A New Approach to Probabilistic Load Flow

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Abstract: This paper describes a new approach to modelling of transmission line uncertainties using complex random variables. For example, if there is a system having n generating units, l transmission lines and m number of load distributions, all possible contingencies can be represented by as many as \(2^n \times 2^l \times 2^m\) states. The method proposed here considers all the \(2^n \times 2^l \times 2^m\) contingencies in an elegant manner using the first four moments based on complex random variables. Generator, transmission line outage events and load variation modelling is described using the complex random variable approach. The CLF algorithm would have to be run \(2^n \times 2^l \times 2^m\) times involving prohibitive amount of calculations with added difficulty of analyzing and synthesizing the solution. The proposed PLF can tackle all the different contingencies in a single load flow. Solution of the stochastic load flow equations is carried out in the moment domain of a composite power system. The input variables such as the total power generated by all the generating units and the system demand are considered complex random variables. Solutions to stochastic voltage and power equations have been found considering all possible contingencies of generation and transmission line outages, load demand variation using the moments of the nodal admittance matrix itself. The algorithm yields the values of the important characteristics of the output variables (bus voltages, line currents and generated power) in terms of the input variables (generating capacities of the different generating units at each bus, load capacities of different load buses and transmission line availabilities).

Key words: probability, capacity, loss of load probability, probability density function, probability mass function, complex random variables, moments, cumulants, slack bus, P-V bus, P-Q bus, load flow solution, bulk power transmission and system planning.

A. INTRODUCTION

The load flow analysis is undoubtedly the most useful study made for designing and operating new power systems as also for planning future extensions to meet increased load demands and define operating practices. The algorithm assesses, the steady state behavior and response of the system being studied for which, adequate data defining the operating conditions have to be provided. The most common techniques used in the past for determining the generating capacity to satisfy the system demand and to have sufficient capacity to perform corrective and preventive maintenance n generation facilities were deterministic in nature. Deterministic methods do not consider the system composition (number, size and forced outage rates) or the load demand characteristics. The use of fixed deterministic criterion in planning capacity reserve, results in a variable risk level over the planning horizon whereas, probabilistic techniques take into account the random nature of both demand and supply of power. This ensures that the actual factors that influence the system reliability are taken into account.

The conventional load flow (CLF) analysis is deterministic in nature and has several drawbacks, some of which may be stated as,

a) Power demands are modeled by single values, usually representing means of certain limits and gives information about a single operating point.

b) Unavailability of generating and transmission equipment is neglected.

c) Calculations involve solution of a given power system network under conditions according as it is operating under balanced and unbalanced conditions. Therefore, numerous load flows under both normal and abnormal (outages of generating units, transmission lines and uncertainties of load) operating conditions will be needed.

As opposed to the deterministic methods, the probabilistic techniques take into account the forced outages, maintenance requirements, variability of hydro inflows etc. Other factors such as uncertainty in load forecasts and statistical variations in hydro unit energy availabilities can also be considered. Also, probabilistic load flow (PLF) determines how the variability range for the inputs affects the range of variations of the output quantities.

In practice, from the probability distribution of the inputs, the PLF provides information on the corresponding distributions for the output. The probability of inadequate transmission capacity in each configuration can be found after performing a load flow study on each, using the appropriate load model. Thus the CLF algorithm would have to be run \(2^n \times 2^l \times 2^m\) times involving prohibitive
amount of calculations with added difficulty of analyzing and synthesizing the solution. These points can be very satisfactorily tackled by applying the probabilistic load flow algorithm.

Another noteworthy point is that the very nature of input variables (generating and transmission equipments and load demands) is random and not deterministic and hence it is imperative that probabilistic techniques be applied in Load Flow Analysis to evaluate the effects of input uncertainties on the steady-state behavior of a power system. A probabilistic load flow algorithm essentially transforms, the input random variables defined in terms of probability density functions (pdf) to terms of output random variables similarly defined.

B. PROPOSED METHOD

In any power system, loads are complex quantities as along with active components (MW) there are reactive components (MVAR) as well, the nature of which depends on the type of load connected and the transmission system feeding it. Thus, even if MW demand remains fixed the MVAR demand of the system may change due to changes in power system (of the loads connected and the transmission system supplying power to the system) thereby raising or lowering the MV A demand. Formulation of the problem using complex random variables mitigates the complexity of considering real and imaginary components separately in order to obtain the true picture.

C. The concept of CRY

A CRY can be expressed in the form Z=X+ jY where both X and Y are real random variables. If Z has a probability mass function $f(Z)$, then it can also be defined in the same probability space with a joint probability mass function, $pmf f(x, y)$. If $Z$ takes up three different values say, $Z_1 = x_1+jy_1$, $Z_2 = x_2+jy_2$ and $Z_3 = x_3+jy_3$ with probabilities of occurrence $p_1$, $p_2$ and $p_3$ respectively. Then $p_1 = f(X=x_1, Y=y_1)$ for $Z_1 = x_1+jy_1$. It may be noted that the real random variables $X$ and $Y$ occur as one event i.e., $X$ always occurs along with $Y$ and obviously $p_1+p_2+p_3 =1$. This can be shown as in Fig 1.

![Fig 1 Discrete probability distribution of CRY Z](image)

Fig 1 Discrete probability distribution of CRY Z

Addition and subtraction of these pdf’s can be carried out using the basic probability concepts to add successive units sequentially to produce the final probability distribution function.

Generator modelling:

Load changes and switching on/off of the generators (due to failure or maintenance) are responsible for variations in the generated powers. The generation plants of a system are modelled using general discrete distribution and the binomial distribution. Fig 2 represents a typical discrete distribution of a generating plant.

![Fig 2 Typical Discrete Distribution of a generator plant](image)

Fig 2 Typical Discrete Distribution of a generator plant

A distribution with a single impulse, with probability 1, the model for the deterministic problem, is a particular case of this distribution. The moment’s are calculated using the expression as below,

$$m_r = \sum_{i=1}^{n} C_i^r p_i$$

where, $m_r$ = moment of order r,

$C_i$ = generating capacity of a unit (in MV)

$A$: $C_i = P_i + jQ_i$

and $p_i$ = probability of having $C_i$ (MV A)

The binomial distribution has been proposed to model generation plants with several identical units. The probability distribution function of the CRV can be found by considering the following outcome of an observation: unit k is working (probability $p$) or is not working (probability $1-p$). If there are n generating units at a generating point then probability that $R = r$ of them are in working state (in service) is given by,

$$p(R=r) = \binom{n}{r} p^r (1-p)^{n-r} \quad \ldots (2)$$

where, $p$ = probability of having a unit in service (equal for every unit)

and $1-p = q$ = probability of having a unit not working.

The rib moment satisfies the recursive relation,
Load Modelling:

where, the coefficients $a_i$ are the coefficients of the expansion $n(n-1)(n-2)(n-3)...$

Distribution, beta distribution, the $r$, the truncated normal, to model any part of the system, such as the binomial completely general and any other distribution could be used etc. In a typical model of a power system, there might be

Load Modelling:

If $x$ and $y$ are the active and reactive components of load powers (at any point) such that, $Z_i = S_{Di} = x_i + jy_i$ then, the moments of load are calculated by,

$$m_r(S_{Di}) = \sum_{i=1}^{n} S_{Di}^r p_i \quad \text{.... (4)}$$

where, $m_r$ = moment of order $r$, $S_{Di}$ = load at a particular bus (in this case it is in MV A: $S_{Di} = P_{Di} + j Q_{Di}$) and $p_i$ = probability of having $S_{Di}$ (MVA)

Transmission Line Modelling:

Transmission lines are represented by their series admittances (taking into account charging admittances also). The admittance of the line is assumed to be a complex admittance (taking into account charging admittances also).

$$Y_{L} = \begin{cases} \text{1 with probability } p_1 \\ \text{0 with probability (1- } p_1) \end{cases}$$

If there are $m$ tie lines then the complex random variable $Y_{L}$ will have binomial distribution with parameters $p_1$. Thus,

$$p(Y_{L} = r) = \binom{m}{r} C_r \left(p_1\right)^r \left(1-p_1\right)^{m-r} \quad \text{.... (5)}$$

Here too moments of each transmission line is calculated by,

$$m_r(Y_{L}) = \sum_{i=1}^{n} Y_{Li}^r p_i \quad \text{.... (6)}$$

where, $m_r$ = moment of order $r$, $Y_{Li}$ = series admittance of line (taking into account charging capacitances and transformer tapping) And $p_i$ = probability of having $Y_{Li}$

Although the distributions presented here are the most commonly used ones, the method of cumulants is completely general and any other distribution could be used to model any part of the system, such as the binomial distribution, beta distribution, the $r$, the truncated normal, etc. In a typical model of a power system, there might be different combinations of distribution functions i.e., generation can be binomial and loads may be represented by normal and discrete distributions.

The knowledge of moments, when they all exist, is for all practical purposes equivalent to knowledge of the distribution function. The implication of using moments and cumulants in power system analysis is that the first four moments give an idea of the mean, standard deviation, skewness and kurtosis, which contains the complete history of any load or generator distribution. Further, we can develop the probability density function with the knowledge of moments.

The $t^{th}$ moment $m_t$ is given by,

$$m_t = E[Z^t] = \sum_{i=1}^{n} Z_i^t p_i ; \quad t=1,2,3... \quad \text{.... (7)}$$

where $Z_i$ is a CRV and $p_i$ its probability mass.

The cumulants are linear combinations of the statistical moments and about an arbitrary point. The relationship between moments and cumulants [19] is given as,

$$m_1 = k_1; \quad m_2 = k_2 + k_1^2; \quad m_3 = k_3 + 3k_2k_1 + k_1^3; \quad \ldots \quad (8) \quad m_4 = k_4 + 4k_3k_1 + 6k_2k_1^2 + 3k_2^2 + k_1^4;$$

Conversely,

$$k_1 = m_1; \quad k_2 = m_2 - m_1^2; \quad k_3 = m_3 - 3m_2m_1 + 2m_1^3; \quad \ldots \quad (9) \quad k_4 = m_4 - 4m_3m_1 + 12m_2m_1^2 - 3m_2^2 - 6k_1^4;$$

In particular, about the mean,

$$k_1 = 0; \quad k_2 = m_2; \quad k_3 = m_3; \quad \ldots \quad (10) \quad k_4 = m_4 - 3m_2^2;$$

Two important properties of moments and cumulants are used for the addition and product of independent CRV's. If $Z_i$ and $Z_2$ were two independent CRV's, then the moment of the product of two CRV's is equal to the product of the moment of each.

$$m_r(Z_1Z_2) = m_r(Z_1)m_r(Z_2) \quad \text{.... (11)}$$

Also, sum of independent random variables are characterized by cumulants, which are the sum the individual CRV cumulants.
In general,
\[ m_i \left( \prod_{i=1}^{n} Z_i \right) = \prod_{i=1}^{n} m_i(Z_i) \quad \text{for } t=1,2,3, \ldots \quad (13) \]
and,
\[ k_i \left( \sum_{i=1}^{n} Z_i \right) = \sum_{i=1}^{n} k_i(Z_i) ; t=1,2,3, \ldots \quad (14) \]

**Slack Bus Representation:**

The probability distribution function of slack bus voltage is as shown in Fig 3. The moments of slack bus are given by,
\[ m_i(V_s) = (V_s)^t ; t=1,2,3, \ldots \quad (15) \]
Its cumulants are given by,
\[ k_i(V_s) = 0, \text{ for } t \geq 2. \quad (16) \]

**Load (P-Q) bus representation:**

It is required to specify only \( P_D \) and \( Q_D \) at such a bus as at a load bus, voltage can be allowed to vary within the permissible limits of 5%. As load model considered here is of the discrete distribution type hence, the pdf of total load \( S_D = P_D + jQ_D \) connected to a bus, is obtained by first calculating the moments using equation (4) and then the respective cumulants using the set of equations in (9) and adding them all using equation (12).

