STUDY OF THE OVERTURNING VIBRATION OF SLENDER STRUCTURES

By

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INTRODUCTION

The aseismic design of structures in Japan is carried out by applying a statical force horizontally to the structure. In the case of some slender structures it is found often that the overturning moment due to the design seismic force is larger than the moment of stability of the structure. Such cases require special consideration in order to prevent the overturning of the structure. This special consideration is usually met by increasing the width of the foundation, anchoring the foundation to piling, or some other acceptable method.

The actual seismic force application to the structure is not a statical force but a dynamical force. The overturning of large structures during even severe earthquakes are unheard-of phenomena. Therefore, we found it advisable to make a dynamic study of the overturning moment problem so as to determine at what condition structures overturn and also to derive some useful data for aseismic design.

This paper describes the dynamic testing of models of slender structures on an elastic foundation and the application of the resultant findings to actual structures using a law of similarity which was proved by the study.

MODELS

The elastic model of a single mass vibrating system was used, see Fig. 1. In the figure the mass W_1 is supported by the flexural bar W_2 which is fixed to a foundation plate W_3. This plate has mild-steel knife edges.

The study of the vibrational behavior of the elastic model is enhanced by comparison with a rigid model. Accordingly, the elastic model was stiffened by use of wire stays and used as the rigid model.

Fig. 2 shows the translation from the uniform structure to a one-mass system. The gravity center of the upper two-thirds of the uniform structure was selected as the replacement mass point in the one-mass system.

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By using such a model we can reproduce any scale of actual structures. For example, if the scale ratio of the model is selected as 1/25, from a law of similarity, as explained hereinafter, the actual corresponding structure has the following physical characteristics:

Height	30 meters
Fundamental period	1 second

The foundation plate shown for the model can be adjusted to a greater or less width. The widths used were 12 cm., 18 cm., 24 cm., and 28 cm. which resulted in equivalent values of k_s, ratio of the half widths of the foundation plate to the height of the gravity center of the model, of 0.15, 0.23, 0.32 and 0.38, respectively.

For the base sheet, rigid, hard or soft ground was represented by use of wood, hard rubber or soft rubber plates, respectively. As a consequence there were four elastic type models, four rigid type models and three kinds of ground giving a total of 24 combinations for experimental use.

Tables 1 and 2 give the model's constants and the symbols used.

RESULTS OF EXPERIMENTS

Free Vibration. This was secured by giving the top of the model an initial horizontal displacement by hand and then loosing it abruptly. The free vibrational displacement of the mass point was recorded on a smoked rotating drum by means of a stylus connected to the model by spring retained cotton cord.

Several examples of the free vibration records are shown in Fig. 3. Their initial displacements are about 30 percent of the statical overturning amplitude A_o which is defined as the displacement of the model when its gravity center comes just vertically above the edge of the foundation plate. This edge is also regarded as the rotation center about which the foundation lifts-up.

Model number 2-A-1, elastic model B = 18cm. on a wood base sheet.

It was found in these tests that the fundamental period of the elastic model with the foundation fixed rigidly was 0.193 sec.

The free vibration has such damping characteristics that the amplitude and period of each wave cycle decreases and converges to 0.193 sec. A small vibration of higher order was found with a period of 0.043 sec.

Model number 2-A-2, rigid model B = 18cm. on a wood base sheet.

The free vibration of this model is similar to that of the elastic model. However, the higher order vibration noticed for the elastic model did not exist.

Model number 2-B-1 and 2-B-2, elastic and rigid models, respectively, B = 18 cm. on a hard rubber base sheet.

The free vibration in these two cases is damped vibration and is similar to those described for the model on the wood base sheet. The recorded wave forms are smooth even in the case of the elastic model. The periods converge to 0.348 sec. and 0.312 sec. for the elastic and rigid models, respectively.

The relationships between the period and the amplitude is shown in Fig. 4. The dashed curves show the theoretical values for lift-up vibration of the foundation about the rotation center for a rigid body on a rigid base. The value A_r shows the displacement coincidental to the instant of foundation lift-up obtained by statical experiment.

In the case of the rigid model (2-A-2) on the wood base sheet, the experimental results agree very well with the theoretical results. In the case of the elastic model (2-A-1) on the wood base sheet, the experimental results also agree well with the theoretical results.

In the cases of the elastic and rigid models (2-B-1 and 2-B-2) on the hard rubber base sheet, the experimental results agree well with the theoretical results at amplitudes larger than A_r. As the amplitude decreases the period converges to a constant value; which value shows the period of vibration of the models on an elastic base sheet and coincides with the fundamental period obtained by analysis statically effected.

Observing these results, it can be noted that the displacement of the models at the point of lift-up coincides closely for the dynamical and statical conditions.

Forced Vibration. The forced vibration experiments use simple harmonic motion imparted by a shaking table. Keeping the amplitude of vibration of the table constant and slowly changing its period from long-to-short or short-to-long, the stationary vibration of the models were recorded on the smoked drum in the form of the relative vibration between the model mass point and the shaking table.

Response curves. Selecting five table double amplitude values, namely, 1.5mm., 3.0mm., 6.0mm., 12.0mm. and 25mm., the period of the table was changed gradually from 0.1 sec. to 0.9 sec. The response curves are shown in Fig. 5.

Model number 2-A-1, elastic model B = 18 cm. on wood base sheet.

In this instance the double amplitude of the table, $2a_0$, is 1.5 mm. The period is changed from long-to-short and then from short-to-long. The response of the mass point is different for these two period changing cases.

Period Changed From Long-To-Short. See Fig. 5.

The curve AB in the Figure shows the elastic vibration with the model phase the same as that of the table (phase angle 0° - 90°). This curve coincides with that plotted from the theoretical analysis, as given hereinafter.

At point B the phase angle of the vibration became about 90 degrees and the foundation commenced rocking up from the base sheet. The maximum amplitude of rocking vibration with separation between foundation and base sheet is represented by point C in the Figure. As the model phase reversed to that of the table (phase angle 90° - 180°), the vibration converged to the stationary state of rocking vibration represented by point D. This type of rocking vibration with separation of foundation and base sheet continued to point D'. The curve DE' traces the elastic type vibration which again occurred.

Period Changed From Short-To-Long. See Fig. 5 also.

