

# Modeling and Simulation of Permanent Magnet Brushless Motor Drives using Simulink

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**Abstract:** Permanent Magnet Brushless Motors (PMBL) are available in two types namely Permanent Magnet Synchronous Motor (PMSM) with a sinusoidal flux distribution, and Permanent Magnet Brushless DC motor (PMBLDC) with trapezoidal flux distribution. This paper presents simulink-based modeling of the complete drive system consisting of the current controlled voltage source inverter (CC-VSI), PI speed controller, motor and the load. The simulation results are given both for transient and steady state operating conditions of PMSM and PMBLDC drive systems.

## I. Introduction

The absence of the field excitation, brushes and the commutator in Permanent Magnet Brushless Motors (PMBL) offers many advantages over other motors, such as high efficiency, small size and quick response time [1-4]. The controller of the PMBL motors consists of a three-phase inverter, position, speed and current sensors and control (which processes the various signals to produce the switching signals for the devices of the inverter). This paper presents the modeling of the complete drive systems with PMSM and PMBLDC motors. The modeling equations are represented by simulink blocks and the simulation results are given both for transient and steady state operating conditions.

## II. PM Brushless Motor Drive System

Figs.1 and 2 show the drive systems with PMSM and PMBLDC motors respectively. It consists of a three-phase Current Controlled Voltage Source Inverter (CC-VSI), the motor and the controller. The inverter, which is connected to the dc supply, feeds controlled power to the motor. The magnitude and frequency of the inverter output voltage depends on the six switching signals, which are generated by the controller. The state of these switching signals at any instant is determined by the rotor position, speed error and winding currents. The controller synchronizes the winding currents with the rotor position. It also facilitates the variable speed operation of the drive, and maintains the motor speed at the reference value even during load and supply fluctuations. From these figures it is obvious that the drive systems for PMSM and PMBLDC motors are almost identical expect for some difference in the controller. Hence, the modeling of these two drive systems will have many things in common, such as inverter and parts of the controller. Therefore, the modeling of complete drive system comprising of a PMSM motor is considered first and for PMBLDC motor drive the modeling of only those parts is given which are different from PMSM drive system.

## III. Modeling of PMBL Motors Drive System

### (a) PMSM Drive System

Fig.1 shows the block diagram of the PMSM drive system. Each component of the drive system is represented by a set of equations. These equations are then represented by suitable simulink blocks to form the complete model of the drive system as shown in Fig. 3. The various parts of the drive system are:

**Speed Controller:** The reference speed ( $\omega_r^*$ ) and rotor speed ( $\omega_r$ ) are the inputs to the PI controller, which together generates the reference value of the torque-producing component of stator current ( $i_q^*$ ). The output of PI controller at any instant ( $k$ ) is

$$i_q^*(k) = k_p \{\omega_e(k) - \omega_e(k-1)\} + k_i \{\omega_e(k)\} \quad (1)$$

Where

$i_q^*$  is the reference q axis current.

$k_p$  and  $k_i$  are the proportional and integral gains of the PI controller

The speed error is:

$$\omega_e(k) = \omega_r^*(k) - \omega_r(k)$$

**dq to abc axis current transformation:** The reference values of the d and q axis currents  $i_d^*$  (which is set to zero in this study as the drive is not operated in the field weakening region) and  $i_q^*$  are converted to three phase reference currents ( $i_a^*$ ,  $i_b^*$ ,  $i_c^*$ ) which are used by the inverter to determine its output. This transformation uses the inverse Park transformation defined below:

$$\begin{bmatrix} i_a^* \\ i_b^* \\ i_c^* \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} i_q^* \\ i_d^* \end{bmatrix} \quad (2)$$

Where  $\theta$  is the rotor position.

