Influence Of Lightning Strike Location On The Induced Voltage On a Nearby Overhead Line

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Abstract—This paper analyses the voltages induced on an overhead line by the electromagnetic effects of the lightning return stroke terminating at various locations near the overhead line. The influence of finite conductivity of ground is considered in the computation of lightning electric field as well as the induced voltage. Ground conductivity decreases the induced voltage at line terminations whereas it increases at the line mid point for a lightning strike location which is equidistant from line terminations. There is also a negative pre-pulse observed in the induced voltage when finite ground conductivity is assumed. It is seen that when lightning strikes at a location equidistant from the line terminations, the induced voltages get reduced with increase in striking distances but not in the same proportion. The rise time of the induced voltage increases with the striking distance. When lightning strikes near one of the terminations, the induced voltage at this terminal is of positive polarity only whereas at all the other observation points the induced voltage goes negative and increases in magnitude as the other line termination is approached.

Keywords—Lightning, induced voltage, overhead line, ground conductivity.

I INTRODUCTION

Power distribution networks are subject to many forms of transient phenomena. Sudden changes in voltage and current cause system overvoltages. Lightning is responsible for 70% (approximately) of the overvoltages. Transient overvoltages on overhead power distribution lines can be due to a direct strike or due to a nearby lightning strike (called as indirect strike). Induced overvoltages from indirect strike constitute one of the main causes of medium voltage power distribution system outages.

The indirect strikes, although less energetic than direct strikes, is a cause of significant problem to the power distribution system because of their high frequency of occurrence[1]. As a consequence, the computation of these induced voltages are important in designing and co-ordinating the protection of overhead power distribution lines.

Lightning induced overvoltages are generally calculated in the following way.

1. The lightning return stroke electromagnetic field is calculated making use of a return stroke current model which specifies the spatial temporal distribution of the lightning current along the channel and
2. The electromagnetic field thus obtained is used to calculate the induced overvoltages, by means of a coupling model which describes the interaction between the field and the line conductors.

The ground conductivity influences significantly the magnitude and the polarity of the electromagnetic field radiated by the lightning as it propagates over the ground and hence the induced voltage on overhead lines. The induced voltage on the overhead conductors also depend on the nearby lightning strike location.

Fig. 1. Sketch showing the lightning channel and its image used in calculating the return stroke fields.

II METHOD OF COMPUTATION

I. Lightning Return Stroke Model

The lightning return stroke current at ground level is described by the double exponential expression[2] as

\[ i(0, t) = I_0(e^{-\alpha t} - e^{-\beta t}) \tag{1} \]

The model adopted for lightning return stroke is the modified transmission line (MTL) model[3]. In this model, it is assumed that the current waveform at the ground decreases exponentially with height as it travels up the lightning channel at a constant velocity \( v \). Mathematically this current \( i(z', t) \) at height \( z' \) and at time \( t \) is represented as

\[ i(z', t) = e^{(-z'/\lambda)}i(0, t - z'/v) \tag{2} \]
where $\lambda$ is the decay constant to account for the effect of vertical distribution of charge stored in the corona sheath of the leader channel and subsequent discharge during the return stroke phase.

In this paper, the induced voltages are computed for a ground level current of 9.8 kA peak and maximum time derivative of 100 kA/\mu s. The return stroke velocity $v$ is taken as $1.3 \times 10^8$ m/s. The lightning channel is assumed as straight and vertical.

II. Electric Fields Generated by the Return Stroke

By assuming that the ground is a perfect conductor, the vertical component of electrical field $dE_z(r,z,t)$ and horizontal component of electric field $dE_r(r,z,t)$ due to an infinitesimal length $dz'$ at height $z'$ carrying current $i(z',t)$ is calculated at a general point $P(r,\phi,z)$ by the following equations [3,4]. These equations representing the MTL model for lightning return stroke in time domain are

$$dE_z(r,z,t) = \frac{dz'}{4\pi \varepsilon_0} \left[ \frac{2(z-z')^2 - r'^2}{R^5} e^{-(z'-\lambda}/|\lambda| \right] \int_0^t i(0, \tau - z'/v - R/c) d\tau$$

$$+ \frac{2(z-z')^2 - r'^2}{cR^4} e^{-(z'-\lambda}/|\lambda| i(0, t - z'/v - R/c)$$

$$+ \frac{r}{c^2 R^3} e^{-(z'/\lambda)} \frac{\partial}{\partial t} i(0, t - z'/v - R/c)$$

(3)

$$dE_r(r,z,t) = \frac{dz'}{4\pi \varepsilon_0} \left[ \frac{3r(z-z')}{R^5} e^{-(z'/\lambda)} \int_0^t i(0, \tau - z'/v - R/c) d\tau + \frac{3r(z-z')}{cR^4} e^{-(z'/\lambda)} i(0, t - z'/v - R/c)$$

(4)

where $\varepsilon_0$ is the permittivity of free space and $c$ is the velocity of light. $R = \sqrt{r^2 + (z-z')^2}$ is the distance from the current element to the observation point. The effect of perfectly conducting ground plane on the electromagnetic fields generated by the current element are included by replacing the ground by an image current at a distance $z'$ below the ground. To find the total field, the equations (3) and (4) are integrated along the channel and its image. A sketch showing the lightning channel and its image used in calculating the return stroke fields is shown in figure 1.

III. Effect of Ground on Horizontal Electric Field

The horizontal component of electric field produced by lightning is significantly influenced by the finite conductivity of the ground. The horizontal electric field including the ground conductivity can be computed using Cooray-Rubinstein formula [5]. This formula requires the azimuthal component of the magnetic field produced by the lightning which is given as [4].

$$dH_\phi(r,z,t) = \frac{dz'}{4\pi} \left[ \frac{r}{cR^2} e^{-(z'/\lambda)} \frac{\partial}{\partial t} i(0, t - z'/v - R/c)$$

$$+ \frac{r}{R^3} e^{-(z'/\lambda)} i(0, t - z'/v - R/c) \right]$$

(5)

where $\mu_0$ and $\sigma_g$ are the permeability of free space and the conductivity of the ground respectively. The horizontal electric field including the ground conductivity $E_{hg}(r,z,t)$ is given as [5]

$$E_{hg}(z = h, r) = E_r(z = h, r) - H_\phi(z = 0, r) \frac{\sqrt{\mu_0}}{\sqrt{\varepsilon + \sigma_g/\omega}}$$

(6)

where $E_r(z = h, r)$ is the Fourier-transform of the horizontal electric field at height $h$ and $H_\phi(z = 0, r)$ is the Fourier-transform of the azimuthal component of the magnetic field at ground level. $\varepsilon$ is permittivity of the ground. Both $E_r(z = h, r)$ and $H_\phi(z = 0, r)$ are calculated assuming a perfect conducting ground using equations (4) and (5).

Fig. 2. Equivalent circuit of a single-wire overhead line excited by lightning return-stroke electromagnetic field.

Fig. 3. Geometry showing the lightning striking locations w.r.t the overhead line.
IV. Field-to-Transmission Line Coupling Model

For the field-to-transmission line coupling, a set of time domain differential equations describing the voltages induced as proposed by Agrawal et.al. [6] have been used. The surge propagation is affected by the ground impedance. This is represented in the overhead line parameters as i) per-unit series ground impedance and ii) shunt ground admittance. Both the above quantities are frequency dependent and generally the latter term is neglected. The frequency dependent series ground impedance is represented in the overhead line parameters as i) per-unit series ground impedance and ii) shunt ground admittance. Both the above quantities are frequency dependent and generally the latter term is neglected. The frequency dependent series ground impedance is represented by a convolution integral in the time domain coupling equations which are given as [7]

\[
\frac{\partial}{\partial x_i} [v_i^e(x, t)] + [L_{ij}] \frac{\partial}{\partial x_i} [i_i(x, t)] \\
+ \int_0^t \left[ \xi_{ij}(t-\tau) \right] \frac{\partial}{\partial \tau} [i_i(x, \tau)] d\tau = [E^l_i(x, h_i, t)]
\]

(7)

\[
\frac{\partial}{\partial x_i} [i_i(x, t)] + [G_{ij}] [v_i^e(x, t)] + [C_{ij}] \frac{\partial}{\partial t} [v_i^e(x, t)] = 0
\]

(8)

where \( [E^l_i(x, h_i, t)] \) is the horizontal component of the incident electric field along the conductor at conductor height \( h_i \).

The sub index \( i \) denotes the particular wire of the multiconductor line. \( [L_{ij}], [G_{ij}] \) and \( [C_{ij}] \) are the inductance, conductance and capacitance matrices per unit length of the line respectively. \( [i_i] \) is the line current vector. \( [\xi_{ij}] \) is the transient ground resistance matrix and is equal to the inverse Fourier-transform of \( [Z_{gij}/j\omega] \), i.e., \( [\xi_{ij}(t)] = F^{-1} \{ [Z_{gij}]/j\omega \} \) and \( [Z_{gij}] \) is the ground impedance matrix. The internal impedance of the line is neglected. \( [v_i^e] \) is the scattered voltage vector on the \( i^{th} \) conductor. This is the voltage due to the field produced by the induced currents on the conductors. The scattered voltage is related to the total line voltage \( v_i(x, t) \) by the following relation.

\[
[v_i(x, t)] = [v_i^e(x, t)] + [v_i^i(x, t)]
\]

(9)

where \( [v_i^e(x, t)] \) is the incident voltage.

\[
[v_i^i(x, t)] = -\int_0^{h_i} E_z^l(x, z, t) dz
\]

(10)

\( E_z^l(x, z, t) \) is the incident (or inducing) vertical electric field at \( x \) at a height of \( z \). The voltage at the end of the line is determined by the boundary condition and the current at the two ends of the line, viz., \( i(x_0, t) \) and \( i(x_0 + l, t) \), where \( l \) is the length of the overhead line. The boundary conditions for the scattered voltage are

\[
v_i^i(x_0, t) = -[Z_1][i_i(x_0, t)] + \int_0^{h_1} E_z^l(x_0, z, t) dz
\]

(11)

\[
v_i^i(x_0 + l, t) = [Z_2][i_i(x_0 + l, t)] + \int_0^{h_1} E_z^l(x_0 + l, z, t) dz
\]

(12)

where \( [Z_1] \) and \( [Z_2] \) are the terminating impedance matrices.

Figure 2 shows the equivalent circuit of this model for a single conductor.

III RESULTS AND DISCUSSIONS

The induced voltage is computed at various points on an overhead conductor which is at a height of 10 m above the ground and 1000 m long. The line is terminated by its characteristic impedance at both the ends. The line constants, i.e., inductance and capacitance of the line are 1.538165 \( \mu \)H and 7.223613 pF per meter length. The overhead line represents one of the conductors of a 3\( \phi \), 33 kV power distribution line. The presence of other conductors in the vicinity of the chosen conductor is not taken into consideration in this study. The conductivity \( \sigma_g \) and relative permittivity \( \varepsilon_r \) of the ground are taken as 0.001 S/m
and 10 respectively. The induced voltages are computed for the following lightning strike locations:

- Lightning striking at (i) 50 m (P) (ii) 100 m (Q) and (iii) 200 m (R) away from the conductor and equidistant from the line terminations.
- Lightning striking at 50 m away from one of the line terminations and perpendicular to the conductor (S).
- Lightning striking at 50 m away from one of the line terminations and along the direction of conductor (T).

Figure 3 shows the geometry of different lightning strike locations with respect to the conductor. The induced voltages were computed at the points A, B, C, D and E.

Figures 4 and 5 show the induced voltages computed at line terminations (A or E) and at line mid point (C) respectively, assuming ground as a perfect conductor as well as with finite ground conductivity. From the figures, it is seen that at line terminations, the peak value of induced voltages get reduced and there is an initial negative pre-pulse due to the influence of finite conductivity of ground. At line mid point, the induced voltage is increased due to the influence of finite ground conductivity. This can be explained from the horizontal electric field. Figure 6 shows the horizontal electric field computed at a distance of 500 m from the lightning channel and at a height of 10 m assuming ground as a perfect conductor as well as with finite ground conductivity. The effect of finite ground conductivity on horizontal electric field for the distance considered is to make the polarity of the field negative initially and for late times it approaches a value equal to that of ground as a perfect conductor. Since the horizontal electric field along the conductor at line termination possesses initial negative polarity, the induced voltage also goes negative initially. At line mid point, the horizontal electric field along the conductor is zero for the lightning striking location at ‘P’. Hence the induced voltage at line mid point does not have an initial negative peak [2].

Figure 7 shows the induced voltage at line termination for different lightning strike distances from the mid point of the conductor. The induced voltages have peak values of 34 kV, 21 kV and 14 kV for lightning strike distances of 50 m, 100 m and 200 m respectively. Induced voltages have been measured by Yokoyama et.al. [8] on an experimental 3 conductor overhead line of length 820 m for a lightning striking distance of 200 m. It was reported by them that for a lightning current of 9 kA, the induced voltage measured was 16 kV which closely matches with the computed voltage (see fig.10, curve A which is for a lightning current of 9.8 kA striking at a distance of 50 m for 1000 m long single conductor line) presented in this paper. Figure 8 shows the induced voltages at line mid point. The induced voltages have peak values of 111 kV, 62 kV and 36 kV for lightning strike distances of 50 m, 100 m and 200 m respectively. From the peak values of induced voltages, it is seen that when lightning strike distance is increased by a factor of 2, the induced voltage is reduced by a factor of 0.6 at line termination and reduced by 0.56 at line mid point. Hence it can be noted that with increase in lightning striking distance, the induced voltage gets reduced, but not in the same proportion. It is also observed from the figures that the rise time of the induced voltage increases when the striking distance is increased.

Figure 9 shows the induced voltage at different points on the conductor when lightning striking location is 50 m from the line mid point and equidistant from the line termination. It can be observed from figure 9 that the induced voltage at line termi-
nations (A or E) has an initial negative peak whereas at line mid point (C) it is not present. The induced voltage in between the line termination and mid point (B or D i.e., 250 m from the line terminations) has an initial negative peak of lower magnitude. This may be due to the fact that the horizontal electric field has an initial negative peak and its component along the direction of conductor is more at line terminations and it decreases at different points on the line and is equal to zero at line mid point for the chosen lightning striking location.

Figure 10 shows the induced voltages for a lightning strike location of 50 m from one of the line terminations and perpendicular to the line and figure 11 shows the induced voltages for a lightning strike location of 50 m from one of the line terminations but along the direction of the conductor. From the figures 10 and 11, it is seen that the induced voltage at line terminations does not have an initial negative peak, i.e., it is of positive polarity only whereas at all the other chosen observation points, the induced voltage goes negative and its magnitude increases when the observation point is moved away from the lightning striking location along the conductor.

**IV CONCLUSIONS**

The induced voltages at different points on an overhead conductor due to a nearby lightning strike have been computed. The ground conductivity decreases the induced voltage at line terminations whereas it increases at the line mid point for a lightning strike location which is equidistant from line terminations. There is also a negative pre-pulse observed in the induced voltage when finite ground conductivity is assumed. It is also ob-

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**Fig. 8.** Induced voltage at line mid point (C) due to the lightning striking at three points P, Q and R.

**Fig. 9.** Induced voltage at different points on the conductor when lightning strikes at point ‘S’.

**Fig. 10.** Induced voltage at different points on the conductor when lightning strikes at point ‘T’.

**Fig. 11.** Induced voltage at different points on the conductor when lightning strikes at point ‘P’.
served that when lightning strikes at a point equidistant from the line terminations, the peak value of the induced voltages get reduced with increase in lightning striking distance but not in the same proportion. The rise time of the induced voltage increases when the striking distance is increased. When lightning strikes near one of the line terminations, the induced voltage at this line termination does not have an initial negative peak and is of positive polarity only whereas at all the other observation points, the induced voltage goes negative and increases in magnitude as the other line termination is approached.