Similarly, the pdf of total available capacity at a bus \( S_G = P_G + jQ_G \) may be obtained by convolution of the individual pdf's of the units connected to the bus in question, using the cumulant method.

The cumulants of injected power \( S_i = P_i + jQ_i \) at a bus is obtained from the knowledge of cumulants of \( S_i \) and \( S_G \) by convolving the pdf of generation with the pdf of load demand (regarded as negative generator). Since these CRV's are assumed to be independent, their cumulants add as stated in equation (12). Therefore,
\[ k_i(S_i) = k_i(S_G) - k_i(S_D) \quad (17) \]

The moments and cumulants of the bus voltages for all possible states of transmission lines are obtained from the solution of the basic stochastic nodal matrix equations as detailed in the next section.

**Voltage Controlled (P-V) bus representation:**

At the \( k^{th} \) bus, the moments of \( Q_k \) are obtained from the knowledge of the moments of voltage and admittances of the nodal admittance matrix taking into account both presence of shunt admittances and tapped transformers.

**Solution of Stochastic Nodal Admittance Matrix equations:**

In terms of complex random variables, the network power flow equations, which are basically deterministic in nature, will be referred to as the stochastic power flow equations.

The first step in the solution of the probabilistic load flow problem is to find the net injected power at each bus of the system except the reference or slack bus. At any node there can be different kinds of distributions either discrete or continuous for very large systems. It is found that if a system is very large the discrete distribution of system capacity outages can be approximated by a continuous distribution. Such a distribution approaches the normal distribution as the system size increases. In the proposed method, independence between the input random variable loads and generators is assumed, although it is possible to consider some degree of linear dependence between individual random variables or groups of them. For a slack bus, voltage and phase angle remains constant.

For a P-Q bus, the moments of bus voltages are calculated iteratively such that the \((u+1)^{th}\) iteration gives the moment of voltage at bus \( i \) as,
\[ m_i(V_i^{(u+1)}) = m_i(V_i^{(u)}) + \left[ \frac{P_i^{(u)} - Q_i^{(u)}}{(V_i^{(u)})^2} + k_{i+1} \sum_{l=1}^{u} Y_{kl} V_i^{(u)} - k_u \sum_{l=k+1}^{u} Y_{kl} V_i^{(u)} \right] \]
\[ m_i(V_i^{(u)}) \quad (18) \]

The iterative process is continued till the change in magnitude of bus voltage, \(|\Delta V_i^{(u+1)}| \) between two consecutive iterations is less than a certain tolerance for all bus voltages.
The revised values of $m_i(Q_{(u+1)}^{(u+1)})$ are found from the equation given below:

$$m_i(Q_{(u+1)}^{(u+1)}) = \angle m_i(V_{i(u+1)}^*) = \angle \left( \frac{P_i - jQ_i}{(V_i^*)^*} - \sum_{i=1}^{i=n} Y_{ik} V_i^{(u+1)} - \sum_{i=n+1}^{i=m} Y_{ik} V_i^{(u+1)} \right)$$

$$m_i(Y_{ii})$$

For a P-V bus, the upper limit, $Q_i^{(max)}$, and lower limit, $Q_i^{(min)}$, of $Q_i$ to hold the generation VAR within limits are also usually given i.e., $Q_i^{(max)} < Q_i^{(min)}$. Therefore, first the calculated value of $m_i(Q_{(u+1)}^{(u+1)})$ from equation (3.7) is checked if it is within the given limits i.e., $m_i(Q_{(max)}^{(u+1)}) < Q_i^{(max)}$ and $m_i(Q_{(min)}^{(u+1)}) > Q_i^{(min)}$. If the value of $m_i(Q_{(u+1)}^{(u+1)})$ is within the stated bounds then the new value of $m_i(Q_{(u+1)}^{(u+1)})$ is first calculated using equation (3.9) substituting the values of $m_i[V_{sp}]$ and calculated value of $m_i(Q_{(u+1)})$ in the equation. From this the new value of $V_i$ is calculated using previously specified value ($[V_{sp}]$) and calculated value of $m_i(d_i^{(u+1)})$. Now retaining this new value the next bus is taken up.

If $m_i(Q_{(u+1)}^{(u+1)}) < Q_i^{(min)}$, then substitute $m_i(Q_{(u+1)}^{(u+1)}) = Q_i^{(min)}$ to find the new value of $S_i^{*}$, treating the $i$th bus as a P-Q bus and continuing computations similar to a P-Q bus. Similarly, if $m_i(Q_{(u+1)}^{(u+1)}) > Q_i^{(max)}$ then in same manner taking the reactive power moment $m_i(Q_{(u+1)}^{(u+1)}) = Q_i^{(max)}$ the new value of $S_i^{*}$ is again computed and treating the $i$th bus as a P-Q bus continuing computations similar to a P-Q bus are carried out. The line flows are calculated with the final bus voltages and the given line admittances and line charging using equation given below:

$$m_i(P_{ik} - jQ_{ik}) = m_i(V_i^* (V_i - V_k)_{ik} + V_i^* V_l^* \frac{Y_{ik}}{2})$$

Finally, the slack bus power can be determined by summing the flows of the lines terminating at the slack bus.

**Illustrative example:**

The method for solving the load flow problem is illustrated in the following sample power system given in Fig 4 (Here acceleration factors of 1.4 and 1.4 and tolerances of 0.0001 per unit for the real and imaginary components of voltage are used). The transmission line impedances in per unit on a 100,000-kva base are given in Table 1. The scheduled generation and loads and the assumed per unit bus voltages are given in Table 2.

![Fig 4](https://example.com/fig4.png)

**Table 1**: Line Admittances for the sample system

<table>
<thead>
<tr>
<th>Bus</th>
<th>Admittances</th>
<th>shunadm</th>
<th>p-q</th>
<th>Ypq</th>
<th>q=1-p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>2.000 -j 4.000</td>
<td>0.01</td>
<td>0.800</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>1.000 -j3.000</td>
<td>0.01</td>
<td>0.700</td>
<td>0.300</td>
<td></td>
</tr>
<tr>
<td>2-3</td>
<td>1.000 -j8.000</td>
<td>0.01</td>
<td>0.900</td>
<td>0.100</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2**: Scheduled generation and loads and assumed bus voltages for sample system.

<table>
<thead>
<tr>
<th>Bus Assumed Generation Load Type probability code</th>
<th>Bus</th>
<th>MW</th>
<th>MVAR</th>
<th>MW</th>
<th>MVAR</th>
<th>p_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-q</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3 Moments of $Y_{BUS}$ matrix elements for the sample system