**The Inverter:** This block models the IGBT based three-phase Voltage Source Inverter (VSI). Three phase stator currents are synchronized with the rotor position by providing proper gating signals to the devices of the inverter. The reference value of a phase current (determined by the previous block) and winding current of that phase are used to find the voltage of that phase as shown below:

$$\text{if } (i_a^* + Hb) \geq i_a \quad \text{then } S_a = 1$$

$$\text{else } S_a = 0 \quad (3)$$

$$\text{if } (i_b^* + Hb) \geq i_b \quad \text{then } S_b = 1 \quad \text{else}$$

$$S_b = 0 \quad (4)$$

$$\text{if } (i_c^* + Hb) \geq i_c \quad \text{then } S_c = 1 \quad \text{else}$$

$$S_c = 0 \quad (5)$$

$$v_a = \frac{v_{dc}}{3} (2S_a - S_b - S_c) \quad (6)$$

$$v_b = \frac{v_{dc}}{3} (2S_b - S_a - S_c) \quad (7)$$

$$v_c = \frac{v_{dc}}{3} (2S_c - S_b - S_a) \quad (8)$$

Where,  $H_b$  is the hysteresis band.  $S_a, S_b, S_c$  are switching functions (which are either 1 or zero).  $v_a, v_b, v_c$  and  $v_{dc}$  are voltages of phase winding a, b, c and dc link, respectively.

**abc to dq axis voltage transformation:** Three phase voltages ( $v_a, v_b, v_c$ ) of the inverter are converted to dq axis voltages ( $v_d, v_q$ ) by using Park transformation shown below:

$$\begin{bmatrix} v_q \\ v_d \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (9)$$

**The PMSM:** The stator of the PMSM comprises of balanced three phase winding similar to the wound rotor synchronous motor. The mathematical model of PMSM is derived from that of a wound rotor machine under the assumption that the armature emf is induced by the permanent magnets in place of dc excitation. Assuming that the induced emf is sinusoidal and eddy current and hysteresis losses are negligible, the stator voltage equations in the rotor reference frame are [5]:

$$v_q = R i_q + p \lambda_q + \omega_r \lambda_d \quad (10)$$

$$v_d = R i_d + p \lambda_d - \omega_r \lambda_q \quad (11)$$

Where

$$\lambda_q = L_q i_q \quad (12)$$

$$\lambda_d = L_d i_d + \lambda_{af} \quad (13)$$

Where,  $v_q$  and  $v_d$  are the d,q axis stator voltages.  $i_q$  and  $i_d$  are the d,q axis stator currents.  $L_q$  and  $L_d$  are the d,q axis inductances.  $\lambda_{af}$  is stator flux linkages due to permanent magnets.  $R$  is stator-winding resistance per phase.  $\omega_r$  is rotor speed in rad/sec (electrical).  $p$  is the d/dt operator.

The electromagnetic torque produced is [5]:

$$T_e = (3/2) P \{ \lambda_{af} i_q + (L_d - L_q) i_d i_q \} \quad (14)$$

Where,  $P$  is the number of pole pairs.

The electromagnetic torque is balanced by the load torque, accelerating torque and damping torque of the system and can be represented as:

$$T_e = T_L + B \omega_r + J p \omega_r \quad (15)$$

Where,  $T_L$  is the load torque,  $B$  is the damping coefficient and  $J$  is the moment of inertia.

For the purpose of dynamic simulation the equations (10-15) can be rearranged to give first order nonlinear differential equations in terms of variables as:

$$p i_d = (v_d - R i_d + \omega_s L_q i_q) / L_d \quad (16)$$

$$p i_q = (v_q - R i_q - \omega_s L_d i_d - \omega_s \lambda_{af}) / L_q \quad (17)$$

$$p \omega_r = (T_e - T_L - B \omega_r) / J \quad (18)$$

$$p \theta = \omega_r \quad (19)$$

The simulink-based model of the PMSM motor drive system is developed on the basis of these equations and is shown in Fig 3.

