategy that permits this.

Of all the benefits of microgrids, reliability is arguably one of the most compelling. It is a major consideration in cases where the cost per kW is an issue. Moreover, the ability to install DG at or near load points immediately opens up the opportunity to offer reliability-differentiated services. These considerations have motivated our attempts to develop methodologies for reliability-centered design and planning of systems where DG can be deployed. In recent times, increasing awareness of reliability issues have resulted in reliability being considered as an important factor in network planning [6]–[12].

The development of optimal microgrid architectures, as envisioned by the authors, consists of two aspects: (a) optimal sizing and siting of DG, and (b) optimal network topology, comprising an optimal set of interconnections and associated capacities. In both these aspects, the optimization seeks to determine the most economical architecture that meets stipulated reliability criteria. In prior work [13]–[16], the authors have solved these problems independently. This paper presents a unified method to expand a distribution system into a microgrid that satisfies system-wide reliability guarantees. The expansion consists of adding suitable quantities of DG at suitable locations and increasing feeder capacities or adding new feeders, and is optimal in that the cost of expansion is minimized.

In this paper a scheme for solving the combined reliability-oriented optimal microgrid architecture problem is presented. This scheme is based on Particle Swarm Optimization (PSO) [17]. This paper presents the PSO formulation of the problem and describes the method and the implementation. The method is also demonstrated on a test system.

II. PROBLEM STATEMENT

This paper attempts to develop a self-sufficient, networked (meshed) microgrid from an existing distribution system. The existing system is connected to the main grid at the Point of Common Coupling (PCC). For the purpose of developing this network into a self-sufficient system, it is assumed that the system is disconnected from the utility, i.e., islanded operation is assumed. Under these circumstances, the local load must be met by means of distributed generation. A size and location of DG to be deployed at candidate locations is to be determined. Generally permission is acquired to install generating units, or build DG ‘farms’ or ‘parks’ at various locations. From the available locations, one needs to decide where to finally place the clusters of DG units of optimal size.

Another aspect of microgrid is the optimal network topology, i.e., among all possible rights of way for building or augmenting transmission lines, which ones should be chosen, and once the optimal set of paths are determined, how much capacity to assign to those paths.

The aggregate of the above component problems constitute the optimal microgrid architecture. This paper presents a method to determine both the topology and location of DG such that the cost is minimized and at the same time satisfying reliability stipulations.

The paper mentions a reasonable approach to defining the problem in a generic way. Though complex formulations are possible the technique developed can be expanded to a wide set of problems related to optimization.
III. SYSTEM MODELING

Different components of the microgrid are modeled both for the computation of reliability and developing a cost function. This section defines the models for the system components. 

Generators: These are modeled as two-state devices. Each generator \( i \) is described by its maximum generating capacity \( G_{max} \), and its forced outage rate \( FOR_i \).

Load: For the purpose of system planning, the load has been assumed to remain constant at the coincident peak.

Transmission Lines: The basic element of the transmission network is the unit-link which is a transmission line with the following characteristics:

- A unit-link connecting a given pair of buses is a line of fixed capacity, fixed cost per unit length, fixed impedance per unit length, and of length corresponding to the right of way between the buses.
- Unit-links between different bus pairs will have different lengths, impedances and costs, but same capacity.
- A link between a given pair of buses can consist of one or more (an integral number of) unit-links connected in parallel between that bus pair. The cost of a link is equal to the total cost of the unit-links that constitute the link.

Network Model: A linearized network model in the form of DC Load Flow [18] has been used in this work. This representation is considered appropriate in this method because of the following reasons.

1) For a planning study involving DG, which typically has inadequate VAr capability, it makes sense to plan for real power generation first and then for VAr support. DC flow is more appropriate when dealing with only real power flows, particularly since there is inadequate information about reactive power.

2) In this work, it is assumed that the DG will be installed at buses on three-phase primary feeders, so unbalance is not a concern.

A. Scope of the problem

It is important to understand that this paper defines an architectural framework of the problem and not specifically the design of a microgrid. Microgrid architecture becomes the scope for design of various microgrid applications eg. hospitals or a military bases etc. Architecture is necessary before attempting to design a microgrid. Design elements like unbalanced three phase system, reactive support and DG control characteristics etc. vary with circumstances. The authors have addressed the problem at the architectural level.

