INEQUALITY AND EQUALITY CONstrained POWER SYSTEM STATE ESTIMATION ALGORITHM

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Abstract

In this paper Power System State Estimation (PSSE) incorporating generator Q flow limits and tap limits of the On Load Tap Changing (OLTC) transformer is proposed. Q limits of the generators and tap limits of the OLTC transformers are modeled as inequality constraints with upper and lower limits. The inequality constrained power system state estimator proposed in this paper is based on Lagrangian multipliers approach and the decomposition technique proposed in [1, 2, 3]. Lagrangian multipliers are used to compute the sensitivity of the objective function with respect to the equality and inequality constraints. The proposed PSSE is implemented on various systems, and the results for 89 and 319 bus system are presented. The study cases show that, the inequality constrained PSSE algorithm provides better results compared to equality constrained power system state estimation.

Keywords: Inequality Constraints, Equality Constraints, Decomposition, PSSE, Ordering, etc.

1 Introduction

Conventionally Power System State Estimation problem is formulated as equality constrained non linear least squares problem, where the equality constraints are due to zero injection measurements. Equality constrained power system state estimation problem is formulated as:

\[
\min J(z) = \frac{1}{2} [z - f(x)]^T W^{-1} [z - f(x)]
\]

\[\text{such that } c(x) = 0\]

\[W^{-1}\] is the weight matrix associated with the accuracy of the transducers used for measurements. \(f(x)\) and \(c(x)\) are the non linear functions corresponding to the power flows and zero injections respectively. \(x\) is the state vector consisting of voltage and angle at every bus. \(z\) is the telemetered measurement vector. Measurements are obtained in such a way that the system is observable [4].

Presence of undetectable gross errors in the measurement causes the estimated state to deviate from the actual state. When the power flows are computed using the deviated state, the error gets spread (smeared) across the computed flows. It may result in introducing errors in the computed Q flows of the generators, such that they violate the limits, if not enforced. The flows computed from the estimated state considering the generator Q limits results in better measurement data base. Hence it is essential to model the generator Q limits in estimating the state of the system, resulting to inequality and equality constrained state estimation.

Survey of literature indicates that very few approaches have been developed for Inequality and Equality constrained PSSE. Very few papers have been published on inequality constrained power system state estimation algorithm. Clement et. al [5] (presented in IEEE PES Winter meeting 1992) quote that no literature have appeared on inequality constrained power system state estimation till their publication. They use interior point method to solve the Least Squares based inequality constrained PSSE problem. Later Ali Abur et. al. [6] formulated the problem as Least Absolute Value State Estimator for equality and inequality constrained PSSE problem.

Clements et. al [5] formulate the problems as:

\[
\text{Minimize } \frac{1}{2} r^T W^{-1} r
\]

\[
\text{subjected to: } h(x) + s = 0
\]

\[g(x) = 0\]

\[r - z + f(x) = 0\]

\[s \geq 0\]

Where \(r = z - f(x)\) and \(s\) is a slack vector of slack variables used to convert the inequality constraints to equality constraints. Interior point method is used to solve the equations (4-8) by using logarithmic barrier function.

Recently, Least Absolute Value (LAV) PSSE with equality and inequality constraints have been proposed by Ali Abur et. al. [6]. A linear parameter estimation problem using Liner Programming (LP) is solved at each iteration of PSSE algorithm. The cost function to be minimized is
given by

\[ J(x) = \| z - f(x) \| \]  \tag{9}  

subjected to

\[ f_a(x) + e_a = z_a \]  \tag{10}  
\[ f_e(x) = z_e \]  \tag{11}  
\[ f_s(x) \leq z_s \]  \tag{12}  

Where \( h_a(x), h_n(x) \) and \( h_s(x) \) are the non linear functions corresponding to field measurements, error free measurements and bound measurements respectively. \( x \) is the state vector and \( e_a \) represents the errors in unconstrained measurements. vector \( z \) is given as

\[ z^T = [ z_a^T \ z_e^T \ z_s^T ] \]

where \( z_a \) is unconstrained measurements, \( z_e \) is the measurement with equality constraints and \( z_s \) is the measurements with inequality constraints.