### (b) The Modeling of the PMBLDC Motor Drive System

It can be seen from the block diagram of PMBLDC motor drive system shown in Fig.2 that in this case the PI controller and the inverter blocks are the same as in the PMSM drive system and hence these blocks are not discussed again and only the modeling of PMBLDC motor is discussed. The stator of PMBLDC motor is similar to PMSM and the rotor has permanent magnets. The only difference between them is due to the shape of the air gap flux. The air gap flux distribution is sinusoidal in PMSM and trapezoidal in PMBLDC motor. Hence the d q variable approach cannot be directly applied to model the PMBLDC motor. Therefore, the PMBLDC motor is modeled using the 3-phase abc variables. The volt-ampere equation for each phase of PMBLDC motor is expressed as follows [6]:

$$v_{an} = R i_a + p \lambda_a + e_{an} \quad (20)$$

$$v_{bn} = R i_b + p \lambda_b + e_{bn} \quad (21)$$

$$v_{cn} = R i_c + p \lambda_c + e_{cn} \quad (22)$$

Where  $v_{an}, v_{bn}, v_{cn}$  are the phase voltages.  $e_{an}, e_{bn}, e_{cn}$  are the phase back emfs. The flux linkages of each phase ( $\lambda_a, \lambda_b, \lambda_c$ ) can be expressed as:

$$\lambda_a = L_s i_a - M (i_b + i_c);$$

$$\lambda_b = L_s i_b - M (i_a + i_c);$$

$$\lambda_c = L_s i_c - M (i_a + i_b) \quad (23)$$

Where  $L_s$  and  $M$  are self and mutual inductances, respectively.

The PMBLDC motor has an isolated neutral which means that the sum of three currents ( $i_a, i_b$  and  $i_c$ ) is zero. The flux linkages can now be written as:

$$\lambda_a = (L_s + M) i_a;$$

$$\lambda_b = (L_s + M) i_b;$$

$$\lambda_c = (L_s + M) i_c \quad (24)$$

The voltage of each phase can be expressed in terms of the potential of that phase w.r.t the mid point of dc link ( $v_{ao}, v_{bo}, v_{co}$ ) and the potential of neutral w.r.t the mid point of dc link ( $v_{no}$ ). Hence we get the expressions for the phase voltages as:

$$v_{an} = v_{ao} - v_{an};$$

$$v_{bn} = v_{bo} - v_{an};$$

$$v_{cn} = v_{co} - v_{an} \quad (25)$$

The voltages ( $v_{ao}, v_{bo}, v_{co}$ ) assume one of these three values

$$\left( + \frac{v_{dc}}{2}, - \frac{v_{dc}}{2}, 0 \right)$$

depending on whether the upper switch, lower switch or no switch is turned on.

The potential of neutral point can be expressed as:

$$v_{no} = (v_{ao} + v_{bo} + v_{co} + (e_{an} + e_{bn} + e_{cn})/3) / 3 \quad (26)$$

The back emfs depend on the rotor position  $\theta$ , and can be expressed as:

$$e_a = K_b \omega_r f_a(\theta);$$

$$e_b = K_b \omega_r f_b(\theta);$$

$$e_c = K_b \omega_r f_c(\theta) \quad (27)$$

Where  $K_b$  is the back emf constant and  $f_a(\theta), f_b(\theta), f_c(\theta)$  are functions of rotor position ( $\theta$ ) and  $f_a(\theta)$  is defined as:

$$f_a(\theta) = 1 \quad 0^\circ < \theta \leq 120^\circ \quad (28)$$

$$f_a(\theta) = \frac{6}{\pi} (\pi - \theta) - 1 \quad 120^\circ < \theta \leq 180^\circ \quad (29)$$

$$f_a(\theta) = -1 \quad 180^\circ < \theta \leq 300^\circ \quad (30)$$

$$f_a(\theta) = \frac{6}{\pi} (\theta - 2\pi) + 1 \quad 300^\circ < \theta \leq 360^\circ \quad (31)$$

The functions  $f_b(\theta)$  and  $f_c(\theta)$  will be similar to  $f_a(\theta)$  with a phase difference of  $120^\circ$  and  $240^\circ$  respectively. As shown in Fig.4 these back emf's are represented by using a S-function

block (user defined function), because the available library blocks of simulink could not model these back emf functions.

The electromagnetic torque produced is [6]:  

$$T_e = (e_{an} i_a + e_{bn} i_b + e_{cn} i_c) / \omega_r \tag{32}$$

Which is simplified to:  

$$T_e = K_b \{ f_a(\theta) i_a + f_b(\theta) i_b + f_c(\theta) i_c \} \tag{33}$$

Which is balanced by load torque ( $T_L$ ), friction torque ( $B\omega_r$ ) and accelerating torque ( $Jp\omega_r$ ) as:

$$T_e = T_L + B\omega_r + Jp\omega_r \tag{34}$$

For the purpose of dynamic simulation equations (20-34) can be rearranged to give first order nonlinear differential equations in terms of variables as:

$$p i_a = (v_{an} - e_{an} - i_a R) / (L_s + M) \tag{35}$$

$$p i_b = (v_{bn} - e_{bn} - i_b R) / (L_s + M) \tag{36}$$

$$p i_c = (v_{cn} - e_{cn} - i_c R) / (L_s + M) \tag{37}$$

$$p \omega_r = (T_e - T_L - B \omega_r) / J \tag{38} \quad p\theta = \omega_r \tag{39}$$

The simulink-based model of the PMBLDC motor drive system is developed on the basis of these equations and is shown in Fig 4.

**IV. Results and Discussion**

Fig. 5 shows the simulation results of PMSM motor (parameters given in Appendix) drive system. The motor speed and current waveforms are given for various operating conditions such as starting, reversal, and load perturbation. During transient conditions (starting and reversal) the motor draws high current to attain the reference speed. The rotor speed remains constant with the sudden application and removal of load, showing the closed loop operation of the drive system. The simulated response of the PMBLDC motor (parameters given in Appendix) drive system during starting, load perturbation and speed change is shown in Fig.6. Similar to the response of PMSM motor, the PMBLDC motor also draws high current during starting and reversal. The four-quadrant operation of the drive systems is demonstrated through these simulated waveforms.

**V. Conclusion**

Modeling and simulation of the PMBL motors have been carried out in MATLAB through Simulink

Toolbox. Each component of the drive systems has been modeled and represented by a separate subsystem. These subsystems have been constructed using simulink library blocks. The simulated response of both PMSM and PMBLDC motor drive systems for transient and steady state conditions have confirmed the effectiveness of simulink based models for these PMBL motors.

**VI. References**

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- [6] P.Pillay and R. Krishnan, "Modeling, Simulation, and Analysis of Permanent Magnets Motor Drives, Part II: The Brushless DC Motor Drive", IEEE Trans. on Industry Applications, Vol. 25, No. 2, March/April 1989, pp. 274-279.

**VII. Acknowledgements**

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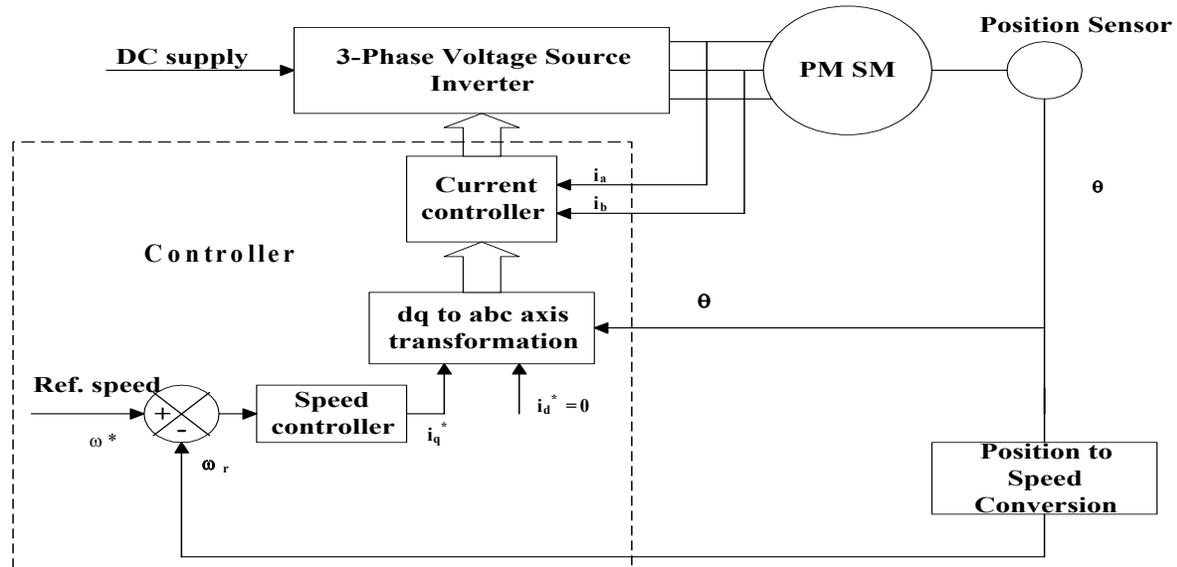
**VIII. Appendix**

