Singular Perturbation: A Promising Tool for Power System Engineers

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Abstract: Singular perturbation was first applied for boundary layer applications in 1905. It is a very powerful tool for control engineers due to advantages of order reduction and time scaling. Though it has found immense applications in diverse fields like fluid mechanics, flight control, chemical engineering, applied mechanics, and optimal control. It is only recently, in the past two decades that this technique has found applications in the field of electrical engineering.

This paper is aimed to serve as an introductory tutorial on the topic while reviewing the literature concerning its applications in electrical engineering.

I. INTRODUCTION

Singular perturbation technique was made known to the world at the third international congress of mathematicians in Heidelberg in 1904 by L. Prandtl through his report on fluid motions with small friction. Perturbation theory first appeared in celestial mechanics while deducing the planetary motions, if the only the sun and one planet are considered, the result is an elliptical motion with sun at the focus. However, this is not so in the presence of other planets because the planets exert gravitational forces on each other and perturb their motions. Perturbation theory refers to accounting for all these perturbations in the final solution. So, one begins with unperturbed solution, that is with purely elliptical motion as a first approximation and then arrive at a final solution by applying a series of corrections to this approximation by computing the forces, these planets would exert on each other.

The perturbation problem mentioned above is categorized into following two cases:

Regular perturbation problem

This problem \( P_\varepsilon(y_\varepsilon) = 0 \) depends on this small parameter \( \varepsilon \) in a such a way that its solution \( y_\varepsilon(x) \) converges to solution \( y_0(x) \) of the\(^1\) limiting problem \( P_0(y_0) = 0 \). Consider, for example, spring mass problem on any finite time interval.

Assuming sufficient smoothness (w.r.t. \( y, x, \varepsilon \)), the solution of regular perturbation problem can be approximated by a former asymptotic power series expansion in \( \varepsilon \) having the leading term \( y_0 \)

Singular Perturbation problem

This type of problem is said to occur whenever the regular perturbation limit \( y_\varepsilon(x) \to y_0(x) \) fails. For example, consider the problem \( \varepsilon \frac{dx}{dt} + x = 1 \) whose solution is given by

\[
x(t, \varepsilon) = 1 + [x(0) - 1] \exp(-t/\varepsilon)
\]

where \( x(0) \) stands for initial value of \( x \) at \( t=0 \).

For \( \varepsilon > 0 \) the solution tends to 1 for any \( t > 0 \) as \( \varepsilon \to 0 \).

As \( \varepsilon \to 0 \) \( x(t, \varepsilon) \) increases monotonically towards the constant limit 1 for each \( t > 0 \) if \( x(0) < 1 \).

For \( t=0 \) \( x(t, \varepsilon) \to x(0) \); \( t > 0 \) \( x(t, \varepsilon) \to 1 \)

Thus \( x(t, \varepsilon) \) has a discontinuous limit as \( \varepsilon \to 0 \). This type of singularity, generally, occurs when \( \varepsilon \) is multiplied with the highest derivative in the system. The model, which is generally used in power system applications, is as below.

\[
\begin{align*}
\frac{dy}{dt} &= f(y, z, \varepsilon) \quad y(0) = y_0 \quad y \in \mathbb{R}^n \\
\varepsilon \frac{dz}{dt} &= g(y, z, \varepsilon) \quad z(0) = z_0 \quad z \in \mathbb{R}^m
\end{align*}
\]

where \( y \) is \( n \)-dimensional vector of slow variables, \( z \) is \( m \)-dimensional of fast variables and \( \varepsilon \) is called singular perturbation parameter, which is always positive.

If \( \varepsilon \) is quite small, we can approximate the system by setting \( \varepsilon = 0 \). So, Eq(2) now becomes an algebraic equation instead of a differential equation. It is now required to solve only n-differential equations thus reducing the order of the system from \((m+n)\) to \(n\). This solution will be much more accurate than the one where we find the solution by totally neglecting the fast transients and approximate the system only by its average or slow response. The reasons of higher accuracy obtained in this case is that the fast states are not totally eliminated but approximated by an algebraic equation instead of a differential equation. This solution obviously will not be correct at the boundary layer (at initial time) since fast states (represented \( z \)) will be given by initial conditions instead of algebraic solutions.

Thus, in fact, Singular Perturbation technique legitimizes the adhoc simplification of models of dynamic system. One way of simplification is to neglect small time constants, masses and other such parameters and thus reduce the order of the system. But this oversimplified model may be erroneous and may result in dissatisfactory performance. Singular perturbation technique, instead, by setting \( \varepsilon = 0 \) reduces the system to simplified model, but, treats this as first step towards final solution. The final solution is then computed by asymptotic expansion.

