An NLP Algorithm for Short Term Hydro Scheduling

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Abstract: This paper presents a method based on nonlinear programming for short term scheduling of hydro power system. The proposed algorithm is based on the solution of an augmented lagrangian function of the scheduling problem using conjugate gradient method. The water transportation delay between cascaded reservoirs is considered. Results concerning this method are compared with those achieved from successive solution of augmented lagrangian function utilizing gradient decent method. It is shown that proposed solution approach is capable of yielding better and fast optimal solution.

I. INTRODUCTION

The short term scheduling of hydro generations has been one of the principal and difficult optimization problems in economic operation and control of interconnected multireservoir power system. In such a system the marginal cost of hydroelectric generation is insignificant. This suggests that the water at hand in system reservoirs should be utilized for hydropower generation in such a way so that overall cost of nonhydraulic power generations is minimized along the planning horizon while satisfying diverse constraints. The usual assumption is that target end point storage levels generally conform to water release schedule earlier set up by mid-term scheduling process in which long term river flow probabilities and demand predictions are taken into account. The short-term scheduler then allocates this water for hydro generation among the various time intervals along the scheduling period so as to minimize the production cost while trying to satisfy different constraints.

The governing constraints of the short term scheduling problem are demand-supply balance, flow balance or continuity equation, bounds on reservoir storage, bounds on water release, limits on spillage and coupling constraint that puts a boundary condition on the initial and final reservoir levels. Supplementary constraints such as flood control, irrigation, recreation, fishing and wild life maintenance etc. could be dictated depending on contractual or legal obligations in respect to hydro system network. The common assumption is that load demand and river inflows are known.

A large number of researchers have extensively investigated the short-term hydro scheduling problem. Main computational techniques that have been employed are maximum principal [1], variational calculus [2,3], dynamic programming [4-8], functional analysis [9], network flow and linear programming [10-12], nonlinear programming [13], mathematical decomposition [14-17], and progressive optimality algorithm [18]. These methods have one or the other limitation such as dimensionality difficulty, large memory requirement and inability to handle nonlinear cost function. In the work presented in this paper, a non-linear programming algorithm based on iterative use of conjugate gradient method has been developed to solve short-term hydro scheduling problem. The scheduled water discharge rate for each hydro plant are taken as variables of the problem. An new strategy has been developed to generate initial estimates for water discharge rates. The proposed algorithm is implemented on multi-chain cascade of four reservoir system and is compared with the algorithm based on gradient decent method.

II. PROBLEM FORMULATION

Notation:
- $F$: composite cost function.
- $P_h^t$: power generation of $j^{th}$ hydro plant at time $t$.
- $x_j^t$: storage volume of $j^{th}$ reservoir at time $t$.
- $q_j^t$: water discharge rate of $j^{th}$ reservoir at time $t$.
- $u_j^t$: turbine discharge rate at $j^{th}$ reservoir at time $t$.
- $P_d^t$: load demand at time $t$.
- $E^t$: error between load demand and total hydro generation at time $t$.
- $P_l^t$: total transmission losses at hour $t$.
- $v_j^t$: volume of spilled water from $j^{th}$ reservoir at time $t$.
- $y_j^t$: side inflow rate of $j^{th}$ reservoir at time $t$.
- $z_j^t$: inflow from upstream plants to reservoir $j$ at time $t$.
- $W_j^t$: total available water in reservoir $j$ over the complete time horizon.
- $C_{p_{j1}}^{mgh},\ldots,C_{p_{j6}}^{mgh}$: power generation coefficients at $j^{th}$ hydro plant.
- $t_{j}$: water transport delay from reservoir $k$ to reservoir $j$.
- $N_u$: upstream plant index.
- $N_h$: number of hydro plants.
- $T$: total time horizon.
- $\bar{x}_j^t$: upper bound on storage of reservoir number $j$.
- $x_j^t$: lower bound on storage volume of reservoir number $j$.
- $q_j^t$: upper bound on water discharge rate of $j^{th}$ reservoir.
- $q_j^t$: lower bound on water discharge rate of $j^{th}$ reservoir.
- $u_j^t$: maximum allowable water discharge rate through turbine at reservoir number $j$.
- $u_j^t$: minimum allowable water discharge rate through turbine at reservoir number $j$.
- $x_j^{in}$: initial storage volume of $j^{th}$ reservoir.
- $x_j^{out}$: storage of $j^{th}$ reservoir at the end of planning horizon.