**Permanent Magnet Synchronous Motor (PMSM)**

- Voltage: 340 Volts
- Power: 1.93 kW
- Current: 8.64 A
- Phase: 3
- Speed: 3327 rpm.
- Pole: 4

**Permanent Magnet Brushless DC Motor (PMBLDC)**

- Voltage: 400 Volts
- Power: 1.5 kW
- Current: 4 A
- Phase: 3
- Speed: 1500 rpm
- Pole: 4



**Fig.1 Block diagram of Closed Loop Speed Control Scheme of PMSM Motor Drive System**

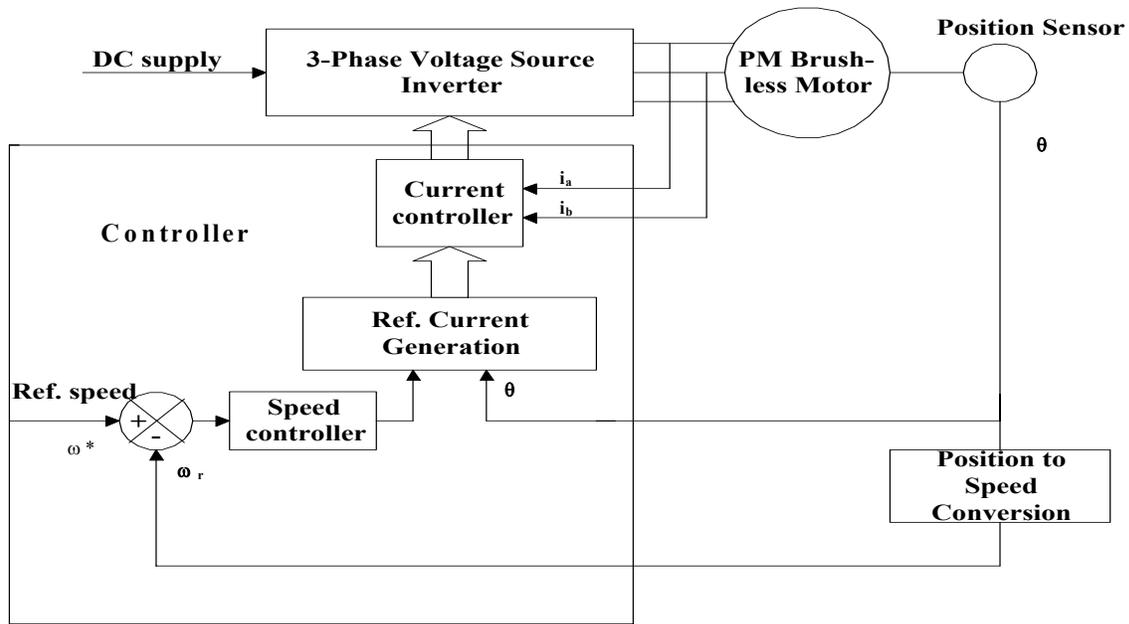


Fig.2 Block diagram of Closed Loop Speed Control Scheme of PMBLDC Motor Drive System