The curves ED' and D'DF show the elastic and rocking vibration with foundation separation from base sheet, respectively. At point F the phase angle difference became about 90 degrees and the rocking vibration occurred and then converged to the stationary elastic state within the same phase.

Model number 2-A-2, rigid model B = 18 cm. on a wood base sheet.

The response curve is generally like that for the model 2-A-1 described above, except that the response between A and B is about zero. The portion of the curve corresponding to the reverse phase vibration coincides closely to the curve plotted from the theoretical analysis as given hereinafter.

Model number 2-B-1 and 2-B-2 on a hard rubber base sheet.

The shape and characteristics of the response curves are similar to those obtained for the models upon the wood base sheet. The dashed line, shown in Fig. 5 and designated as the backbone curve, was obtained from the results of the free vibration tests. This backbone curve can be confirmed also from the theoretical analysis. In every case, the response curves developed on both sides of the backbone curve are separated by the backbone into the portions representing same phase on one side and reverse phase on

the other. In addition, rocking vibration with foundation and base sheet separation occurs in the vicinity of the backbone.

Lift-Up Point. (Point of separation between base sheet and foundation).

In order to study in more detail the vibration in which lift-up occurs, the double amplitude of the shaking table was varied from zero to 60 mm. The models reaction on the wood base sheet and the relationships between the period and the amplitude of vibration of the shaking table at which lift-up vibration starts is shown in Fig. 6.

As the amplitude of the shaking table gets large, the curves approach the chained lines k. These k lines represent the statically overturning coefficients on each model on this base sheet. Multiplying the chained line value of acceleration by the mass of the model, gives the force which causes statical overturning of the model. Along the curve BC, the lift-up vibration diverges and the model overturns. The curve ABC for the case of the model on the wood base sheet can be represented by the following formula.

$$\frac{a_0}{\varphi^2 - 1} = \text{const.} \qquad (1)$$

Where: $\varphi_1 = T_0 / T_1$

a_0 = Amplitude of the table

T_0 = Period of the table

T_1 = Fundamental period of model before foundation-base sheet separation (lift-up).

The form of equation (1) shows the value of the response curve. This equation means that the lift-up occurs when the elastic deformation of the model becomes equal to a certain value; which certain value is equivalent to the overturning moment of the model.

Table 3 shows the value of the constants of equation (1) multiplied by the coefficient of the normal function βu and the relative amplitude A of elastic vibration of the 1st order obtained by analysis when the moment at the foundation of the model becomes equal to the statically overturning moment. The good agreement between them means that the jump from the same phase to the reverse phase occurs when the moment at the foundation of the model becomes equal to the statically overturning moment. The curve DE in Fig. 6 shows the lift-up vibration from the reverse phase to the same phase.

The Maximum Amplitude During Lift-Up Vibration

The maximum amplitude, A_{max}, during the lift-up vibration from the same phase to the reverse phase is related to the amplitude, a_o, of the shaking table as shown in Fig. 7. The value of A_{max} is expressed in terms of the statically overturning amplitude, A_o, and that of a_o is expressed in terms of the minimum amplitude, a_{oL}, to overturn the model. By choosing those terms, all the results of experiments can be plotted to a certain curve.

The empirical formula concerning the upper limit of the amplitude was found to be:

$$\frac{A_{max}}{A_o} = \left(\frac{a_o}{a_{oL}}\right)^{\frac{5}{7}} \tag{2}$$

The maximum amplitude, $\bar{A}_{max}$, during the lift-up vibration from the reverse phase to the same phase is related not only to the amplitude of the shaking table but also to the statically overturning coefficient k as shown in Fig. 8.

The following empirical formulas were obtained.

$$\bar{A}_{max} = 0.046\ k^{-1.8} \cdot a_o^{0.7} \quad \text{elastic model}$$
$$\bar{A}_{max} = 0.055\ k^{-1.8} \cdot a_o^{0.7} \quad \text{rigid model on wood base sheet} \tag{3}$$

Overturning Vibration. When the amplitude of the shaking table exceeded a certain value during the lift-up vibration progression from the same phase to the reverse phase, overturning of the model occurred. During experiments of the several types of models on the wood base sheet, the minimum amplitude, a_{oL}, of the shaking table to just overturn the models had a linear relationship to the statically overturning coefficient, k_s, as shown in Fig. 9. The following empirical formula was obtained.

$$a_{oL} = 9.2\ K_s \ (\text{cm.}) \tag{4}$$

From Fig. 7, it is supposed that the overturning of the model occurs when the value of the table acceleration expressed in terms of gravity acceleration, g, becomes almost equal to the statically overturning coefficient on an elastic base sheet, k. Then the following equation is derived:

$$K \cdot g = a_{oL} \left(2\pi / T_{oL} \right)^2 \tag{5}$$

From this is derived:

$$T_{oL} = 2\pi\sqrt{a_{oL} / K \cdot g} = 2\pi\sqrt{9.2\ K_s / K \cdot g} \tag{5'}$$

Since the value of k is almost equal to that of k_s, in the case of the elastic model on the wood base sheet, we find that the overturning period T_{oL} is:

$$T_{oL} = 0.61 \text{ sec} \tag{5''}$$

The values from the experiments are shown in Table 4. It appears that they are in good agreement with the values from use of equation (5'').

We assume, from this good agreement of results, that the empirical formula (4) and our other assumptions defines the actual behavior fairly well. During these experiments we did not find any cases of overturning occurring during the reverse phase. This suggests that overturning during the reverse phase is difficult and not to be expected.

ANALYSIS OF VIBRATION OF RIGID MODEL ON A RIGID BASE SHEET

The rigid model on the rigid (wood) base sheet was selected for theoretical analysis as the response curve for this model and the elastic model on the wood base sheet are very similar.

Assumption. We assume that there is no slip between the rigid model and the base sheet, so that horizontal movement of the point O shown in Fig. 10 does not occur.

Equation of Vibration. The equation of vibration is found as follows:

$$m r_o^2 \frac{d^2\theta}{dt^2} + mgr\sin(\alpha-\theta) = -mr\cos(\alpha-\theta)\frac{d^2y_o}{dt^2} \tag{6}$$

The first term and the second term of the left side and the terms of righd side shows the inertia force, the restoring force and external force, respectively.