The power system discussed here, compose of a cascaded multireservoir system and interconnection lines to the neighboring utilities through which electric power may be exchanged. A general system representation for $j^{th}$...
reservoir is shown in figure 1. The problem of short term scheduling of hydro generations in such a system can be stated as to find out the water release from each reservoir and through each power house over all the planning time intervals so as to minimize the total cost of energy import or energy export while satisfying diverse hydraulic and load balance constraints. Here the energy import refers to the shortage in hydro generated energy to meet the load demand. On the other hand the energy export is stated as the surplus of hydro generated energy over the load demand. Typically the total planning period is one day or one-week and time interval is one hour.

A. Objective Function and Constraints

The objective is to minimize the summation of the production cost of energy import or energy export over the scheduling period. The energy cost has been taken as a square function \[18\] of the error between the demand and scheduling period. The energy cost has been taken as the surplus of hydro energy export while storage is allowed only when water release from reservoir exceeds the maximum discharge limit through each power house over all the planning time intervals so as to minimize the total cost of energy import or energy export while satisfying diverse hydraulic and load balance constraints. Here the energy import refers to the shortage in hydro generated energy to meet the load demand. On the other hand the energy export is stated as the surplus of hydro generated energy over the load demand. Typically the total planning period is one day or one-week and time interval is one hour.

subject to the following constraints.

B. System active load balance:

In a power system total power generated is equal to the total power demand including losses in each time interval. This equality for time interval \(t\) is expressed as

\[
\sum_{i=1}^{N_h} P_{h_i} = P_d^t + P_l^t
\]  

(2)

C. Reservoir flow balance:

Reservoir flow balance or continuity equation is expressed as equality constraint. In this constraint water transportation delays between reservoirs are taken into consideration. As is evident from figure 1 for \(j^\text{th}\) reservoir, the flow balance equation relates the previous interval storage with the storage, net inflow and net outflow during the current time interval. Mathematically the equality is expressed as

\[
x_j = x_j^{t-1} + y_j^{t-1} - u_j^{t-1} - v_j^{t-1} + \sum_{m \in M_h} (u_m^{t-1} - v_m^{t-1})
\]

(3)

E. Spillage modeling:

Spillage is allowed only when water release from the reservoir exceeds the maximum discharge limit through the turbines. From figure 1, water spilled from reservoir during time interval \(t\) is written as follows

\[
v_j = \begin{cases} 
q_j^{t-1} - u_j; & \text{if } q_j^{t-1} > u_j \\
0; & \text{otherwise}
\end{cases}
\]

(4)

F. Physical constraints:

Reservoir storage volume and turbine discharge rates are bound between maximum and minimum by physical limitations. Net reservoir release may also be constrained between maximum and minimum limits. Thus for reservoir number \(j\) we have

\[
x_j^{min} \leq x_j^{t-1} \leq x_j^{max}
\]

\[
u_j^{min} \leq u_j^{t-1} \leq u_j^{max}
\]

\[
q_j^{min} \leq q_j^{t-1} \leq q_j^{max}
\]

(5)

G. Coupling constraint:

Terminal reservoir volumes are previously set by midterm scheduling process. This constraint implies that the total quantity of available water should be used. Coupling constraint for reservoir number \(j\) is expressed as

\[
x_j^0 = x_j^{begin}
\]

\[
x_j^T = x_j^{end}
\]