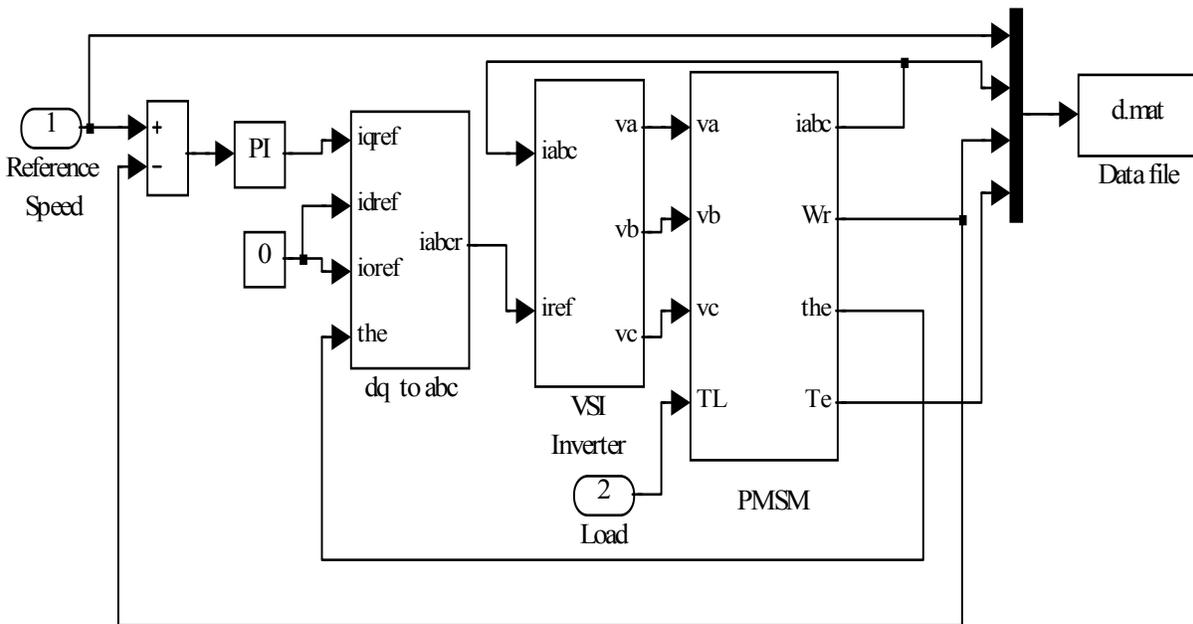
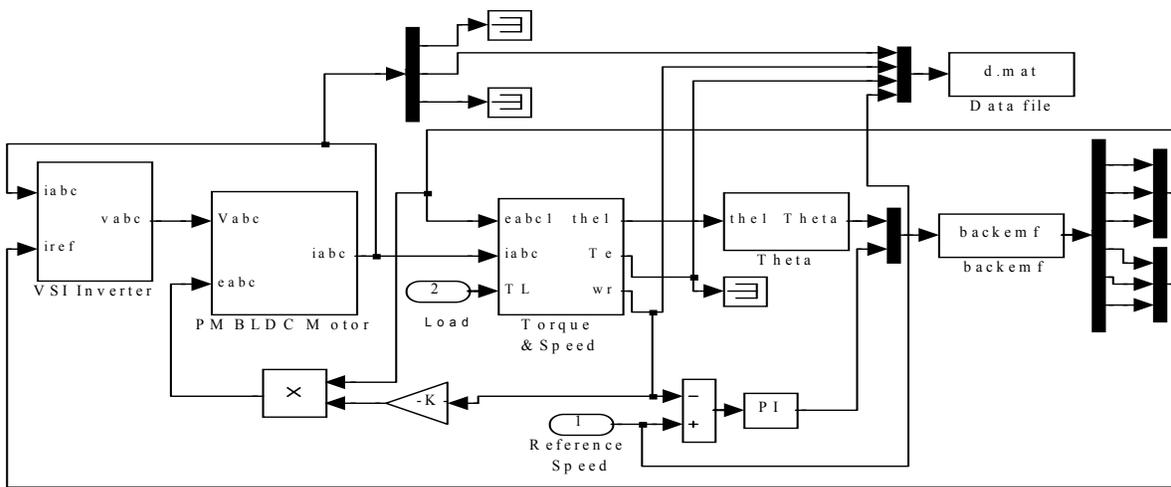