IV. PROBLEM FORMULATION

Cost optimal microgrid expansion is defined here. The objective function is the system expansion cost for the microgrid. Reliability is modeled as a constraint so that the microgrid satisfies a system-wide reliability guarantee. The cost of building the microgrid consists of the following parts:

(a) The cost of the transmission network:

Let

\[
J_i = \text{cost of a unit-link along the } i^{th} \text{ right of way} \\
u_i = \text{number of unit-links in parallel along} \\
\text{the } i^{th} \text{ right of way} \\
N_{\ell} = \text{number of rights of way for building transmission lines}
\]

Then, the cost of the transmission network is given by

\[
J_T = \sum_i (J_i \times u_i) \quad \forall \ 1 \leq i \leq N_{\ell}
\] (1)

(c) Other costs such as T&D losses, O&M etc can also be addressed. The formulation and optimization technique are so developed to solve any complex cost function. The simulated annealing based optimization explained in this paper is capable of solving highly non-linear objective functions and constraints. The problem formulation is made generic with the possibility of adding additional costs in the objective function. For the sake of demonstration a simple linear objective function is defined here. In this paper the change in the costs due to T&D losses and O&M costs are considered negligible and not included.

The total cost is therefore:

\[
J = J_T + J_G
\] (3)

The aim is therefore, to determine

1) The vector \( u = < u_1, u_2, \ldots, u_{N_{\ell}}, d_1, d_2, \ldots, d_{N_{\ell}} > \), which satisfies the above objective function. The solution vector has two components. The first part \( < u_1, u_2, \ldots, u_{N_{\ell}} > \) is the (integral) number of unit links to be allocated along each right of way. This will determine the network topology as well as the optimal capacities of transmission lines. The capacity of a line can be found by multiplying the capacity of a unit link by the number of unit links the line is comprised of.

2) The second part \( d = < d_1, d_2, \ldots, d_{N_{\ell}} > \), which is the capacity of DG cluster to be deployed at candidate locations. Each component of this vector is a number pointing to the capacity of DG at a particular node from the set of nodes where we have the permission to install DG.
The components $\mathbf{u}$ and $\mathbf{d}$ together specify the solution vector $\mathbf{x} = < \mathbf{u}, \mathbf{d}>$. Our aim is to find that solution vector which

Minimize

$$J = J_T + J_G$$

subject to

$$EIR > R_0$$  \hspace{1cm} (4)

where

$EIR$ = system-wide energy index of reliability

$R_0$ = minimum required system-wide reliability

The energy index of reliability, $EIR$, is defined as the ratio of energy served to the total system energy demand, and is given by equation (10). This index is determined as described in section V(c).

V. SOLUTION STRATEGY

A. Particle Swarm Optimization

The problem of microgrid expansion is a non linear optimization problem. The combinatorics, especially due to the topology part of the problem, makes the problem very computation-intensive. Further, different deployment strategies invariably lead to different network topologies and the boundary of feasible solution becomes extremely discontinuous.

In order to solve this problem, we use the method of Particle Swarm Optimization in this work. This method was chosen because it has been found to be very fast in solving unconstrained optimization problems. It has also been successfully employed in many engineering areas (constrained optimization) including power systems [19]. In the power systems area, it has particularly been used in distribution state estimation [20], dynamic security analysis [21] and AGC tuning [22].

The method is implemented as simulation of social behaviour, birds in search of food etc., which is described in detail in [23]. The movement of the particles, which represent potential solutions, is governed by three factors: (1) Inertia, (2) Personal Best and (3) Group Best. An interaction of the above three components generates a vector that determines the direction and magnitude of movement for each particle. This is given by the following equations:

$$v[i] \leftarrow \alpha \cdot v[i] + r_1 \cdot c_1 \cdot (pbestx[i] - presentx[i]) + r_2 \cdot c_2 \cdot (pbestx[gbest] - presentx[i])$$  \hspace{1cm} (5)

where

$v[i]$ is a vector of velocities in each direction in an $(N_T + N_D)$-dimensional space where the particles are free to move.

$pbestx[i]$ is the vector of co-ordinates (or the point in the space) which gives the personal best encountered by the particle in its history.

$presentx[i]$ is the present position of the particle.

$pbestx[gbest]$ is the position which gives the best fitness value among all the particles in the group. This is also referred to as the group-best.

$r_1$ and $r_2$ are uniformly distributed random numbers between 0 and 1, which account for randomness in the social behavior.

$c_1$ and $c_2$ are parameters that need to be carefully chosen for each application.

In the above equation, the first term represents the inertia. It helps the particles to move out of local minima. This term is also multiplied by a damping coefficient $\alpha$ ($0 \leq \alpha \leq 1$) so that as the search process proceeds, the impact of the inertia diminishes gradually. This factor is also necessary in order to keep the velocities of the particles from diverging. The second term represents attraction towards the personal best while the third term towards the group best.

The velocities or direction of movement is computed for each particle using the above equation. Then, the following equation is used for each particle to update the position.

$$presentx[i] \leftarrow presentx[i] + v[i]$$  \hspace{1cm} (6)

B. Application to Microgrid Architecture

This section explains the application of the PSO technique mentioned before to the problem of microgrid expansion. This problem which is similar to many engineering problems is constrained in nature. Therefore a technique is developed to handle constraints.