(6)

H. Hydropower generation characteristics

Hydropower generation is a function of net head and turbine discharge. Constant water head is generally assumed in short period scheduling formulation. However this assumption is true only in case of large capacity reservoirs. Head variation can not be ignored if there is strong relationship between inflow and capacity. Since net head is a function of volume of stored water, hydropower generation can be written in terms of turbine discharge rate and storage, and the frequently used expression is \([16]\)

\[
\begin{aligned}
Ph_j^t &= C_j(x_j^{t-1})^2 + C_{j2}(u_j^{t-1})^2 + C_{j3}(x_j^{t-1})(u_j^{t-1}) \\
&+ C_{j4}x_j^{t-1} + C_{j5}u_j^{t-1} + C_{j6}; \; j \in Nh
\end{aligned}
\]

(7)

III. PROBLEM REFORMULATION:

The aim to reformulate the problem is to express the objective function (1) entirely in terms of reservoir release variables, thus reducing the number of variables and constraints in the scheduling problem considerably. The modeling of spill in (4) permits to use reservoir release \(q_j^{t-1}\) in place of \(u_j^{t-1}\) in the hydropower generation function (7).

\[
\begin{aligned}
Ph_j^t &= C_j(x_j^{t-1})^2 + C_{j2}(q_j^{t-1})^2 + C_{j3}(x_j^{t-1})(q_j^{t-1}) \\
&+ C_{j4}x_j^{t-1} + C_{j5}q_j^{t-1} + C_{j6}; \; j \in Nh
\end{aligned}
\]

(8)

That is, whenever reservoir release exceeds the maximum turbine discharge limit, \(u_j\) is replaced by \(u_j\) in (8). Thus it is evident from (8) that hydro generation is a function of storage volume and release of the reservoir. It is also easy to express storage entirely in terms of reservoir releases thus eliminating storage from (8). Using (4) the resulting expression for storage is given by

\[
x_j = x_j^{begin} + \sum_{i=1}^{T} (y_j^{i} + \sum_{m \in M_h \& k \in T_j^{i-1} \; 0} q_m^{t-1} - q_j^{t-1})
\]

(9)

Substituting \(x_j\) from (9) in (8) we get
\[ P h_j = C_{j1}(z(q))^2 + C_{j2}(q_j)^2 + C_{j3}(z(q))(q_j') + C_{j4}z(q) + C_{j5}q_j' + C_{j6} ; \ j \in N_h \]  

(10)

Now using equation (2) the non hydraulic power is expressed in terms of hydro generations and losses that are also a function of hydro generations, thus yielding the thermal power generation as a function of only release variables.

\[ E' = Pd' - P_t'(q) - \sum_{i=1}^{NB} P h'_i(q) \]  

(11)

Thus the problem reformulation yields an objective function entirely in terms of reservoir release variables and the load balance constraint is not treated explicitly but rather implicitly.

The reformulated problem in compact form is given by,

\[ F = \sum_{t=1}^{T} \left( E'_t \right)^2 \]  

(12)

subject to

\[ x_i \leq x_i(q) \leq \bar{x}_i \]  

\[ q_i \leq q_j' \leq q_i \]  

(13)

\[ v'_j = \begin{cases} q_j' - \bar{u}_j ; & \text{if } q_j' > \bar{u}_j \\ 0 & \text{otherwise} \end{cases} \]  

(14)

\[ x'_j(q) = x_j^{\text{new}} \]  

(15)

In equations (13-15); \( j=1,2,...,N_h \); \( t=1,2,...T \).