The maximum value of α shown in Fig.10, is 0.38 in this study. We can put $\sin(\alpha-\theta) \fallingdotseq \alpha-\theta$, $\cos(\alpha-\theta) = 1$ in equation (6), for the difference between α and $\sin\alpha$ is 2.3% and that between unity and $\cos\alpha$ is 6.3% at their limits. Then the equation of vibration can be simplified as follows:

$$mr_o^2\frac{d^2\theta}{dt^2} + \left(-\frac{M_o}{\alpha}\theta \pm M_o\right) = -mr\frac{d^2y_o}{dt^2} \tag{7}$$

when $\theta \gtrless 0$ and where $M_o = mgr\alpha$ is the statically overturning moment.

The relationship between the restoring force and the rotation angle θ is shown in Fig. 11. The restoring force in moment takes the maximum value at $\theta = 0$, which coincides with the statically overturning moment M_o: decreases as θ increases with a linear relationship, and takes a negative value for $\theta > \alpha$. We assume that

the restoring force takes a value from M_0 to $-M_0$ discontinuously at $\theta = 0$.

Free Vibration. Putting $n^2 = M_0/mr_0^2\alpha = r\delta/r_0^2$ the following equation for free vibration is obtained from equation (7):

$$\frac{d^2\theta}{dt^2} - n^2\theta + \alpha n^2 = 0$$

and the general solution is found as follows:

$$\theta = \alpha + A\cdot\sinh(nt) + B\cdot\cosh(nt) \tag{8}$$

Consider the following conditions:

$$\begin{aligned} t = 0 &: \quad \frac{d\theta}{dt} = 0 \\ t = T/4 &: \quad \theta = 0 \end{aligned} \tag{9}$$

Eliminating A and B from equations (8) and (9), the period T and rotation angle θ are as follows:

$$T = \frac{4}{n}\cosh^{-1}\frac{\alpha}{\alpha - \theta_{max}} \tag{10}$$

$$\theta = \alpha\left(1 - \cosh nt / \cosh\frac{nT}{4}\right)$$

$$0 \leq t \leq T/4$$

$$\theta_{max} = \left(1 - 1/\cosh\frac{nT}{4}\right)$$

Forced Vibration: Here we consider the stationary vibration as forced by a simple harmonic ground motion y_0.
Putting $y_0 = a_0 \cos pt$, and using n as in the case of free vibration, equation (7) becomes:

$$\frac{d^2\theta}{dt^2} - n^2\theta \pm \alpha n^2 = \frac{ra_0}{r_0^2}p^2\cos pt \tag{11}$$

The general solution of equation (11) is:

where $$\theta = (\pm\alpha + A\sinh nt + B\cosh nt) - \frac{ra_0}{r_0^2}\cdot\frac{1}{1+\varphi^2}\cos pt \tag{12}$$

When the ground motion y_0 and the rotation angle θ are both in the same phase or reverse phase in a system with no damping and the curve of the restoring force versus the rotation angle is symmetrical about the Y-axis, then we may consider the vibration as follows:

Forced vibration of the same phase. Putting:

$$t = 0 : \quad \frac{d\theta}{dt} = 0 \qquad (13)$$
$$t = \pi/2p : \quad \theta = 0$$

and taking the positive sign solution in equation (12), we find the stationary solution as:

$$\theta = \alpha\left(1 - \frac{\cosh nt}{\cosh \frac{\pi}{2}\varphi}\right) - \frac{ra_0}{r_0^2}\cdot\frac{1}{1+\varphi^2}\cos pt \qquad (14)$$
$$\theta_{max} - \bar{\theta}_{max} = -\frac{ra_0}{r_0^2}\,\frac{1}{1+\varphi^2}$$

where $\bar{\theta}_{max}$ is the value of the amplitude obtained by substituting the period of ground motion for the period of free vibration in equation (10).

Forced vibration of the reverse phase. The stationary solution in the reverse phase can be obtained in the same manner, as follows:

$$\theta = -\alpha\left(1 - \frac{\cosh nt}{\cosh \frac{\pi}{2}\varphi}\right) - \frac{ra_0}{r_0^2}\,\frac{1}{1+\varphi^2}\cos pt \qquad (14')$$
$$\theta_{max} - \bar{\theta}_{max} = \frac{ra_0}{r_0^2}\,\frac{1}{1+\varphi^2}$$

There is a possible solution where $\theta = 0$. Also, a higher order form of subharmonics with special wave forms exists. The solution of the reverse phase is stable and the same phase unstable.

Response Curves. The response curves are shown in Fig. 12. The upper part and the lower part of the backbone curve represents the vibration of the reverse phase and the same phase, respectively. In the vibration of the reverse phase the amplitude exceeds , that is, it enters the area of the negative restoring force.

The response curves of the model from this analysis are shown in the case of 2-A-2 in Fig. 5. They are considered as coinciding with the experimental results.

APPLICATION TO ACTUAL STRUCTURES

In this study on lift-up vibrations and the overturning phenomenon, small scale models were used. The results of experiments were confirmed theoretically, as shown hereinbefore. In order to extend the results to actual structures, we introduce a law of similarity:

Refer to Fig. 13. Case 1 of Fig. 13 shows that with models in which the height remains constant, changing the width of the foundation results in no change in the period of the overturning moment.

This theoretical law of similarity is verified by the results of experiment. Therefore, we can confirm and use this law of similarity in general cases.

Case 2 in Fig. 13 shows extension from the test results to actual structures, namely, where both the width and height of structures change simultaneously. In this general case, we find a law of similarity such that the amplitude is proportional to the scale ratio of structures and the period is proportional to the square root of the scale ratio. For example, by this law we get the necessary limiting ground motion to overturn 30m height structures in the same phase of vibration, considering that the scale ratio of the models is 1/25, as follows:

$$a_{oL} = 9.2 \times 25\ k_s \quad \text{(cm.)}$$

$$T_{oL} = 0.61 \times 25 = 3.1 \text{ (sec.)}$$

There have been no overturning phenomena occur in the experiments in the reverse phase vibration. However, assuming that the overturning occurs when A_{max} in formula (3) becomes equal to the statically overturning amplitude, we get the ground motion necessary to overturn the 30m height structure as shown in Table 5.

It should be recognized that the ground motions necessary to overturn structures are of very long periods and large amplitudes. These ground motions cannot be expected in actual earthquakes.

CONCLUSIONS

In this study we conducted research on the lift-up vibrations of several elastic and rigid models. By analyzing the results of these experiments theoretically, we can establish a law of similarity as follows:

"WITH THE SCALE RATIO OF SIMILARITY DESIGNATED BY S, THE LAW FOR PERIODS IS PROPORTIONAL TO THE SQUARE ROOT OF S AND FOR AMPLITUDES IS PROPORTIONAL DIRECTLY TO S."

When we refer to structures 30m in height, the periods and the amplitudes of the ground motion used are equivalent to 0.5 sec. to 4.5 sec. and 0cm. to 75cm., respectively.

We arrived at the following conclusions concerning lift-up vibration.

1. Lift-up vibration can be analyzed by assuming that the restoring force is a maximum at the beginning of lift-up and zero when the gravity center comes vertically over the center of foundation rotation, as shown in Fig. 11.

2. The response curve is separated into two portions according to the period value of the shaking table. One part is the vibration in the same phase and the other is the vibration in the reverse phase (model to table).

3. The response curve obtained by theoretical analysis, using the restoring force versus the displacement gives a curve as shown in Fig. 11, which curve coincides with the experimental results.

4. When the moment at the base of the structure, in the same phase of vibration, becomes equal to the statically overturning moment, the vibration changes to the reverse phase. During this vibration phase change, larger amplitudes than those of the stationary vibration in the reverse phase, appear transiently.

5. During the vibration change from the same to the reverse phase, the overturning phenomemon occurs. The amplitude of ground motion which is necessary to make the structure overturn is proportional to the ratio of the width of base to height of model. The minimum overturning value of amplitude can be found as follows:

The minimum overturning amplitude	$a_{oL} = 9.2\ k_s$	(cm.)
The period	$T_{oL} = 0.61$	(sec.)

Extending the results of the experiments to the h actual meter height of structures by the law of similarity, we find.

$$a_{oL} = 7.7h \cdot K_s \qquad \text{(cm.)}$$

$$T_{oL} = \frac{1}{5}\sqrt{7.7\,h\,K_s/K} \qquad \text{(sec.)}$$

From these experiments and studies, it would appear that slender buildings, similar to the standard, multi-story, ferro-concrete apartment buildings in Japan, will not overturn easily in any earthquakes of the same magnitude as those experienced in the past.

ACKNOWLEDGEMENT

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NOMENCLATURE

Lift-up vibration. Vibration during which the foundation rocks-up or lifts-up from the base sheet about a rotation point (definitively shown in Fig. 10) causing a separation between foundation and base sheet.

A Amplitude of elastic vibration of model just before point of lift-up, obtained by theoretical analysis.

A_{max} Maximum amplitude during the lift-up vibration from the same phase to the reverse phase in the case of forced vibration.

A_{max} Maximum amplitude during the lift-up vibration from the reverse phase to the same phase in the case of forced vibration.

A_o Statically overturning amplitude.

A_r Displacement of model mass point just before point of lift-up obtained by statical experiment.

a_o Amplitude of shaking table.

a_{oL} Minimum amplitude of shaking table to just overturn the models.

α Angle between OG and vertical line normal at rest position, see Fig. 10.

B Width of foundation plate.

βu Coefficient of the normal function of elastic vibration.

d Ratio of total energy of half wave to that of the next.

h Total height of actual structures, in meters.

k Statically overturning coefficient on each base sheet.

k_s Statically overturning coefficient on the rigid base sheet (k_s is B/H) of a uniform monolithic body, where B and H are width and height, respectively of the body.

m Mass of the rigid body, see Fig. 10.

r Distance between gravity center G and either edge of balanced foundation plate B, see Fig. 10.

r_o Radius of gyration about O, see Fig. 10.

1/S Scale ratio of model to actual structures.

T Period of lift-up vibration during free vibration test.

T_1 Fundamental period of model before lift-up occurs.

T_o Period of shaking table.

T_{oL} Period of shaking table when its amplitude is a_{oL}.

$W_1 = Al$ Weight of the supporting flexural bar of model.

W_2 Weight of the mass point of model.

W_3 Weight of foundation of model.

y Displacement relative to shaking table.

y_0 Horizontal vibration of shaking table.

θ The rotation angle of rigid body.

θ_{max} The maximum rotation angle.

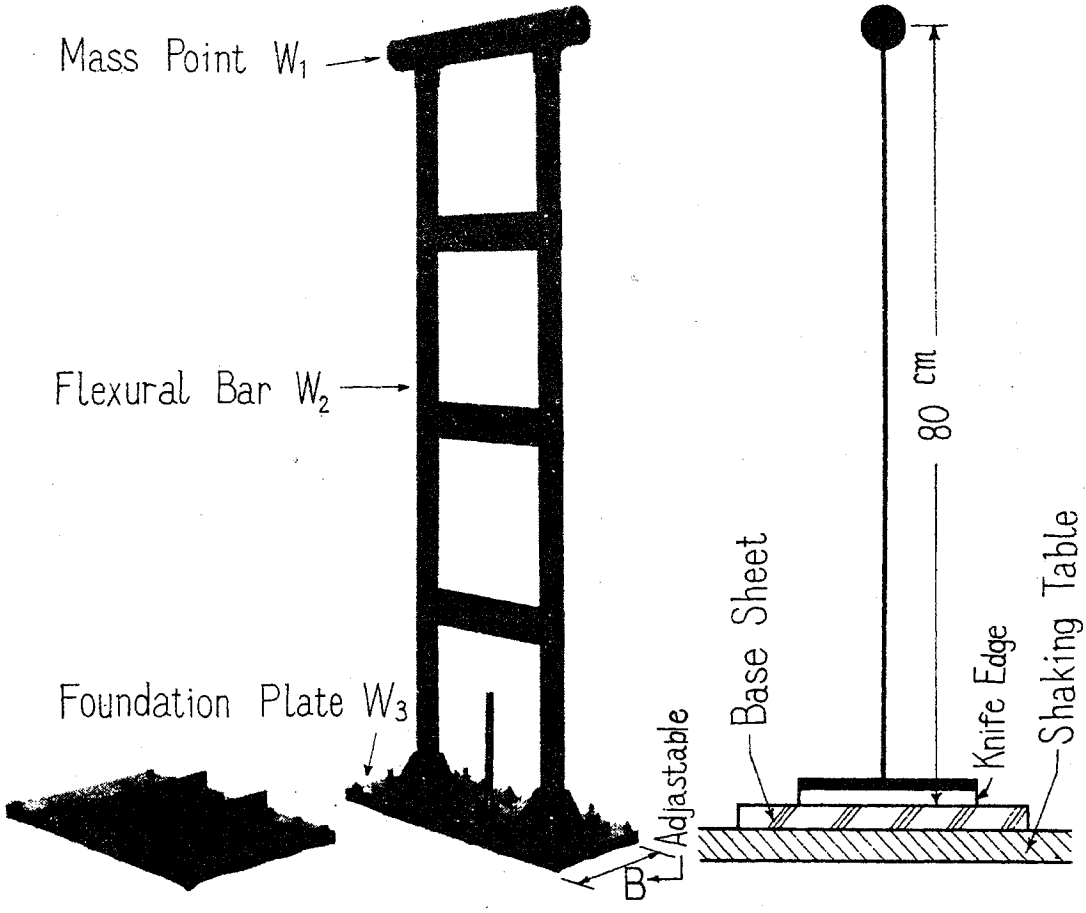


Fig. 1 Model

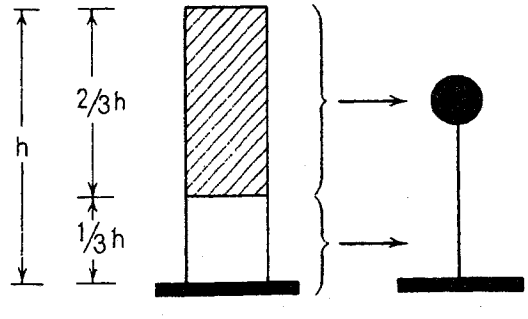


Fig. 2 Model of structural equivalence

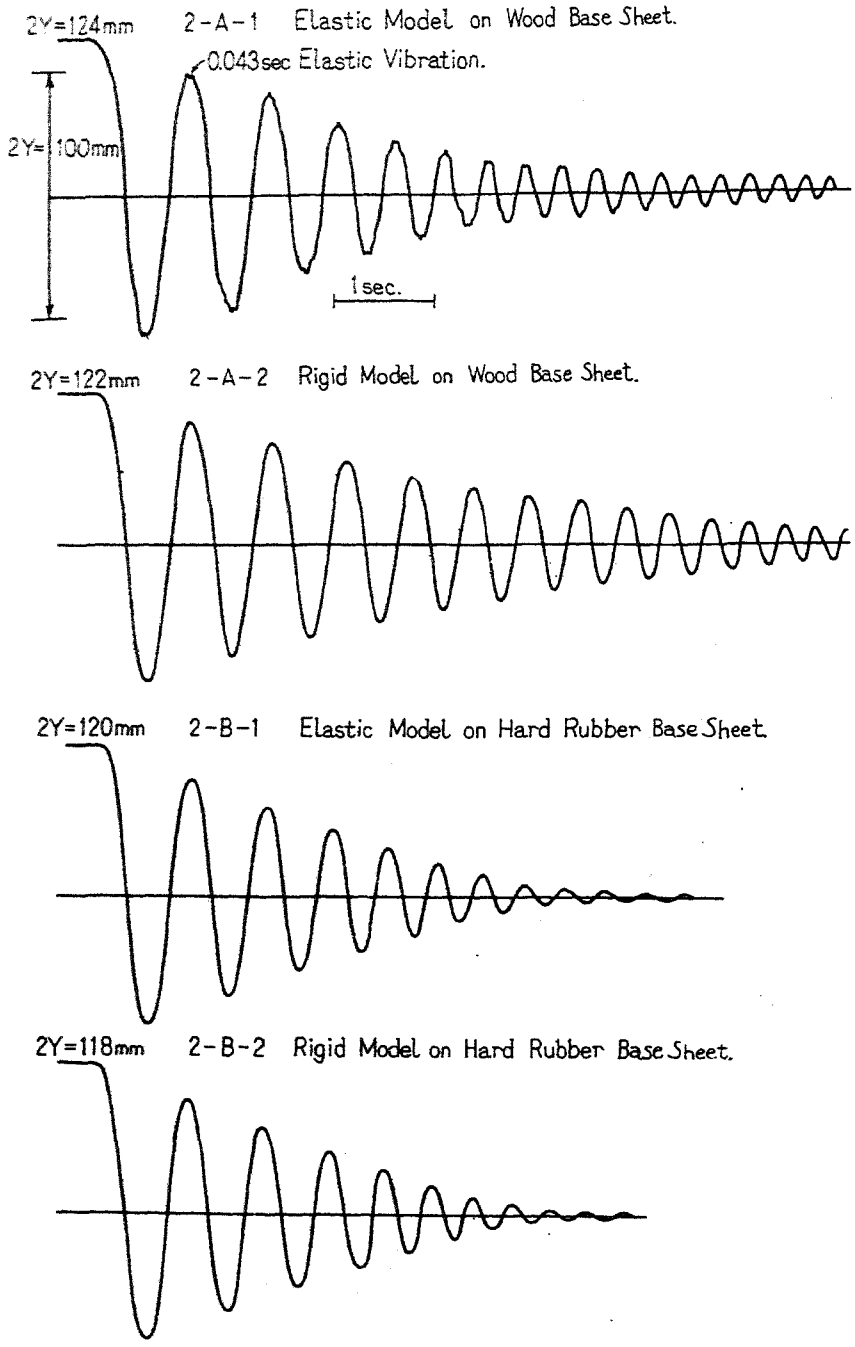


Fig. 3 Examples of free vibration records

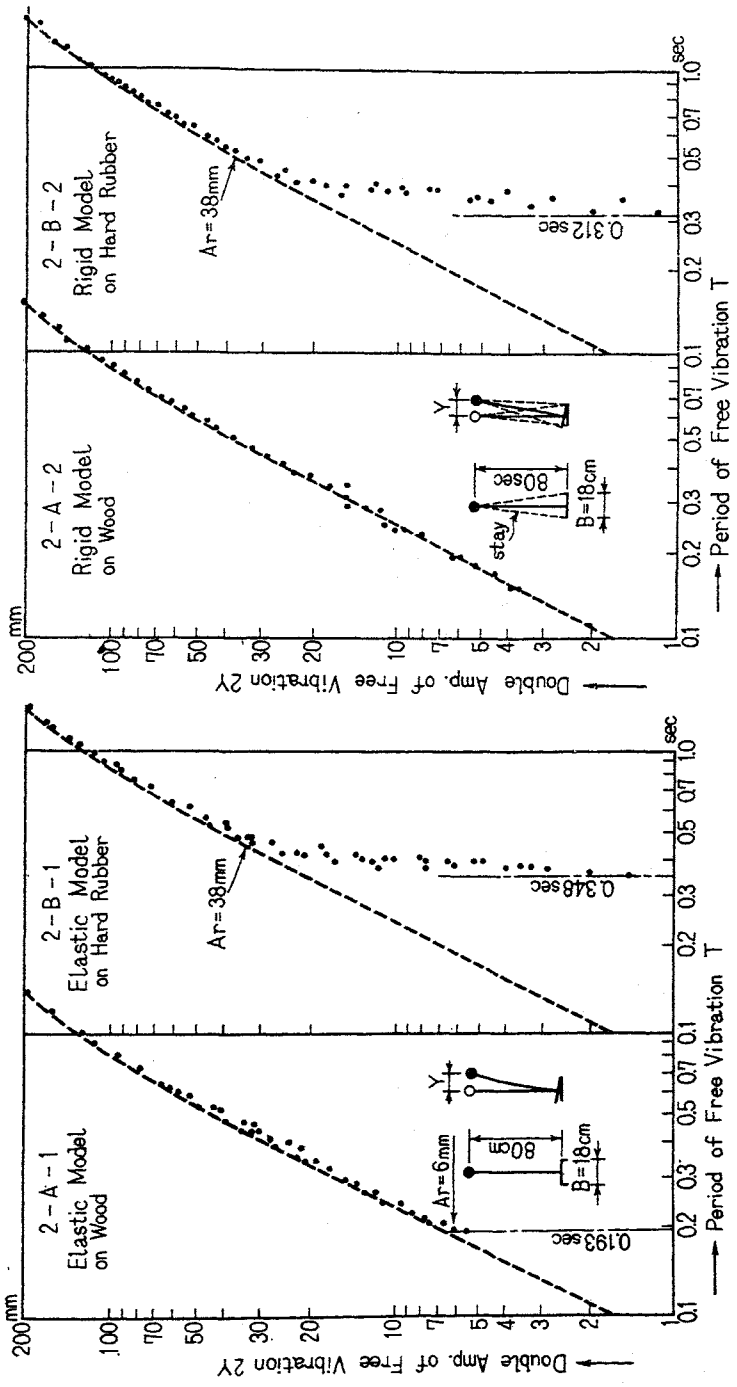


Fig. 4 **Period-amplitude curves of free vibration**

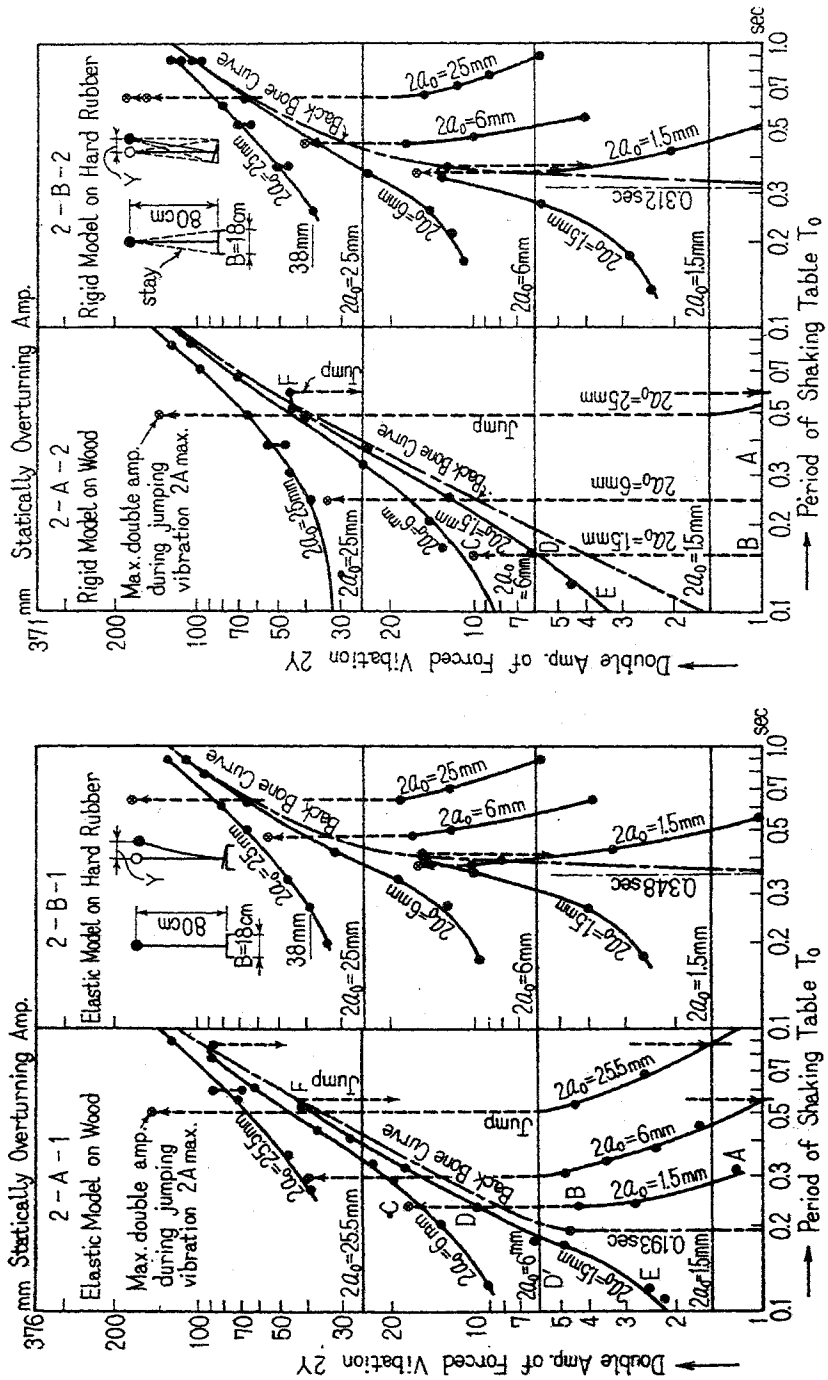


Fig. 5 Response curves

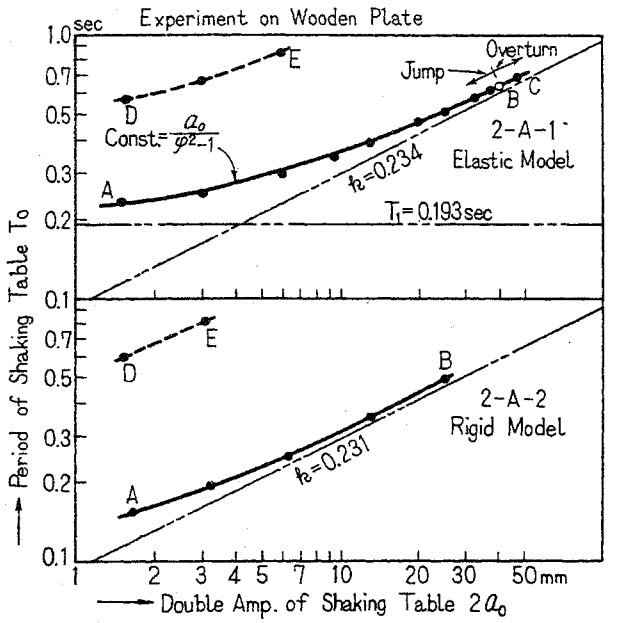


Fig. 6 **Period-amplitude curves of shaking table at lift-up point**