Problem (9) is formulated as a LP problem by introducing residual variables and adding appropriate linear constraints on the LP variables. The LAV estimators provide a better approximation than LS estimators when the errors in the measurement set have unknown distribution and also when the sample size is small. Thus, the LAV estimators are expected to perform better than LS estimators in presence of bad data in the measurements set. This is due to the fact that LAV chooses only \( n \) measurements (number of nodes) from the over determined \( m \) measurements, where \( m \gg n \). Selection of \( n \) nodes is made iteratively to minimize the errors. These estimators have not been widely used because they require excessive memory space and computing time. However in on line applications, where the estimation has to be performed fast with acceptable accuracy, Least Squares based estimators are preferred. The next section details proposed inequality constrained PSSE, formulated as an inequality constrained least squares problem.

The LAV and interior point methods require indefinite linear system solvers and are slow; in this view, decomposition based power system state estimator [2, 1, 3] is extended to inequality constrained power system state estimator. Lagrangian multipliers are used to compute the sensitivity of the inequality constraints. Ordering strategy COP2 [7] for Given rotations based power system state estimator is implemented for inequality and equality constrained PSSE. Use of QR factorization approach for solving LS problems results in excellent numerical stability characteristics. Simulation results for 89 and 319 bus systems with various test cases are presented in section 3.

2. Decomposition based inequality and equality constrained PSSE

State estimation formulated by considering the generator Q limits becomes:

\[ \min J(x) = \frac{1}{2} [ z - f(x) ]^T W^{-1} [ z - f(x) ] \]  \tag{13}  

such that \( c(x) = 0 \)  \tag{14}  
\[ h(x)_{\text{min}} \leq h(x) \leq h(x)_{\text{max}} \]  \tag{15}  
and \( t_{\text{min}} \leq t \leq t_{\text{max}} \)  \tag{16}  

Where \( h(x)_{\text{min}} \) and \( h(x)_{\text{max}} \) are the lower and upper limits of the Q flows of the generators. \( h(x) \) is the computed Q flow from the estimated state. \( t_{\text{min}} \) and \( t_{\text{max}} \) are the minimum and maximum limits of the OLTC transformer taps. The non linear problem (13) with constraints (14, 15 and 16) is solved by solving the following linearized problem iteratively.

2.1 Linearization

\[ \min \frac{1}{2} \| [ W^{-1} ] [ H^k \Delta x^k - \Delta z^k ] \| \]  \tag{17}  

such that

\[ C^k \Delta x^k = \Delta c^k \]  \tag{18}  
\[ \Delta q_{\text{min}}^k \leq A^k \Delta x^k \leq \Delta q_{\text{max}}^k \]  \tag{19}  

Where \( k \) is the iteration index, \( C^k \) is the Jacobian of \( c(x^k) \) and \( A^k \) is the Jacobian of \( h(x^k) \). \( \Delta q_{\text{min}}^k \) and \( \Delta q_{\text{max}}^k \) are the lower and upper bound violation vector of generator Q flows. \( \Delta x^k \) and \( \Delta c^k \) are the error vectors corresponding to flow measurements and zero injections respectively. These vectors are computed as:

\[ \Delta x^k = z - f(x^k) \]  \tag{20}  
\[ \Delta c^k = 0 - c(x^k) \]  \tag{21}  
\[ \Delta q_{\text{min}}^k = h(x)_{\text{min}} - h(x^k) \quad \text{if} \quad h(x^k) < h(x)_{\text{min}} \]  \tag{22}  
\[ \Delta q_{\text{min}}^k = h(x)_{\text{max}} - h(x^k) \quad \text{if} \quad h(x^k) > h(x)_{\text{max}} \]  \tag{23}  
\[ \Delta q_{\text{max}}^k = 0.0 \quad \text{if} \quad h(x)_{\text{min}} \leq h(x^k) \leq h(x)_{\text{max}} \]  \tag{24}  

Constraint set defined by equation (15) contains all the inequality constraints, out of which some of the constraints are satisfied and other violated. The set containing violating constraints are called active set and they remain active. The active set contains the constraints which are either violated or fixed at the bounds due to violation in the previous iteration. Equation (19) is computed only for the active set of constraints. The Q flows of the generator which satisfies (15) are not included in computation of \( A^k \), \( \Delta q_{\text{min}}^k \) and \( \Delta q_{\text{max}}^k \).
Equations (17-19) are solved by formulating the objective function as:

\[
\min_{\Delta x^k, \mu^k, \lambda^k} L = \| \hat{H}^k \Delta x^k - \Delta z^k \|_2 - \mu^T (C^k \Delta x^k - \Delta d^k) - \lambda^T I (\Delta x^k)^T
\]  

(25)

Where:

- \( \mu = (\mu_1, ..., \mu_j)^T \) are Lagrangian multipliers for equality constraints, corresponding to zero-injection measurements.
- \( \lambda = (\lambda_1, \lambda_2, ..., \lambda_{2g})^T \) are the Lagrangian multipliers for inequality constraints (Q-injection limits on generators) and \( I(\Delta x^k) \) is given as:

\[
I(\Delta x^k) = \left[ \begin{array}{c} -A^k \Delta x^k \\ \Delta x^k \end{array} \right] \geq \left[ \begin{array}{c} q_{(1)\min} - q_{(1)\max} \\ q_{(2)\min} - q_{(2)\max} \\ q_{(g)\min} - q_{(g)\max} \\ -q_{(1)\max} + q_{(1)\min} \\ -q_{(g)\max} + q_{(g)\min} \end{array} \right]
\]  

(26)

For convenience 1 to \( g \) are assumed generators.

The Jacobian for \( h(x) \) due to generator is given as:

\[
A^k = \left[ \begin{array}{cccc} \overline{\delta q_{11}} & \overline{\delta q_{12}} & \cdots & \overline{\delta q_{1n}} \\ \overline{\delta q_{21}} & \overline{\delta q_{22}} & \cdots & \overline{\delta q_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\delta q_{n1}} & \overline{\delta q_{n2}} & \cdots & \overline{\delta q_{nn}} \end{array} \right]
\]  

(27)

Let

\[
f(\Delta x^k) = \| \hat{H}^k \Delta x^k - \Delta z^k \|_2
\]  

(28)

Substituting (28) in (25)

\[
\min_{\Delta x^k} L = f(\Delta x^k) - \mu^T (C^k \Delta x^k - \Delta d^k) - \lambda^T I (\Delta x^k)
\]  

(29)

The Kuhn-Tucker necessary condition for an optimal solution of a stationary point of the Lagrangian function is:

\[
L_{\Delta x^k}(\Delta x^k, \mu^k, \lambda^k) = \partial f_{\Delta x^k}^k - C^k \mu^k - A^k \lambda^k = 0
\]  

(30)

\[
\frac{\partial f_{\Delta x^k}^k}{\partial \Delta x^k} \text{ in equation (30) is given as:}
\]

\[
\frac{\partial f_{\Delta x^k}^k}{\partial \Delta x^k} = \left[ \hat{H}^{kT} \right] \left[ \hat{H}^k \Delta x^k - \Delta z^k \right]
\]  

(31)

Substituting (31) in (30)

\[
\hat{H}^{kT} \hat{H}^k \Delta x^k - \hat{H}^{kT} \Delta z^k - [C^{kT}, A^{kT}] \begin{bmatrix} \mu \\ \lambda \end{bmatrix} = 0
\]  

(32)

i.e.

\[
[C^{kT}, A^{kT}] \begin{bmatrix} \mu \\ \lambda \end{bmatrix} = \left[ \hat{H}^{kT} \hat{H}^k \Delta x^k - \hat{H}^{kT} \Delta z^k \right]
\]  

(33)

First \( \Delta x^k \) is solved using decomposition proposed in [1, 2, 3], which solves the LSE problem (17) and (18), then using (33) \( \lambda \) is computed. When some of the constraints are violating the bounds, they are fixed to the bounds by treating them as equality constraints. The constraints are fixed to the appropriate bound by assigning large weight \( (10^3) \) to the violating constraint. \( \lambda \) is computed only for the constraints which are in the active set. The value of \( \lambda \) is not used till all the constraints are either at bounds or within bounds. When none of the constraints are violating, then the computed \( \lambda \) for the active set (the constraints which are forced at the bounds) is used to relax the enforced constraints. The forced constraints are relaxed (freed) so that the relaxed constraint moves inside the bounds from the bounds. One constraint at a time is chosen for relaxing depending upon the value of \( \lambda \). The constraint which has most negative value of \( \lambda \) is chosen for relaxing at each iteration. The algorithm terminates when all the constraints are within bounds or some of the constraints are at bounds and cannot be relaxed or the algorithm diverges.

### 2.2 OLTC Transformer tap modeling

Taps of OLTC transformers are modeled as the state variable and the tap is estimated at each iteration from the branch power flow and power injections. The derivative of the flows with respect to the state variable (taps) are incorporated in the Jacobian matrix. The state variables are the voltages, angles and taps of the OLTC transformer. Once the state of the system is estimated including the tap position of the OLTC transformers, the tap value of each OLTC transformer is checked with the bounds. If the tap is violating the bounds then the state vector is corrected by setting the tap to the appropriate bounds. The flows are then computed based on the corrected state vector, which has the tap positions within the bounds.

The algorithm IECSE for inequality and equality constrained PSSE is given below:

**Algorithm IECSE**

1. Set iteration count \( k=0 \), initialize all voltages to 1.0 and angles to 0.0 and set \( \epsilon \) for convergence, max_iter_count and \( J^{-1} = 100.0 \)

2. Compute

\[
\hat{H}^k = \begin{bmatrix} H^k \\ C_k \end{bmatrix}, C_k, \Delta z^k \text{and} \Delta c^k
\]

3. Solve the LSE problem using the decomposition [1, 2, 3]

\[
H^k \Delta z = \Delta z^k
\]

such that \( C_k \Delta z^k = \Delta c^k \)
3 Test Results

Algorithm IECSE discussed in previous section is implemented to 89 and 319 systems. 89 bus system contains 12 generators and 319 bus system contains 22 generators. Bad data is simulated by introducing gross errors in the line flows and generator Q measurements. The results are obtained for both inequality constrained state estimation proposed in this paper as well for equality constrained state estimation (ECSE) proposed in [1, 2, 3]. Both the algorithms use COP2 ordering strategy proposed in [7] and are implemented in C++ on IBM RS6000 workstation.

3.1 89 Bus System

Both the ECSE and IECSE algorithms are executed without creating artificial bad data in the system. Both the algorithms converged in 4 iterations and has objective function value of 0.155083, the time taken by ECSE and IECSE are 0.43 and 0.58 seconds respectively. As there was no error in the data both the algorithms converged to the same objective function and to the same state. But IECSE algorithm has taken more time to converge due to computation of Lagrangian multiplier. The performance of the algorithms are observed on two cases of creating gross bad data in the systems.

3.1.1 Case 1

Forward Q flow in line 83-3 is changed from -104.863 MVAR to -304.863 MVAR.

The following results are obtained by ECSE and IECSE algorithms:

<table>
<thead>
<tr>
<th>Description</th>
<th>ECSE</th>
<th>IECSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converged Objective function</td>
<td>2.05719</td>
<td>1.74383</td>
</tr>
<tr>
<td>Total Execution time (secs)</td>
<td>0.44</td>
<td>0.64</td>
</tr>
<tr>
<td>Total number of iterations</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P and Q flow in line 83-3</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Measured flow</td>
<td>ECSE</td>
</tr>
<tr>
<td>------------------------------</td>
<td>--------------</td>
<td>--------</td>
</tr>
<tr>
<td>Forward P (MW)</td>
<td>-178.889</td>
<td>-175.107</td>
</tr>
<tr>
<td>Forward Q (MVAR)</td>
<td>-304.863</td>
<td>-156.061</td>
</tr>
<tr>
<td>Backward P (MW)</td>
<td>179.897</td>
<td>176.350</td>
</tr>
<tr>
<td>Backward Q (MVAR)</td>
<td>124.763</td>
<td>180.98</td>
</tr>
</tbody>
</table>

node 3 is a generator node and the Q flows in generator is given as:
| Lower limit  | 0.000 MVAR          |
| Upper limit  | 120.000 MVAR        |
| measured Q flow | 124.763 MVAR |
| By ECSE algorithm | 180.980 MVAR |
| By IECSE algorithm | 120.000 MVAR |

It can be observed from the results that the generator Q flow computed by algorithm ECSE is violating the upper bound by 60.98 MVAR whereas Q flow computed by IECSE algorithm is same as the upper bound. Also the backward Q flow of line 83-3 computed ECSE algorithm has an error of -56.217 MVAR, whereas algorithm IECSE has an error of -4.763 MVAR.

3.1.2 Case 2
In this case generator Q flow in generator 6 is changed from 90.623 MVAR to 290.623 MVAR.
The results obtained by ECSE and IECSE algorithms are given as:

<table>
<thead>
<tr>
<th>Description</th>
<th>ECSE</th>
<th>IECSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converged Objective function</td>
<td>1.64684</td>
<td>0.18802</td>
</tr>
<tr>
<td>Total Execution time (secs)</td>
<td>0.41</td>
<td>0.57</td>
</tr>
<tr>
<td>Total number of iterations</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

The flows at generator 6 is given as:

| Lower limit  | 0.000 MVAR          |
| Upper limit  | 100.000 MVAR        |
| Measured Q flow | 290.623 MVAR |
| True Q flow   | 90.623 MVAR         |
| By ECSE algorithm | 156.360 MVAR |
| By IECSE algorithm | 90.623 MVAR |

The results show that the Q flow computed using ECSE algorithm has an error of 56.360 MVAR compared to the upper bound and 66.360 MVAR compared to the true flow, whereas IECSE algorithm computes the Q flow same as the true flow.

3.2 319 Bus system
Both the ECSE and IECSE algorithms are executed without creating artificial bad data in the system. Both the algorithms converged in 4 iterations and has objective function value of 0.320472, the time taken by ECSE and IECSE are 9.78 and 10.50 seconds respectively. The performance of the algorithms are observed on two cases with simulated gross bad data in the systems.

3.2.1 Case 1
Forward Q flow in line 303-78 is changed from 129.698 MVAR to 329.698 MVAR.

The following results are obtained by ECSE and IECSE algorithms:

<table>
<thead>
<tr>
<th>Description</th>
<th>ECSE</th>
<th>IECSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converged Objective function</td>
<td>1.70451</td>
<td>2.05944</td>
</tr>
<tr>
<td>Total Execution time (secs)</td>
<td>4.99</td>
<td>5.17</td>
</tr>
<tr>
<td>Total number of iterations</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

P and Q flow in line 303-78

<table>
<thead>
<tr>
<th>Description</th>
<th>Measured flow</th>
<th>ECSE</th>
<th>IECSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward P (MW)</td>
<td>-228.438</td>
<td>-230.985</td>
<td>-228.090</td>
</tr>
<tr>
<td>Forward Q (MVAR)</td>
<td>-112.509</td>
<td>-165.575</td>
<td>-107.904</td>
</tr>
<tr>
<td>Backward P (MW)</td>
<td>228.902</td>
<td>231.884</td>
<td>228.800</td>
</tr>
<tr>
<td>Backward Q (MVAR)</td>
<td>329.698</td>
<td>187.128</td>
<td>125.000</td>
</tr>
</tbody>
</table>

Node 78 is a generator node and the Q flows in generator is given as:

| Lower limit  | -60.000 MVAR          |
| Upper limit  | 125.000 MVAR          |
| measured Q flow | 129.698 MVAR |
| By ECSE algorithm | 187.128 MVAR |
| By IECSE algorithm | 125.000 MVAR |

The generator Q flow computed by ECSE has an error of 62.128 whereas IECSE algorithm has fixed the Q flow to the upper bound. Node 303 is a zero injection node, the injections computed at node 303 is 0.00000 (up to 5 decimal places) in case of both ECSE and IECSE algorithms.

3.2.2 Case 2
The Q flow in measurement at generator 78 is changed from 129.698 MVAR to -129.698 MVAR

The results obtained by ECSE and IECSE algorithms are given as:

<table>
<thead>
<tr>
<th>Description</th>
<th>ECSE</th>
<th>IECSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converged Objective function</td>
<td>2.23438</td>
<td>0.332677</td>
</tr>
<tr>
<td>Total Execution time (secs)</td>
<td>4.96</td>
<td>5.29</td>
</tr>
<tr>
<td>Total number of iterations</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

The flows at generator 78 is given as:
Though the computed Q flow in both the algorithms are within the limits, the ECSE algorithm has an error of 124.4178 MVAR, whereas IECSE algorithm has computed the Q flow equal to the upper bound.

Gross errors are introduced in Q flows of the OLTC transformers for both 89 and 319 bus systems and found that the algorithm IECSE forces the taps of the OLTC transformers to be with the bounds by appropriately minimizing the error introduced in Q flows.

The results presented for four cases in total for 89 and 319 bus system show that the proposed algorithm IECSE performs better than ECSE in terms of the generator Q flow and has the same speed and stability characteristics. As both the methods are based on the decomposition [1, 2, 3] both enforce the equality constraints.

4 Conclusions

The proposed decomposition based equality and inequality constrained PSSE using COP2 ordering strategy is a numerically stable and fast implementation. The inequality constraints due to generator Q flows are strictly enforced and improves the quality of the state estimated. The line loading and unmeasured loads (within known range) can also be incorporated as an inequality constraint and can also be solved in the same framework. As the proposed algorithm is fast and numerically stable it is most suitable for on line applications of EMS in Energy Control Centers.

References