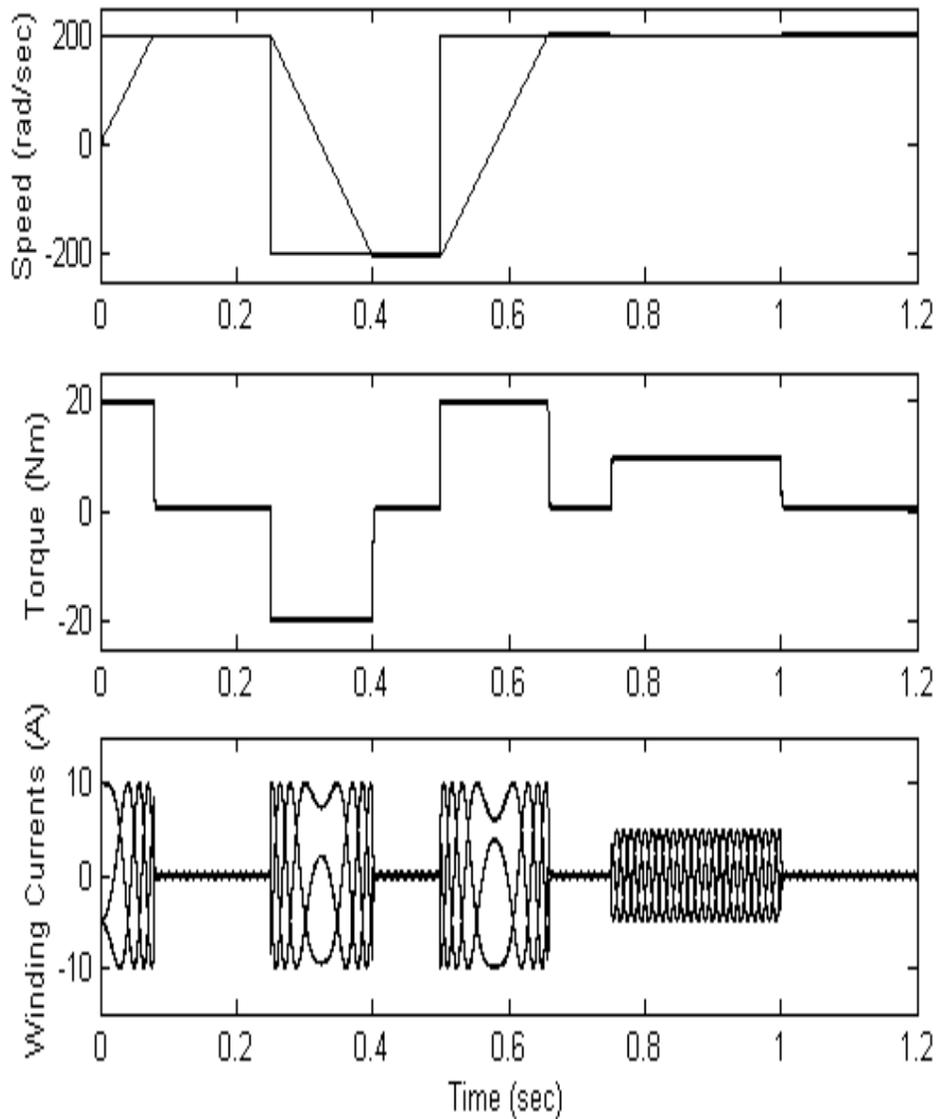