Solution Space: The solution vector comprises of two parts: the transmission network, and the deployment of DG. Both the parts are discrete in nature. The solution space is, therefore, a lattice in $(N_T + N_D)$ dimensions.

Each axis in the solution (the vector $\mathbf{u}$) represents the integral number of units allocated to the corresponding expansion option (either right-of-way or candidate location). Though the granularity of the solution space can not be reduced so as to make the problem amenable for computation, a higher resolution can be used when handling the solution. The final solution, however, is reported in integral multiples of a unit-link by rounding the solution coordinate to the nearest integer.

The $(N_T + N_D)$ vector specifies the complete solution vector.

Boundary Conditions: Boundary conditions arise due to the following reasons: (a) Physically, the number of unit links cannot be negative. In order to accommodate this, a fictitious “wall” is assumed along each axis corresponding to the topology part of the solution vector. If the motion of the particles makes them transgress this wall, then the particles bounce back into the positive solution space. (b) Because of
the numbering scheme for the locations where DG can be deployed, the particles cannot go out of the range of allocated indices. Therefore, proceeding as above, walls are constructed at each end point. (c) The minimum reliability stipulations impose significant constraints on the particles. The particles (solutions) can be either in the feasible region or in the infeasible region. The penalty function approach was found to perform better to handle constraints. A penalty function is constructed for each constraint (or reliability guarantee). The construction of the penalty function is described below.

**The Modified Objective Function:** After including the penalty functions, the new cost function now becomes:

\[ J = J_T + J_G + \phi_0(x_2|x_1) \]  

(7)

where,

\[ \phi_0(x_2|x_1) \] is the penalty function for violating the global reliability stipulation. It is the penalty for the infeasible solution vector \( x_2 \) based on the most recent feasible solution \( x_1 \).

If there are additional constraints, then the above equation can be extended by adding more penalty factors. The construction of the penalty factor is described in detail later in this section.

For any given solution, if the solution is feasible, then the value of the penalty function is zero. If the solution is not feasible, then the penalty depends on two factors:

1. How far the last known feasible solution was from the boundary.
2. How far the current infeasible solution is from the boundary.

This is illustrated below.

Let the constraint boundary be represented by

\[ EIR = R_0 \]  

(8)

where \( R_0 \) is the minimum reliability required.

Let the particle move from a feasible solution \( x_1 \) to an infeasible solution \( x_2 \). Because of this movement, the particle has crossed the boundary of the feasible space.

Let the \( EIR \) indices for the solutions \( x_1 \) and \( x_2 \) be \( R_1 \) and \( R_2 \) respectively and the costs \( J_1 \) and \( J_2 \) respectively. Now, the cost of the infeasible solution must be raised by imposing a suitable penalty. This penalty is calculated using the following equation:

\[ \phi(x_2|x_1) = 2 \times \left( \frac{R_0 - R_2}{R_1 - R_2} \right) \times |J_1 - J_2| \]  

(9)

Essentially, this equation penalizes the infeasible solution in ratio of the distances of the feasible and infeasible solutions from the boundary \( R_0 \).

Further, let us assume that the particle moves to another infeasible solution \( x_3 \) in the next step. In this case, the penalty will be calculated in proportion to the last known feasible solution i.e., \( x_1 \). If \( x_3 \) were a feasible solution, then this would serve as the last known feasible solution for the subsequent transitions to infeasible solutions.

The penalty factors for other constraints are calculated in a similar way by choosing the appropriate value of the boundary.

**Number of particles:** The choice of number of particles depends on many factors. If there are very few particles, then the solution space would not be adequately covered. On the other hand, if the number of particles is large, then initializing the particles to the feasible space becomes difficult. For this application, a number of particles equal to the number of nodes in the system was found to perform well.

**PSO parameters:** Values of parameters \( \alpha \), \( c_1 \) and \( c_2 \) were determined after trial and error. The values used are reported in the “Demonstration” section.

**C. Reliability Evaluation**

In a distribution network, a measure of reliability based on the energy supplied is considered more appropriate. Therefore, the energy index of reliability (EIR) is chosen as the index to be specified. The EIR is obtained as follows: first the expected minimum curtailment is obtained by evaluating the system or local, as is the case gives the energy index of unreliability, which subtracted from unity yields the EIR.

As mentioned before, a linearized power flow model has been used to determine the minimum curtailment for any given contingency. This is implemented in the form of a Linear Programming problem [18]. The reliability evaluation is described in detail in [13], [14], [23].