II. LITERATURE SOURCES

Introductory mathematical background for perturbation techniques is excellently covered in references [1-5]. For the application of singular perturbations to control theory the books/monographs in [6-9] serve as a good reference. Ref.
III. APPLICATIONS TO POWER SYSTEMS:
The controllability of singularly perturbed linear system was established by Kokotovic et.al. in 1974[16]. This was done by establishing the controllability of the slow and fast subsystems. It was then demonstrated that the fast-slow separation can be accomplished in a general formulation of time optimal control problem. The application of singular perturbation theory to power systems determining the high frequency oscillations were reported by Chow et.al. [17].

While giving an analogy of mass-spring damper example, where the spring is stiff, the authors showed that it can be viewed as a perturbation of a rigid rod. Similarly they analysed the intermachine oscillations in a multimachine system by treating it as a perturbation from coherent (swinging almost in unison) machine. It is shown that if the intermachine oscillations were neglected, then the power angles of tightly coupled machines are coherent. The decomposition of the system into a slowly varying subsystem and a fast oscillatory subsystem further gives an additional advantage of reduction in order of computation needed as we are required to deal with lower order subsystem rather than a full system. Kokotovic et.al [18] demonstrated clearly that, though order reduction and separation of time scales can be achieved using techniques like aggregation and modal analysis, singular perturbation technique (hereafter referred to as SPT), resolves the inconsistencies in quasi-steady-state (QSS) model described in other techniques. The authors further explain the idea of stretching of time, so that fast phenomenon has adequate time to reach its steady state while still treating the slow variables as constants during that period. Also the approximation obtained by treating perturbation parameter \( \varepsilon \) can further be improved iteratively by constructing asymptotic expansions in \( \varepsilon \). The authors have considered an Single Machine – Infinite Bus (SMIB) system with voltage regulator and have shown that by considering \( e'q \) and \( Rf \) as slow states and \( ed' \), \( Efd' \) and \( VR \) (where symbols have their usual meaning) as fast states, quite accurate results are obtained after one iteration. The difficulties encountered while extending the above model in [18] to large power systems were studied by Winkelman et.al. [19]. It was demonstrated that for a three generator, nine bus system, although subsystem models are state separable into slow and fast states, thereby making them ideal candidates for singular perturbation studies, the interconnection of subsystems may introduce new phenomena and change the speeds of some of the states. Thus a new choice of state variables is needed to make the interconnected model state separable. A procedure of validation of reformulated model and its improvement is outlined. The results obtained on linear coherency and aggregation were extended to non-linear electromechanical models of power system and other similar dynamic networks by Peponides et.al.[20].

Naidu et al. [21] applied the singular perturbation method for transient analysis of step input to a two winding transformer. Since coefficient of coupling \( k \) is nearly equal to unity, perturbation parameter \( h = 1 - k^2 \) is made use of, to model the transformer equations into singular perturbation form. The total series solutions is obtained in terms of outer, inner and intermediate series solution for zeroth and first order approximations which agree quite well with the exact solution. The SPT was applied to sparse dynamic networks by Chow et.al.[22]. Here they differed from the traditional view of referring to weak connections as synonymous with weak coupling. So even the system where the individual inter-area external connections were as strong as internal connections within the area but much sparser than internal connections were considered as weakly coupled. In fact, application of SPT resulted in finding out the unknown sparsity pattern of a large network. The idea of representing interconnected multimachine power system as \( I = \sqrt{Y} \) type interface equations as the simplest approximation of slow time scale dynamics in time decomposed model was explored by Sauer et.al. [23]. Chow et.al. [24] discussed the possible power system applications including Transient stability analysis. The methods for improving the linear models to any desired accuracy for carrying out small signal stability analysis were also reported by them. Durec et.al. [25, 26] considered the complete synchronous machine model with damper windings and complete excitation system [ IEEE type 1 AVR and PSS] and applied SPT successfully on four machine system in Serbia. X.Xu et. al. [27] further improved upon this model to include both governor and turbine models. Also switching between full model and reduced model was performed depending upon rate of change. SPT application for modeling effects of system frequency variations in induction motor dynamics was studied by Xu et.al. [28]. In induction motor, the system frequency change would cause the stator voltage to vary at the difference of frequency between actual and nominal frequencies. Thus it is not sufficiently accurate to neglect the stator transients for computing starting response. Slip is treated as slow variable. It has been shown that singularly perturbed model saves 50% to 60% of computing time as compared to full model. De Leon-morales et.al. [29] combined the sliding mode technique alongwith singular perturbation technique to track the reference rotor angle for a synchronous generator.

IV. FUTURE SCOPE
The entire literature reported above shows the enormous potential of singular perturbation techniques that can be applied for studying different aspects of power systems. Although the studies have been carried out on smaller size power systems, applying the same to realistic large-scale power system seems quite promising although quite
challenging. Also dynamic equivalencing and inter-area control are other areas where the singular perturbation technique may prove useful.

V. CONCLUSIONS

Basics of singular perturbation is explained and its application related to power systems are reviewed in detail along with other useful literature in books and survey papers. The future applications are also indicated.

References: