THEORETICAL FOUNDATION OF INVESTIGATION OF EARTHQUAKE RESISTANCE OF STRUCTURES ON MODELS

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A b s t r a c t

The report comprehends the results of the theory of enlarged similarity of solid deformable bodies applicable to the modelling of structures investigated on seismic influences. It is proved that when a modelling material is used, that models the mechanical properties of the original fairly accurately, quite a reliable modelling is expected when dealing with problems of engineering seismology and the earthquake resistance of structures, including the stage of crack formation and failure. It is especially convenient to carry out the experimentation when there is equality of accelerations in the original and in the model since in this case there arises no more need of special charging or experimenting in an artificial field of gravity.

Let us keep to the following basic designations.

The primed system of marking is applied to models, while the unprimed - to the originals. The condition of geometric similarity is

$$\ell' = \alpha \ell \qquad (1)$$

where ℓ and ℓ' are homologous dimensions of the bodies A and A'. α — is the scale factor of the geometric similarity.

The scale factor for similarity for time is ξ, so that

$$t' = \xi t \qquad (2)$$

The conditions of similarity of the stresses σ and σ' at homologous points of the bodies A and A' and at homologous moments of time t and t' are

$$\sigma' = \beta \sigma \qquad (3)$$

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where β is the scale factor of similarity of the stresses.

The condition of similarity for the strains ε and ε' at homologous points of bodies A and A' and at homologous moments of time t and t' is

$$\varepsilon' = \gamma \varepsilon, \qquad (4)$$

where γ is the scale factor of similarity for strains.

The scale factor of similarity for densities is δ, so that

$$\rho' = \delta \rho \qquad (5)$$

It should be realised that in σ and ε all the components of stress and strain are involved.

The scale factor of similarity γ, introduced for strain, is a dimensionless value which should equal unity, according to the theory of dimensionality. Special studies revealed the fact that the scale factor can be taken as distinct from unity for small strains with the degree of accuracy corresponding to the classical theory of elasticity (Rocha M.1952, Nazarov A.G. 1957-1963). In large displacements, in contact problems with small area of contact and in problems of stability and dynamic stability, the scale factor γ must invariably be taken equal to unity.

The scale factor of similarity for times ξ we establish for such a class of cases when the ageing of bodies, creep, relaxation, shrinkage or swelling can be overlooked while dynamic charging is in progress. Practically speaking, in earthquakes these conditions are always met.

Two materials M and M' are called similar if at homologous moments of time t and t' the conditions (3) and (4) are fulfilled under any law of charging. The materials will be quite similar if the conditions (3) and (4) are abided by also in the yield point or failure. In this case the modelling of constructions in any stressed conditions is quite possible. In particular, the linearly elastic materials are similar if their Poisson's ratios ($\nu = \nu'$) are equal. The moduli of elasticity comply with the conditions:

$$E' = \frac{\beta}{\gamma} E, \quad G' = \frac{\beta}{\gamma} G \qquad (6)$$

where the ratio $\frac{\beta}{\gamma}$ can equal the arbitrary but fixed number.

According to Prandtle's law, two materials are similar if their Poisson's ratios are equal and

$$\beta = \frac{\sigma_s'}{\sigma_s} \qquad (7)$$

where σ_s and σ_s' are the yield point.

$$\gamma = \beta \frac{E}{E'} . \qquad (8)$$

In the general case of similarity of materials we have to be content with the approximate conditions (3) and (4), β and γ being determined on the basis of some extremal signs, say, by the method of least squares (Nazarov A.G.1962).

Two geometrically similar bodies A and A' are called similar if, at homologous points, they are made of similar materials M and M'.

The following is the emerging theorem.

If two similar bodies A and A', fixed in the same way, at homologous moments of time t and t' variable distribution forces with the tensity σ and $\sigma' = \beta\sigma$ and variable volumetric forces the intensity of which K and $K' = \frac{\beta}{\alpha} K$ are applied, then fulfilling the condition.

$$\xi = \alpha \sqrt{\delta \frac{\gamma}{\beta}} , \qquad (9)$$

at homologous moments of time and at homologous points of both bodies, the stresses are equal to σ and $\sigma' = \beta\sigma$ the strains are equal to ε and $\varepsilon' = \gamma\varepsilon$ the displacements are equal to $\bar{u}$ and $\bar{u}' = \alpha\gamma\bar{u}$, i.e. there exists a dynamic similarity.

Let us quote below a number of basic results from the condition of similarity.

1. If the bodies A and A' possess initial stresses σ_0 and σ_0' and initial strains ε_0 and ε_0' at moments of time t_0 and $t_0' = \xi t_0$ to ensure a similarity of their conditions the following requirements are to be met:

$$\sigma_0' = \beta \sigma_0, \qquad (10)$$
$$\varepsilon_0' = \gamma \varepsilon_0$$

2. The linear forces $\bar{q}$ and $\bar{q}'$, distributed along the homologous curves C and C' on the surfaces Σ and Σ' of the bodies A and A' should comply with the condition

$$\bar{q}' = \alpha\beta\bar{q} \qquad (11)$$

3. The concentrated forces $\bar{P}$ and $\bar{P}'$ applied to homologous points of bodies A and A' should comply with the condition

$$\bar{P}' = \alpha^2\beta\bar{P} \qquad (12)$$

4. The concentrated moments m and m' applied to homologous points of bodies A and A' comply with the condition

$$m' = \alpha^3\beta m \qquad (13)$$

5. If the boundary conditions for the body A are given in the displacements $\bar{u}$ of the surface points, to provide for a similar condition it is necessary to apply the displacements to the homologous points of the body A $\bar{u}' = \alpha\gamma\bar{u}$.

6. If the boundary conditions for the body A are partly given in the stresses σ and partly in the displacements $\bar{u}$, then the stresses $\sigma' = \beta\sigma$ and the displacements $\bar{u}' = \alpha\gamma\bar{u}$ are to be given in the homologous points of the body A'.

7. The deformed bodies A and A' retain in all respects the geometrical similarity only on condition that $\gamma = 1$. Therefore the above-given theorem holds good only for small displacements, when practically one can consider that the geometrical similarity of the bodies is not disturbed and the orientation of the active forces regarding the bodies A and A' remains virtually unchanged. In satisfying the condition $\gamma = 1$ the results of the basic theorem hold good for large displacements too, if the forces acting on the bodies A and A' change their orientation in the same way.

8. The coefficients of the coulomb friction for similar bodies should be identical.

9. The theorem applies also to compound similar bodies C and C' , i.e. to bodies made up of bodies A_i and A'_i of various mechanical properties under the following conditions:

a) the scale factors of similarity for all the bodies A'_i should be the same

b) the bodies A'_i should have the same position as the bodies A_i and conjugated in a way quite similar to the bodies A_i making up the body C.

10. While considering the contact problems when contact between the bodies is effected along points, lines or small areas, the conditions of similarity as put forth in point No.9 hold good for the case of the necessary requirement $\gamma = 1$

11. The conditions of similarity for the bodies A and A' remain also valid while considering the problems of stability and dynamic stability in case of the necessary requirement $\gamma = 1$.

12. If crack formation is not accompanied by physico-chemical processes and is basically mechanical by nature (which is the case for most of the building materials), all the conditions of similarity, dealt with above, remain valid; the cracks in similar bodies are located similarly, while the width of crack openings Δl and $\Delta l'$ comply with the condition

$$\Delta l' = \alpha \gamma \Delta l \qquad (14)$$

13. The velocities $\bar{U}$ and $\bar{U}'$ of homologous points of the bodies A and A' are related by the equation

$$\bar{U}' = \frac{\alpha \gamma}{\xi} \bar{U} \qquad (15)$$

14. The accelerations $\bar{W}$ and $\bar{W}'$ of homologous points of the bodies A and A' are related by the equation

$$\bar{W}' = \frac{\alpha \gamma}{\xi} \bar{W} \qquad (16)$$

15. If the resistance of the external medium related to the surface unit of the body A, is the function of velocity, i.e. $p = f(v)$ then for the body A' it must be

$$\frac{P'}{\beta} = f\left(\frac{\xi}{\alpha\gamma} v'\right) \qquad (17)$$

16. The velocities of propagation of wave disturbances a and a' of the bodies A and A' comply with the condition

$$a' = \frac{\alpha}{\xi} a \qquad (18)$$

A similar condition exists also in velocities of propagation of cracks in similar bodies.

The modelling is particularly convenient if an equality of accelerations in the systems of the original and the model is obtained. In fact, the study of the model can be carried out, in this case, in a natural field of gravity and without any charging which factors compensate the non-compliance of the volume weight of the model with the requirements of the theory of similarity.

In this case the conditions of similarity will be expressed as follows:

length $\ell' = \alpha \ell$
stress $\sigma' = \alpha\delta\sigma$
strain $\varepsilon' = \gamma\varepsilon$
density $\rho' = \delta\rho$
time $t' = \sqrt{\alpha\gamma}\, t$
mass $m' = \alpha^3\delta m$
modulus of elasticity $E' = \frac{\alpha\delta}{\gamma} E, \quad G' = \frac{\alpha\delta}{\gamma} G,$
linear charge $\bar{q}' = \alpha^2\delta\bar{q}$
concentrated charge $P' = \alpha^3\delta P,$ (19)
volumetric force $K' = \delta K,$
moment of force $m' = \alpha^3\delta m,$
displacement $\bar{u}' = \alpha\gamma\bar{u}$
velocity $\bar{v}' = \sqrt{\alpha\gamma}\,\bar{v}$
acceleration $\bar{w}' = \bar{w}$
crack opening $\Delta\ell' = \alpha\gamma\Delta\ell,$
velocities of strain propagation $a' = \sqrt{\frac{\xi}{\gamma}}\, a$
work related to the unit body volume $u' = \alpha\delta\gamma u$

Solving problems of earthquake resistance connected with large displacements or dynamic stability, it is necessary to take γ equal to 1.

It is interesting to note that when

$$\xi = \sqrt{\alpha\gamma} = 1 \qquad (20)$$

not only the accelerations are identical for both the model and the original, but the displacements and the velocities as well. In this particular case accelerogrammes, velocigrammes and seismogrammes are identical both for the model and the original.

When $\xi = 1$ the accelerogrammes for models coincide with those for the original at moments $t' = \xi t$, i.e. the accelerogramme for the model is deformed as compared to that for the original along the axis of time.

It can be deduced from what has been said that when similarity of materials is satisfactorily abided by, it is possible, in principle, to observe the similarity of condition of two similar bodies including the similarity of limited conditions along carrying capacities, strain and crack formation. The closer the relation between the original and the models gets to the relation between similar bodies, in accordance with their definition, the better the results yielded by carrying into effect such a possibility. As a rule, to achieve satisfactory results great difficulties are to be overcome in the technology of model making and the technique of experimentation. We have dwelt in detail on these problems in our studies. We should like to make a passing mention, in the present paper, of the selection of material for dynamic charge. The opening of the loop of hysteresis is of great significance to dynamic processes in building materials, as their internal friction depends largely on the area of the restricted loop. On the other hand, it is well-known that viscous resistance, contingent on the velocity of the strain, is of no significant value to ordinary building materials. In the case of exact similitude between the indicator curve of the materials of the original and the model, one indicator curve may be converted into another with the help of affine transformations.

$$\sigma' = \beta\sigma, \qquad \varepsilon' = \gamma\varepsilon \qquad (21)$$

In this case similarity holds good for mechanical processes as well. Yet the fulfilment of the condition (21) in the structure of the loop of the hysteresis for both materials remains practically an open question. It is advisable to make use, in the given case, of the results of Pissarenko who has

shown that the details of the structure of the loop of the hysteresis are of no value in the first approximation for the damping of the elastic free vibrations (Pissarenko G.C. 1955).

The decrement of the damping of vibrations is rather exactly characterized by the area, restricted by the loop of the hysteresis. Let u designate the maximum elastic energy in the case of cyclic charges applied to the unit of the volume of the material M, while the absorbed energy is marked by W; then we should have

$$U' = \beta\gamma U, \qquad W' = \beta\gamma W \tag{22}$$

for the material M'.

Thus the coefficients of absorption ψ of the materials M and M' coincide and are equal to

$$\psi = \frac{W'}{U'} = \frac{W}{U} \tag{23}$$

hence the coincidence of the decrements of damping. That is why the following simplified criterion of similarity between the materials can be suggested. Beams are to be made of the materials M and M'. Scale factor similarities β and γ are to be obtained for them by means of direct testing. Next, free vibrations are applied to those beams for which they are to be suspended and given shocks. If the decrements of damping of their free vibrations are identical, they comply with the conditions of similarity.

If axial stresses prevail in the modelled construction, it is better to make the samples in the form of beams and subject them to bending and, if possible, to longitudinal free vibrations. If, however, displacement strains prevail in the construction, it is better to give the samples the form of a cylinder or a hollow cylinder and subject them to free torsional vibrations. This criterion of similarity is fairly practical, especially for uniaxial stressed condition. We do not dwell here on the problem of energy absorption in compound stressed condition as we are unaware of the experimental studies, at least, from engineering standpoint. We have discussed a number of theoretical considerations in the work (Nazarov A.G., 1959).

In case when the modelling material displays lesser damping properties, the possibility of engaging an additional damping device, compensating the insufficiency of energy absorption, is not ruled out.

In such investigations it should also be borne in mind that the possibility of much more viscosity for the model material, especially of low modulus, than for the original material, is not ruled out. In this case damping may occur in the model not only through static hysteresis but through viscosity of the material as well.

In that case the decrement of damping of free vibrations of samples from the model material will augment with the increase of the frequency of vibrations. It is therefore necessary to try to get nearly identical decrements of damping, at least within the range of their basic frequencies of free vibrations.

In alternating charges in the sphere of elastic-plastic strains a direct establishment of similarity of the materials is much simpler in the sense of experimentation technique since this enables us to make a direct comparison of the forms of the loops of hysteresis and their areas.

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