The reliability of the network is given by:

\[ EIR = 1 - \frac{EPNS}{D_T} \]  

(10)

where

\[ EIR = \text{Energy Index of Reliability} \]

\[ EPNS = \text{Expected Power Not Served} \]

\[ D_T = \text{Total Power Demand} \]

**D. Algorithm**

At the beginning of the solution process, the particles are initialized to the feasible space. This is necessary otherwise the penalty for the initial solution cannot be computed meaningfully. After initialization, at each subsequent iteration, the objective values for each particle is computed. Also, the respective EIRs are also computed. Based on feasibility or infeasibility of the current solution, a penalty factor is imposed on the cost of the solution. After this, the group best is identified and personal bests are updated as necessary. Also, the last known feasible solution for each particle is updated as necessary. The velocities and next positions are computed using equations (5) and (6).

At each iteration, the best feasible solution obtained so far is kept track of. The process is continued until a maximum number of iterations has been reached or the coefficient of
variation of the cost of the best feasible solution falls below a given threshold.

VI. DEMONSTRATION

The particle swarm optimization based method is demonstrated on an 8-bus test system.

A. System Description

The test system is a radial standalone system and islanded operation is considered. There are five load buses with a coincident peak electrical load of 6 MW. The system is disconnected from the grid at bus 7 which is the point of common coupling. The test system is shown in Fig. 1.

For the purpose of demonstration the capacity of each unit link as well as a unit DG is taken to be 0.01 MW. The cost of deploying a unit of DG is taken as 50000 $ on an average at the buses. The cost of a unit link between the nodes is taken to be proportional to the length of the line with an average of 60000 $ per unit link per mile.

Five new rights of way are identified as shown in the table 1 where new feeders can be installed. Some existing feeders are also chosen as candidate upgrade options.

B. Results

A reliability target of $EIR \geq 0.96$ is used. The results obtained from applying the optimization method to the test system are shown here.

A pictorial representation of the results can be seen in the Fig. 2. The lines closer to bus 7 have higher capacity than those at the terminal buses as would be expected in a radial system. DG are generally located close to the load centers for maximum benefits. This will cause power to flow in the reverse direction away from terminal buses. Hence the capacity of the lines closer to the terminal nodes is strengthened in the optimization. Though the length of the line connecting bus 3 and 6 is longer which means higher cost, the reliability benefit offered by loop closing dominates the cost in the optimization. Loop closing lines are however less which would be expected as the test system looks radial. Further an engineering decision can be made to see if the results obtained by the method

---

**TABLE I**

<table>
<thead>
<tr>
<th>Start bus</th>
<th>End bus</th>
<th>Length (Miles)</th>
<th>Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>0.75</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>2.3184</td>
<td>2.2</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>1.75</td>
<td>1.85</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>1.25</td>
<td>0.9</td>
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<tr>
<td>5</td>
<td>6</td>
<td>1.25</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2.16</td>
<td>New ROW</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2.64</td>
<td>New ROW</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1.25</td>
<td>New ROW</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1.25</td>
<td>New ROW</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.56</td>
<td>New ROW</td>
</tr>
</tbody>
</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Bus</th>
<th>Electrical load (MW)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
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<tr>
<td>4</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>1.37</td>
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</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Bus</th>
<th>DG capacity (MW)</th>
<th>Bus</th>
<th>DG capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.2</td>
<td>4</td>
<td>1.8</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>5</td>
<td>2.0</td>
</tr>
</tbody>
</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>Start Bus</th>
<th>End bus</th>
<th>New line capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>0.54</td>
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<tr>
<td>2</td>
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<td>2</td>
<td>0.26</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.16</td>
</tr>
</tbody>
</table>

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Fig. 1. Eight-bus test system

Fig. 2. Microgrid with optimal deployment
can be made more simple. For example, if the optimization determines only one unit link along many new rights of way, it could be decided if such a solution is practically viable and appropriate action can be taken.

VII. CONCLUSION

This paper presented a rational approach to designing cost optimized microgrid architectures that are subject to system wide reliability stipulations.

This paper presented a formulation of the problem of developing optimal microgrid architectures that meet reliability stipulations and a solution technique based on particle swarm optimization. The method simultaneously optimizes the network configuration and the location of DG.

The method was demonstrated by applying it to a test system. Useful insights were gained from the experience. Further work will consist of developing improved and expanded formulations of the problem, and of determining suitable solution techniques. Expanded formulations to be explored will include optimal mix of different DER candidate technologies.

It is expected that this method will provide a more rational approach to enabling the evolution of a distribution network than ad hoc expansion in response to load growth. The method presented is likely to be suitable for dense urban areas. While this method is equally applicable in rural areas or otherwise sparsely connected distribution systems, it is likely that similar results may be obtained by simpler methods. Further work on this project is in progress, and will be reported in due course.

REFERENCES


VIII. BIOGRAPHIES

Mallikarjuna R. Vallem received his B.Tech. (Hons.) from Indian Institute of Technology, Kharagpur. He is currently pursuing a doctoral degree in EE at New Mexico State University. His research interests are in the area of power system reliability, Distributed resources and deployment optimization. (e-mail: mallikv@nmsu.edu)

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