IV. SOLUTION TECHNIQUE

Consider a constrained nonlinear programming problem of the form

\[ \min f(Q) \]  

(16)

subject to constraints

\[ g_j(Q) \leq 0 \]  

(17)

\[ h_t = 0.0 \]  

(18)

where \( Q = [Q_1, Q_2, ..., Q_n] \) is a vector of \( n \) variables of problem; \( j = 1,2, ..., r ; i = 1,2, ..., e ; \) \( r \) is the number of inequality constraints; \( e \) is the number of equality constraints;

The hydro scheduling problem has nonlinear objective function. Equality as well as inequality constraints are present in the problem. The vector \( Q \) corresponds to the discharge variables and is the transpose of \( [q_1^1, q_1^2, ..., q_1^M, ..., q_r^1, ..., q_r^M] \).

Let us define a augmented Lagrange function [7]

\[ L_u = f(Q) + \sum_{j=1}^{r} \lambda_j g_j^1 + \sum_{i=1}^{e} \mu_i h_i + \frac{s}{2} \left[ \sum_{j=1}^{r} (g_j(Q))^2 \right] + \sum_{i=1}^{e} (h_i^2) \]  

(19)

where, \( s \) is constant penalty parameter. \( g_j(Q) = \max [0,g_j(Q)] \); i.e., \( g_j(Q) \) is the magnitude of violation of the \( j^{th} \) equality constraint in the problem , where \( 1 \leq j \leq r \). \( h_i(Q) \) is the \( r^{th} \) equality constraint in the problem \( P \) where \( 1 \leq i \leq e \). \( \lambda_j \) and \( \mu_i \) are the lagrange multipliers with inequality and equality constraints respectively. It may be noted that \( h_i(Q) \) may be positive or negative because under both the conditions it represents constraint violation.

The augmented lagrangian (19) has transformed the scheduling problem into the successive process. Within each iteration, \( Q_n \) is found using conjugate gradient method [19]. Then \( \lambda^i_s \) and \( \mu^i_l \) are updated as follows for the \((n+1)^{th}\) iteration.

\[ \lambda_{n+1} = \lambda_n + s \lambda'(Q_n) \]  

(20)

\[ \mu_{n+1} = \mu_n + sh(Q_n) \]  

(21)

The process is terminated when either the feasible solution is obtained or the change in the variables during successive iterations is less than the specified tolerance.

A. Generating the initial estimates for water discharge rates

The control variables, that are water discharge rates, require initialization in order to start the optimization operation by conjugate gradient method. As total available water is known a priori, an new attempt is made here for its allocation in various hours of the planning period in such a way so that coupling constraints are satisfied as well as overall nonhydraulic generation cost is presumed to be minimum. These requirements are approximately met by considering hourly water discharge rate proportional to square of load demand in that hour. Define a vector \( p \) such that

\[ p_t = \left( \frac{pd_t^2}{\sum_{t=1}^{T} (pd_t^2)} \right) t = 1,2,...T \]  

(22)

\[ q'_j = p_t W_j ; t = 1,2,...T \text{ & } j = 1,2,...,N_h \]  

(23)

In this way all the available water is allocated and hourly water discharge rates are proportional to the square of corresponding load demand. Even though some of the constraints may be violated initially, yet the proposed algorithm conveniently handles the infeasible state trajectory by way of penalty functions.

B. Algorithm

1. Scan input data and topology of interconnected hydro system.

2. Generate initial solution for discharge vector \( Q \) and assume parameter value \( s \).

3. Set vectors \( \lambda = 0 \) & \( \mu = 0 \).

4. Initialize iteration count \( n = 0 \).

5. Calculate storage of the reservoirs using current values of water discharge rates.

6. Using water discharge rates and storage, determine hydro plant generations.

7. Calculate shortfall or surplus power generation using load balance equation.

8. Evaluate equality and inequality constraints using current values of discharges and reservoir contents.
9. Calculate gradients of both the objective function as well as equality and inequality constraints.

10. Compute new values of water discharge rates using conjugate gradient method.

11. Check convergence.

12. If convergence is reached then print output quantities such as water discharge rates, reservoir storages, hydroplant generations and nonhydraulic power import or export for each hour and stop; else go to next step.

13. Increment \( n \) i.e. \( n = n + 1 \) and go to step 5.

V. EXAMPLE

For substantiating the efficacy of the proposed algorithm, test system considered is shown in figure 2. The system consists of a multi-chain cascade of four reservoir type hydroplants connected electrically to network having imported power, exported power and load demand. The scheduling period is 24 hours.

The hydro generation data used for the present work is supplied in tables (1-3). Load demand data for 24 hours is given in table 1, while table 2 provides the matrix of hydro generation coefficients. Bounds on reservoir storage volume, water discharge rates and boundary conditions on reservoir storage volume are presented in table 3. In table 3 the units of storage are \( 10^3 \) m\(^3\) while units of water discharge rate are \( 10^3 \) m\(^3\)/hour. The vector \([10,8,1,0]^T \times 10^3 \) m\(^3\)/hour gives reservoir hourly side inflows. The water transportation delays considered are \( r_{1} = 1 \) hour; \( r_{2} = 2 \) hours; \( r_{3} = 2 \) hours.

Table 3 Characteristics of hydro stations

<table>
<thead>
<tr>
<th>Plant</th>
<th>( x )</th>
<th>( \overline{x} )</th>
<th>( q )</th>
<th>( \overline{q} )</th>
<th>( x_{begin} )</th>
<th>( x_{end} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>150</td>
<td>5</td>
<td>15</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>120</td>
<td>6</td>
<td>15</td>
<td>80</td>
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<td>170</td>
<td>170</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>160</td>
<td>13</td>
<td>25</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

A. Results and Discussion

The optimal water discharge, reservoir storage, hydropower generation schedule, and overall hydropower generation, shortfall of power generation have been calculated by implementing the algorithm. Viewing figure 3 it can be observed that the trajectories of water discharge for plants 1 and 2 closely follow the pattern of load demand because they are the upstream plants. Water discharge from plant 3 is the largest in the beginning because it has the highest storage and the hydro generation is proportional to both the water discharge rate and the reservoir storage. Water discharge continues to be high until hour 18, when the load demand starts to drop and \( q_3 \) starts to drop in order to meet terminal storage conditions. There is a sharp fall in reservoir storage of plant 4 during first 2 hours. This happens because of simultaneous presence of three factors, which are; (1) water transport delay of 2 hours from reservoir 3 to reservoir 4; (2) plant 4 is constrained to meet minimum water discharge rate limit of \( 1.3 \times 10^3 \) m\(^3\)/hour; (3) side inflow to reservoir 4 is zero.

Water discharge from plant 4 continues at minimum limit until hour 9 due to relatively low load demand during this period. Then \( q_4 \) follows the load curve to supply peaking load for next three hours. After hour 12 \( q_4 \) continues to increase because reservoir 4 is downstream reservoir to reservoir 3. Consequently hydro generation \( Ph_4 \) continues to increase keeping the shortfall down as shown in figures table 4. From table 4, it can also be observed that overall hourly hydropower generation from the hydro system network closely follow the pattern of load demand thereby keeping the scheduling error at minimum.
In order to validate the results achieved with the proposed solution technique we have also implemented the said algorithm with the gradient decent method [19]. The set of schedules obtained using both the methods are in tandem with each other. Whereas with the conjugate gradient method the computation time is 7 seconds, it is 20 seconds with the gradient decent method.

VI. CONCLUSIONS
A novel method based on nonlinear programming approach has been developed to find optimal schedule for hydro and thermal generations in an interconnected hydro-power system. The water flow rate in an earlier hour affects the future water discharge rates in a hydro scheduling problem. Thus it is favourable to consider all the control variables simultaneously in search of a good optimal solution. The results of the algorithm can be used in setting up the hydro generation and import/export targets for each plant in a interconnected hydro system. In comparison with gradient decent method, the algorithm based on conjugate gradient method has yielded fast solution for the short-term hydro scheduling problem.

VII. REFERENCES