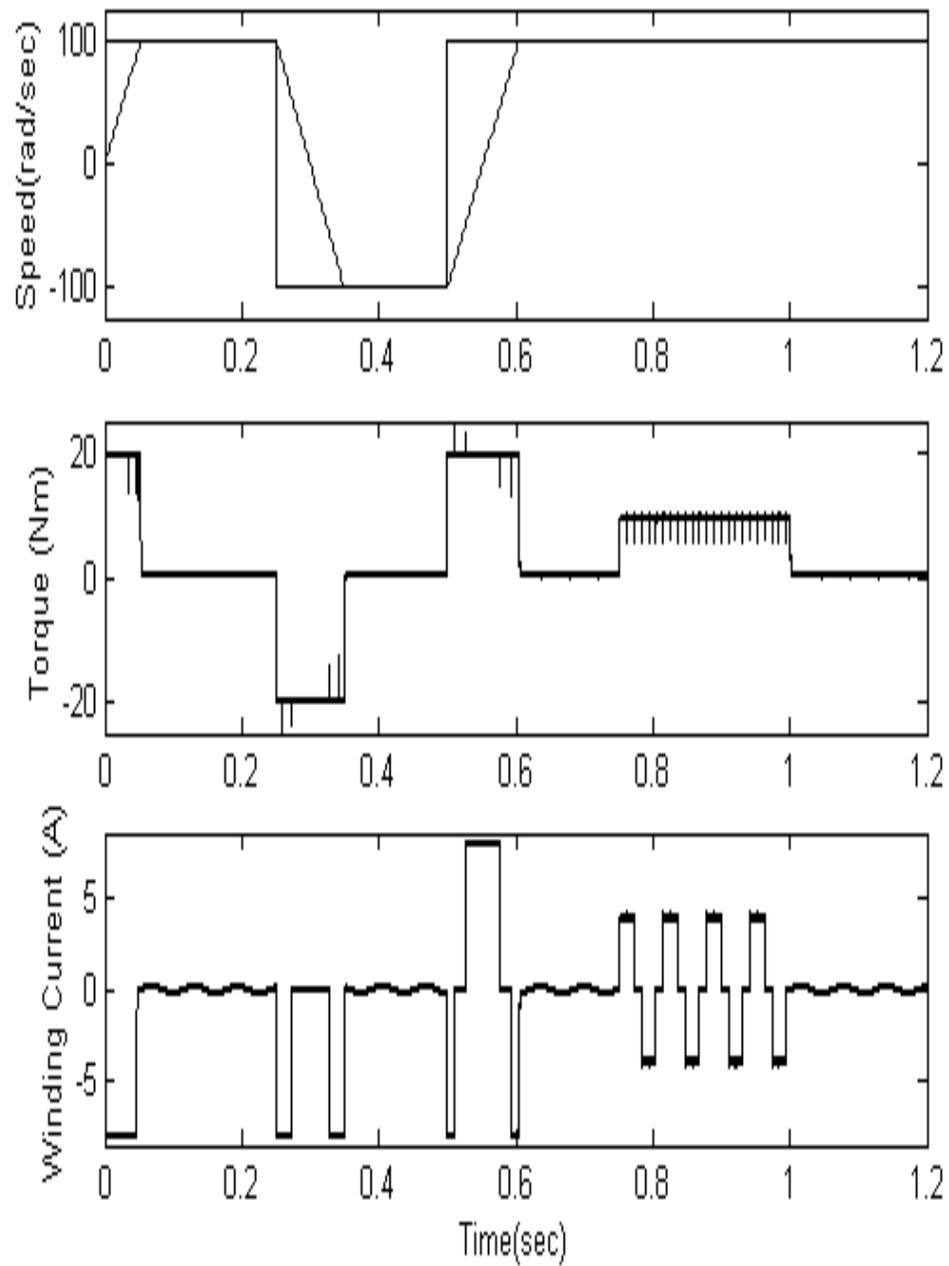
Fig.3 Simulink Based Model of PMSM Motor Drive System



**Fig.4 Simulink Based Model of PMSM Motor Drive System**



**Fig.5 Simulated Response of PMSM Drive System for Starting, Reversal and Load Perturbation.**



**Fig.6 Simulated Response of PMBLDC Drive System for Starting, Reversal and Load Perturbation.**