<table>
<thead>
<tr>
<th>Elements</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y_{11}</td>
<td>2.300</td>
<td>26.400</td>
<td>256.60</td>
<td>114.80</td>
</tr>
<tr>
<td></td>
<td>j3.20</td>
<td>j28.20</td>
<td>j70.28</td>
<td>j1987</td>
</tr>
<tr>
<td>Y_{22}</td>
<td>2.500</td>
<td>109.50</td>
<td>955.10</td>
<td>10048</td>
</tr>
<tr>
<td></td>
<td>j3.97</td>
<td>j56.00</td>
<td>j913.97</td>
<td>j14390</td>
</tr>
<tr>
<td>Y_{33}</td>
<td>1.600</td>
<td>91.28</td>
<td>505.73</td>
<td>8408.85</td>
</tr>
<tr>
<td></td>
<td>j9.28</td>
<td>j34.60</td>
<td>j58.31</td>
<td>j7937.51</td>
</tr>
<tr>
<td>Y_{12}</td>
<td>-1.60</td>
<td>-0.90</td>
<td>0.28</td>
<td>49.45</td>
</tr>
<tr>
<td></td>
<td>j3.20</td>
<td>j7.20</td>
<td>j12.80</td>
<td>j169.57</td>
</tr>
<tr>
<td>Y_{13}</td>
<td>-0.70</td>
<td>-2.24</td>
<td>-4.73</td>
<td>-11.91</td>
</tr>
<tr>
<td></td>
<td>j2.10</td>
<td>j6.80</td>
<td>j6.06</td>
<td>j40.85</td>
</tr>
<tr>
<td>Y_{23}</td>
<td>-0.90</td>
<td>-45.36</td>
<td>79.07</td>
<td>-213.87</td>
</tr>
<tr>
<td></td>
<td>j7.20</td>
<td>j11.52</td>
<td>j127.92</td>
<td>j116.12</td>
</tr>
</tbody>
</table>

Table 4 Moments of Bus Voltages for the sample system

<table>
<thead>
<tr>
<th>Voltages</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_1</td>
<td>0.990</td>
<td>0.980</td>
<td>0.970</td>
<td>0.961</td>
</tr>
<tr>
<td></td>
<td>j0.00</td>
<td>j0.00</td>
<td>j0.00</td>
<td>j0.00</td>
</tr>
<tr>
<td>V_2</td>
<td>1.0290</td>
<td>0.554</td>
<td>0.287</td>
<td>1.102</td>
</tr>
<tr>
<td></td>
<td>j0.052</td>
<td>j2.216</td>
<td>j0.022</td>
<td>j6.828</td>
</tr>
<tr>
<td>V_3</td>
<td>0.9867</td>
<td>1.256</td>
<td>0.426</td>
<td>3.519</td>
</tr>
<tr>
<td></td>
<td>j0.059</td>
<td>j2.191</td>
<td>j0.274</td>
<td>j1948</td>
</tr>
</tbody>
</table>

Table 5 Moments of injected power at bus for the sample system

Table 6 Moments of line flows for the sample system

Table 7 Transmission state probabilities
States | Lines out | Probability = \( p(B_j) \)
---|---|---
1 | 0 | \( p_1 p_2 p_3 = 0.504 \)
2 | 1 | \( q_1 p_2 p_3 = 0.126 \)
3 | 2 | \( p_1 q_2 p_3 = 0.216 \)
4 | 3 | \( p_1 p_2 q_3 = 0.056 \)
5 | 1,2 | \( q_1 q_2 p_3 = 0.054 \)
6 | 1,3 | \( q_1 p_2 q_3 = 0.014 \)
7 | 2,3 | \( p_2 q_2 q_3 = 0.024 \)
8 | 1,2,3 | \( q_1 q_2 q_3 = 0.006 \)

Note: 1,2,3 refer to lines 1-2, 1-3 and 2-3

It is seen that even assuming that there is no load variation and outage of generating units i.e., installed capacity and demand being fixed the conventional load flow (CLF) has to be carried out 8 times.

Table 8 shows the bus voltages for each of the eight different configurations in the load flow carried out separately for each case. Similarly, \( Y_{BUS} \) matrix elements and line flows can be obtained.