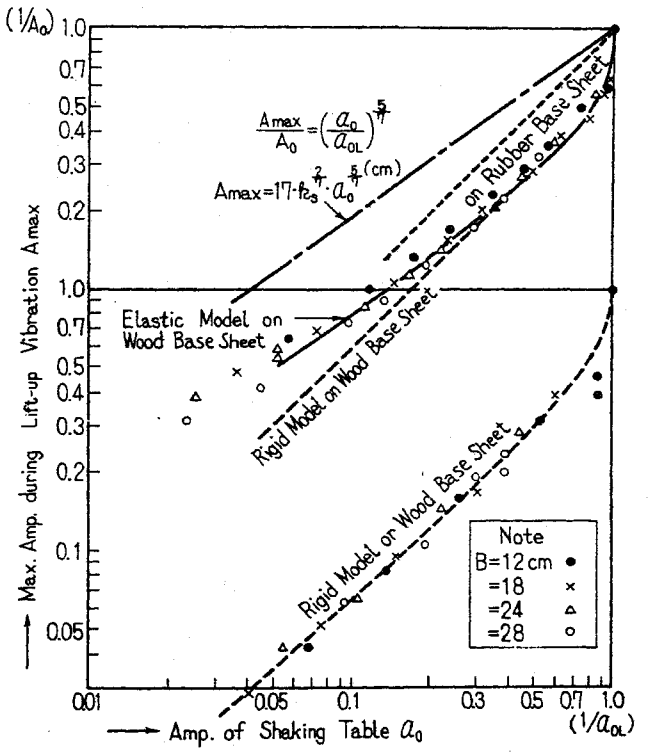


Fig. 7 **Maximum amplitude during lift-up vibration from the same phase to the reverse phase**

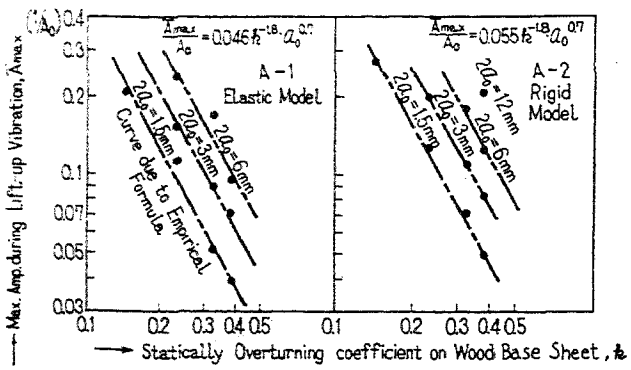


Fig. 8 Maximum amplitude during lift-up vibration from the reverse phase to the same phase

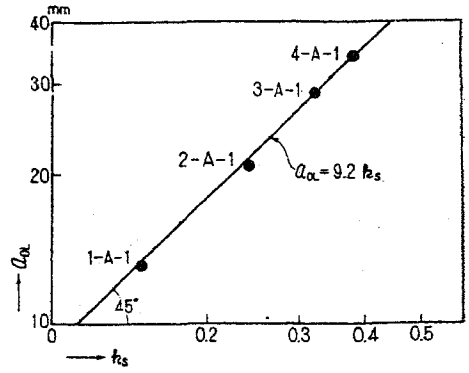


Fig. 9 Minimum amplitudes of shaking table to overturn the model

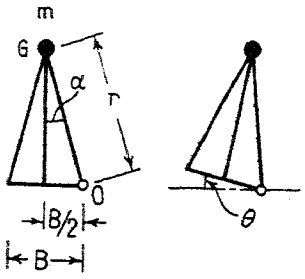


Fig. 10 Symbolic designations

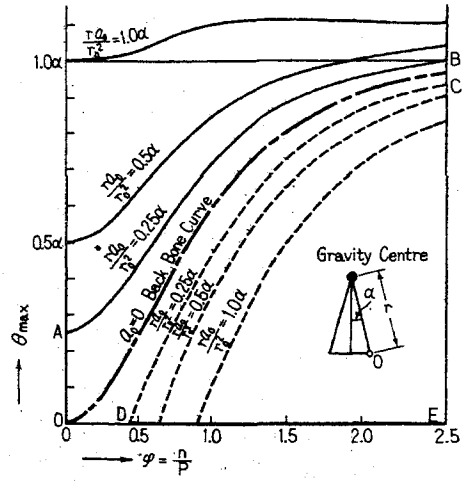


Fig. 12 Response curves

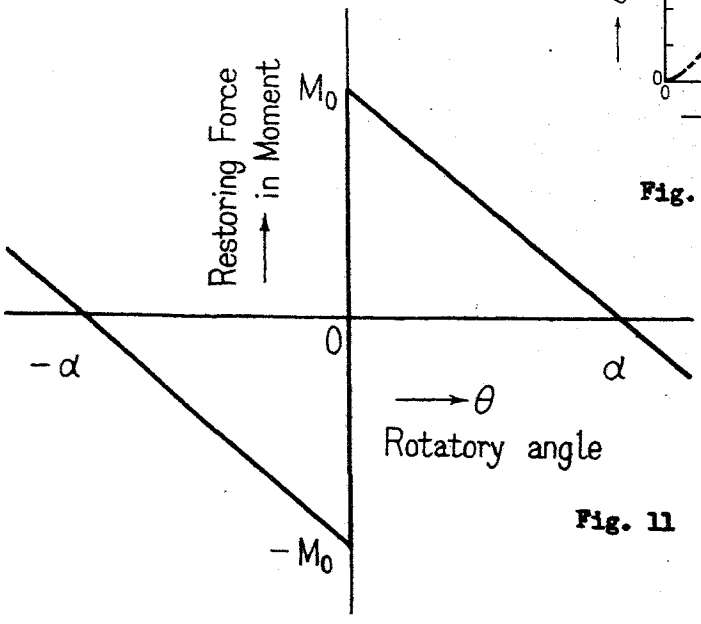


Fig. 11 Curve of restoring force vs. displacement

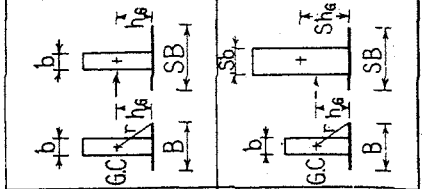
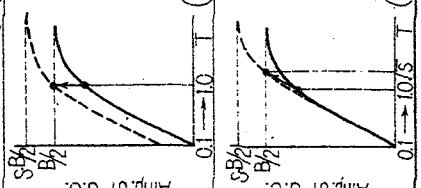
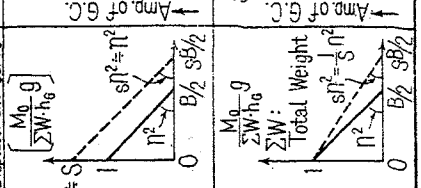
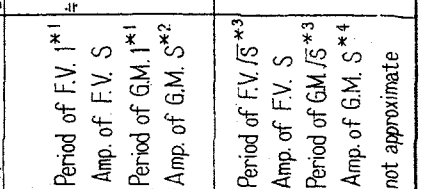
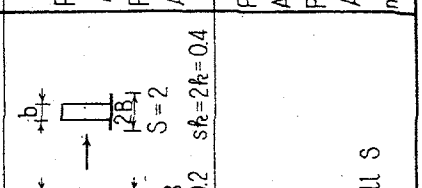
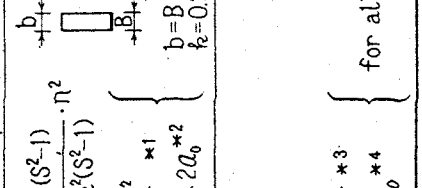
		Examples	Approximate L. of S.	Restoring Force Displacement Curve	Back Bone Curve in Log-Log Scale
1		$s\Omega^2 = \frac{1+\frac{h_G}{r}k^2(S^2-1)}{1+\left(\frac{h_G}{r_0}\right)^2 k^2(S^2-1)}\cdot n^2$ $s\Omega^2 = 1.030n^2$ *1 $sT = 0.985T$ $sa_0 = 0.917\times 2a_0$ *2 $b=B$, $k=0.2$, $sk=2k=0.4$, $S=2$	Period of F.V. 1*1 Amp. of F.V. S Period of G.M. 1*1 Amp. of G.M. S*2		
2		$s\Omega^2 = \frac{1}{S}n^2$ $sT = \sqrt{S}\,T$ *3 $sa_0 = S\cdot a_0$ *4 for all S	Period of F.V. $\sqrt{S}$ *3 Amp. of F.V. S Period of G.M. $\sqrt{S}$ *3 Amp. of G.M. S*4 not approximate		

Fig. 13 Laws of similarity

Table 1

Constants of Models

B	W_1 kg	W_2 kg	W_3 kg	W kg	I_o kg.cm^2	k_s
12cm	2.94	2.71	2.92	8.57	29.2	0.149
18	"	"	3.29	8.94	67.7	0.234
24	"	"	3.50	9.15	127.0	0.319
28	"	"	3.74	9.39	186.5	0.381
12S	3.65	"	2.92	9.28	29.2	0.148
18S	"	"	3.29	9.65	67.7	0.231
24S	"	"	3.50	9.86	127.0	0.317
28S	"	"	3.74	10.10	186.5	0.378

S: rigid model
W: weight of each part
I_o: moment of inertia of foundation about its center

Table 2

Symbols of the experiments

B	WOODEN Plate	Hard Rubber	Soft Rubber
12	1-A-1	1-B-1	1-C-1
12S	1-A-2	1-B-2	1-C-2
18	2-A-1	2-B-1	2-C-1
18S	2-A-2	2-B-2	2-C-2
24	3-A-1	3-B-1	3-C-1
24S	3-A-2	3-B-2	3-C-2
28	4-A-1	4-B-1	4-C-1
28S	4-A-2	4-B-2	4-C-2

Table 3

The Displacement of the Mass Point at the Lift-Up Point

B_{cm}	Const. β u_{mm}	A (analysis)$_{mm}$
12	3.0	3.2
18	4.5	5.1
24	6.4	6.9
28	7.7	8.3

Table 4

The Period of Overturning Vibration of Shaking Table

	T_{oL} sec.
1-A-1	0.64
2-A-1	0.63
3-A-1	0.65
1-A-2	0.60

Table 5

The Ground Motion to Overturn Structures 30m High During the Reverse Phase Vibration

Statically Overturning Coef.	Amplitude of G.M. cm	Period of G.M. sec.
0.5	12.5	35.5
0.3	25	32
0.2	72.5	27.5