### Table 8 Bus Voltages for all eight line contingencies

<table>
<thead>
<tr>
<th>Line Contingency</th>
<th>( V_1 )</th>
<th>( V_2 )</th>
<th>( V_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0.990 + j 0.000</td>
<td>1.04987 + j 0.016489</td>
<td>1.004797 + j 0.05793</td>
<td></td>
</tr>
<tr>
<td>1 0.990 + j 0.000</td>
<td>1.04922 + j 0.040554</td>
<td>0.004736 - j 0.04056</td>
<td></td>
</tr>
<tr>
<td>2 0.990 + j 0.000</td>
<td>1.04987 + j 0.016489</td>
<td>0.014183 - j 0.04056</td>
<td></td>
</tr>
<tr>
<td>3 0.990 + j 0.000</td>
<td>1.048764 + j 0.050936</td>
<td>0.0852335 + j 0.022152</td>
<td></td>
</tr>
<tr>
<td>1,2 0.990 + j 0.000</td>
<td>1.04922 + j 0.040554</td>
<td>0.013532 - j 0.055905</td>
<td></td>
</tr>
<tr>
<td>1,3 0.990 + j 0.000</td>
<td>-10 - j 40</td>
<td>0.0852335 + j 0.022152</td>
<td></td>
</tr>
<tr>
<td>2,3 0.990 + j 0.000</td>
<td>1.052173 + j 0.069085</td>
<td>20 + j 80</td>
<td></td>
</tr>
<tr>
<td>1,2,3 0.990 + j 0.000</td>
<td>-10 - j 40</td>
<td>20 + j 80</td>
<td></td>
</tr>
</tbody>
</table>

The first moment or expected value of bus voltage at each bus for all the different line states is obtained as,

\[
Ex(V_i) = \sum_{j=1}^{n} V_i p_j \quad \ldots(24)
\]

Thus, the value for the above for the bus voltages at bus 1, 2 and 3 are, 

\[
Ex(V_1) = (1.4987 + j 0.016489) \times 0.504 + (1.04922 + j 0.040554) \times 0.126 + (1.04987 + j 0.016489) \times 0.216 + (1.048764 + j 0.050936) \times 0.056 + (1.04922 + j 0.040554) \times 0.054 + (1.052173 + j 0.069085) \times 0.024 = 1.029 + j 0.024
\]

Value obtained by probabilistic load flow, \( m_l(V_2) \) = 1.029 + j 0.028.

\[
Ex(V_3) = (1.004797 - j 0.05793) \times 0.504 + (1.004736 - j 0.04056) \times 0.126 + (1.014183 - j 0.07915) \times 0.216 + (0.852335 - j 0.22152) \times 0.056 + (1.013532 - j 0.055905) \times 0.054 + (0.852335 - j 0.22152) \times 0.014 = 0.970 + j 0.060
\]

Value obtained by probabilistic load flow, \( m_l(V_3) \) = 0.986 + j 0.060

From the calculation above it is observed that the expected value of each bus voltage in each of the deterministic cases can be obtained from the first moment of the bus voltages in the probabilistic load flows analysis. Similarly, the same is true for the injected powers at buses and line flows. Therefore, the eight load flows for each abnormal condition of lines is unnecessary if knowledge of the complete history of the output distribution functions can be obtained from one load flow.

### II. CONCLUSION

The method for solution of the stochastic load flow problem with the complex random variable modelling using the method of moments as shown therefore reduces prohibitive amount of calculations combined with added difficulty of analyzing and synthesizing the solution in the case of very large systems. The algorithm is applicable when arbitrary functions are used to model loads of the system. The input variables are considered to be complex random, thereby a more realistic representation of the system is obtained. The modeling method can be used with the segmentation of load method, given in [13] to calculate the loss of load probability and the expected generation by the machines (the two most important parameters required for generation planning) from the knowledge of the first and the zeroth moments of the load impulses before and after loading of the machines. Thus, the application of the present modeling procedure with the load segmentation method proposed by Ghosh and Mishra would give a more accurate representation of the power system and hence more accurate results can be expected.