Overturning Ratio under one sine pulse

M. Chiriatti
Graduate Research Assistant,Department of Structural, Geotechnical & Building Engineering (DISEG), Politecnico di Torino, 10129 Turin, ITALY

G. P. Cimellaro
Graduate Research Assistant Professor, Department of Structural, Geotechnical & Building Engineering (DISEG), Politecnico di Torino, 10129 Turin, ITALY

SUMMARY
Over the last decade the amount of population injured due to overturning of furniture is around 40% and this trend is expected to continue. Therefore a fundamental topic in the seismic protection is to limit the excessive motion of non-anchored bodies. The aim of this study is to analyze the behaviour of a rigid body under a sine pulse, particularly the rocking behaviour that is established during the pulse.The results are shown in the acceleration-frequency plane which can be constructed for all types of geometry and provides information on the type of wave that can cause the overturning.

Keywords: rocking, sine-pulse, overturning, response

1. INTRODUCTION

Although a strong earthquake motion does not always cause severe damage of structures,there may be a possibility that a lot of people are injured by an overturning accident of furnitureinside structures.Therefore a fundamental topic in the seismic protection is to limit the excessive motion of non-anchored bodies.

The motion of a non-anchored body can be partitioned in 6 basic conditions: rest, slide, rock, slide-rock, free flight and impact. These motions have been analysed by different authors (Ishiyama, Housner, Shenton, Yim).

The purpose of this study is to analyze the behaviour of a rigid body under a sine pulse, particularly the rocking behaviour that is established during the pulse. As demonstrated by the analysis after the input excitation a rigid body can have different behaviours depending on the system parameters. In particular we can distinguishif a rigid body start a rocking motion but it doesn't overturn or if it overturn with distinct modes: by exhibiting one or more impacts, and without exhibiting any impact.

The results obtained are shown in the acceleration-frequency plane that can be created for any geometry, where there are two main areas: a safe zone where the free-standing block doesn't overturn, and the other where the rigid body overturn exhibiting impacts or without exhibiting any impact.

2. REVIEW OF ROCKING RESPONSE OF FREE-STANDING BLOCK

Consider the model shown in Figure 1, which can oscillate about the centers of rotation *O* and *O'* when it is set to rocking. Its center of gravity coincides with the geometric center, which is at adistance *R* from any corner. The angle α of the block is given by $tan(\alpha) = b/h$.

Depending onthe value of the ground acceleration and the coefficient of friction, μ, the block may translate withthe ground, enter a rocking motion or a sliding motion. A necessary condition for the block toenter a rocking motion is $\mu > b/h$ (Aslam et al. 1980, Scalia and Sumbatyan 1996). The possibilityfor a block to slide during the rocking motion has been investigated by Zhu and Soong (1997),and Pompei et al. (1998). In this study, it is assumed that the coefficient of friction between theblock and its base is sufficiently large to prevent sliding at any instant in the rocking motion.

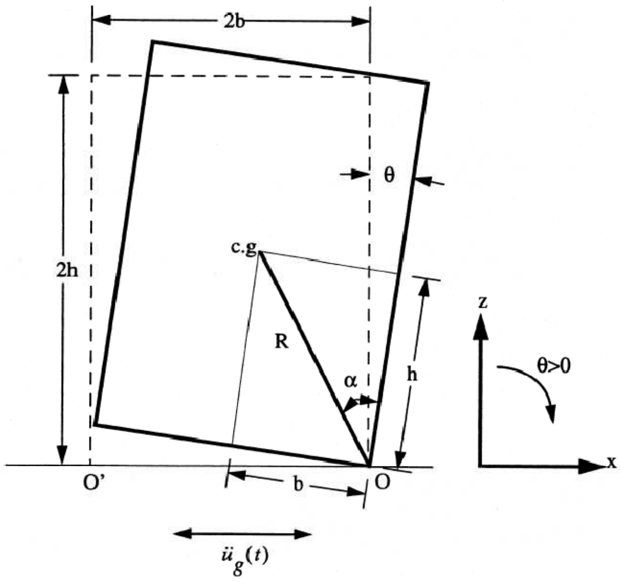

Figure 1. Schematic of rocking block.

Under a positive horizontal ground acceleration, $\ddot{u}_g$, the block will initially rotate with a negativerotation, $\theta<0$, and, if it does not overturn, it will eventually assume a positive rotation and so forth.Assuming zero vertical base acceleration ($\ddot{v}_g(t) = 0$), the equations of motion are

$$I_0\ddot{\theta} + m\,g\,R\,sin(-\alpha-\theta) = -m\,\ddot{u}_g R\cos(-\alpha-\theta) \qquad , \qquad \theta < 0 \tag{2.1}$$

and

$$I_0\ddot{\theta} + m\,g\,R\,sin(\alpha-\theta) = -m\,\ddot{u}_g R\cos(\alpha-\theta) \qquad , \qquad \theta > 0 \tag{2.2}$$

where for rectangular blocks,

$$I_0 = \frac{4}{3} m\,R^2 \tag{2.3}$$

and equations (2.1) and (2.2) can be expressed in the compact form

$$\ddot{\theta}(t) = -p^2\left\{\sin\big(\alpha\,sgn[\theta(t)] - \theta(t)\big) + \frac{\ddot{u}_g}{g}\cos\big(\alpha\,sgn[\theta(t)] - \theta(t)\big)\right\} \tag{2.4}$$

Where $p = \sqrt{\frac{3\,g}{4\,R}}$ is a quantity with units in rad/sec. The larger the block is (larger *R*), the smaller *p* is. The oscillation frequency of a rigid block under free vibration is not constant, since itstrongly depends on the vibration amplitude (Housner, 1963). Nevertheless, the quantity *p* is ameasure of the dynamic characteristics of the block.

When the block is rocking, it is assumed that the rotation continues smoothly from point O to O'.

3. ROCKING RESPONSE UNDER ONE-SINE PULSE

The analysis presented in this section focuses on the overturning potential of a one-sine pulse shown in Figure 2, therefore the ground acceleration is

$$\ddot{u}_g(t) = a_p \sin(\omega_p t + \psi), \quad -\frac{\psi}{\omega_p} \leq t \leq (2\pi - \psi)/\omega_p \tag{3.1}$$

$$\ddot{u}_g(t) = 0, \quad otherwise \tag{3.2}$$

where$\psi = sin^{-1}(\alpha g/a_p)$ is the phase angle when rocking starts.

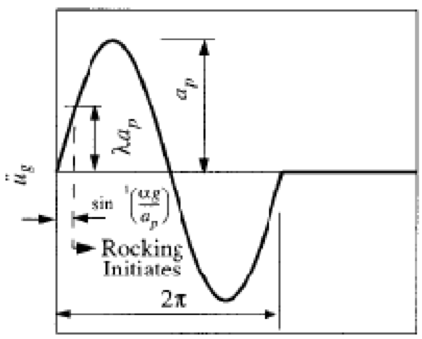

Figure 2. One-sine pulse.

With one-sine pulse input is possible to solve equation (2.4) and to observe the variation of the position of the center of gravity, and consequently the variation of the angle θ. During this phase the rocking response is determined by the ratio between the angle θ and angle α. If the ratio is smaller than *1* the rigid body is in a condition of rocking but if the ratio become bigger than *1* the center of gravity is out of the base of the rigid body so the equilibrium is not satisfied and the object overturns.
For certain acceleration and frequency values the rigid body can start the rocking response with one bouncing before overturning.

The problem is therefore governed by three major variables: the geometric nature of the system, the acceleration and the frequency of the input.The behaviour of a free standing bodycan be defined after defining these variables.

This type of problem is independent from the mass of the rigid body, in fact the mass m is present in all the terms of equation (2.1) and (2.2), and consequently results are equally valid for each rigid body with this particular geometry.

3.1. Nonlinear formulation

Considering the non linear nature of the problem,for evaluating the various overturning boundaries,equation (2.4) must be rewritten in a system:

$$\{y(t)\} = \begin{Bmatrix} \theta(t) \\ \dot{\theta}(t) \end{Bmatrix} \tag{3.3}$$

and the time-derivate vector is

$$f(t) = \{\dot{y}(t)\} = \begin{Bmatrix} \dot{\theta}(t) \\ -p^2 \left[sin[\alpha\, sgn\,[\theta(t)] - \theta(t)] + \frac{\ddot{u}_g(t)}{g} cos[\alpha\, sgn\,[\theta(t)] - \theta(t)] \right] \end{Bmatrix} \tag{3.4}$$

This system can be solved by a numerical integration. One option for performing this integration is to use the standard Ordinary Differential Equation (ODE45) solver available in MATLAB®, thereforeequation (3.4) was implemented in Matlab, and results found for a given geometry have been shown in the acceleration-frequency plane.

4. ANALYSIS OF THE RESULTS

In order to obtain results is necessary determining the geometry of the problem. If a slender block is considered, thenα*=0.25 rad* can be assumed. This means that the value of the measure of the dynamic characteristics of the block is

$$p = 2.14 \; rad/s \tag{4.1}$$

Thenthe other two variables (acceleration and frequency) that govern the system are ranged and results are summarized in the acceleration-frequency plane.
The results obtained can be grouped into several classes: (i) overturning without bouncing, (ii) overturning with one bouncing, (iii) overturning with two bouncing, (iv) no overturning.

Overturning without bouncing: in this case the input doesn't enable the rocking motion of the rigid body and the absolute value of the ratio θ/αquickly becomes greater than 1. This means that the rigid body overturns because the equilibrium is not satisfied. A cross is placed in the acceleration-frequency plane where overturning occurs. For example, results obtained with a normalized acceleration equal to $a_p/\alpha g = 2$ and with the normalized frequency $\omega_p/p = 1$ are shown in Figure 3, where the y-axis is the ratio θ/α, while the x-axisshows the time.

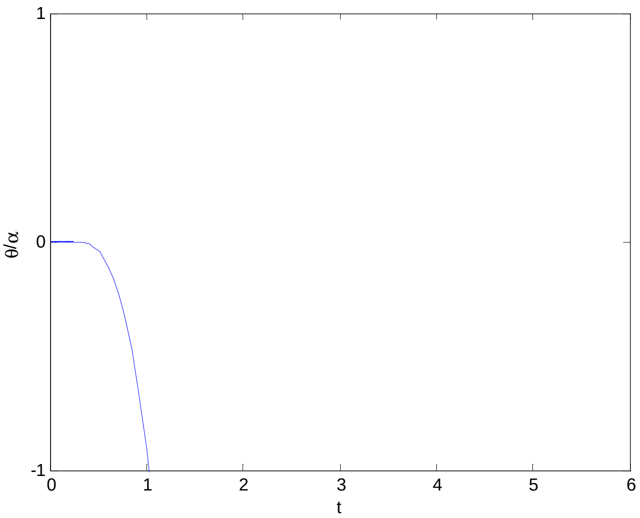

Figure 3. Overturning without bouncing.

Overturning with bouncing: in this case rigid body bounces on a vertex then changes direction of motion and overturns on the other side. This behaviour occurs in a specific area of the acceleration-frequency plane, between the previous zones where the reversal occurs and a safe zone where the rigid body doesn't overturn. In Figure 4 are shown the results obtained with $a_p/\alpha g = 2$and $\omega_p/p = 2$.

In few places there is a different behaviour from previous ones. With a normalized acceleration $a_p/\alpha g = 12$ and a normalized frequency $\omega_p/p = 10$ Figure 5; the overturn occurs after two impact and this is represented in the acceleration-frequency plane with a filled circle. Despite this behaviour is important to note that even in this case the rigid body overturns, hence these points must be inserted in the area unsafe of the acceleration-frequency plane.

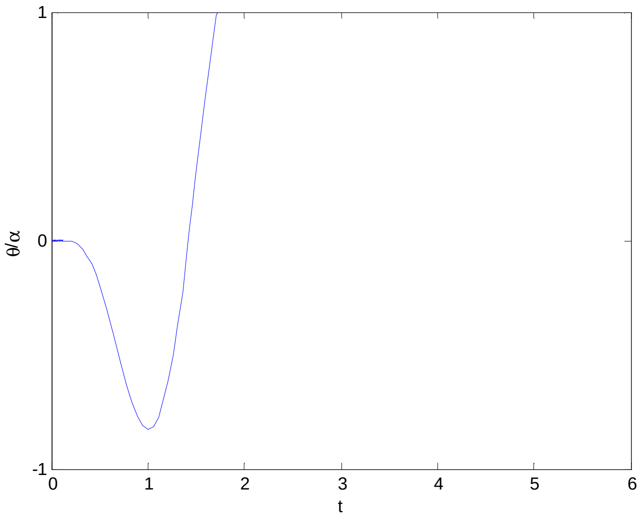

Figure 4. Overturning with one bouncing.

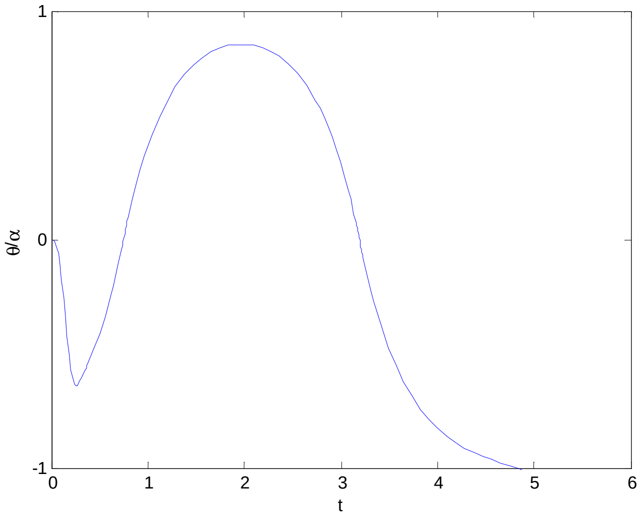

Figure 5. Overturning with two bouncing

No overturning: in this case the rigid body starts a rocking motion and the absolute value of the ratio θ/α remains always less than one, therefore the rigid body does not overturn. It happens in the third main area of the acceleration-frequency plane. This behaviour is represented in the plane by a little circle. An example of this result is shown in Figure 6 realized with the value of acceleration $a_p/\alpha g = 2$ and the frequency $\omega_p/p = 5$.

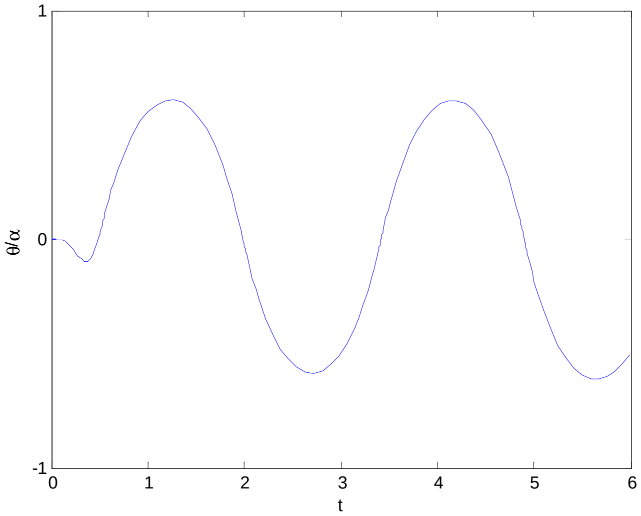

Figure 6. Rocking motion.

Figure 7 is the acceleration-frequency plane, where you can clearly distinguish three different zones. The first area is situated in the lower left of the plane and represents the safe area where the overturn does not occur. In the rest of the plane a free standing block can overturn with two different modes: by exhibiting one or more impacts; and without exhibiting any impact.

The area where the bouncing overturning occurs is between the rocking area and the overturning without bouncing area and the transition between these modes is abrupt.

Another way of approaching the problem may be to vary the type of integration by using other differential equation solver, in fact, as you can see in Figure 7, the result obtained for high acceleration and high frequencies can be affected by several integration errors, one on all the integration step because the input in this zone of the acceleration-frequency plane is very strong and develops in a short amount of time, so it would be appropriate to increase the integration step.

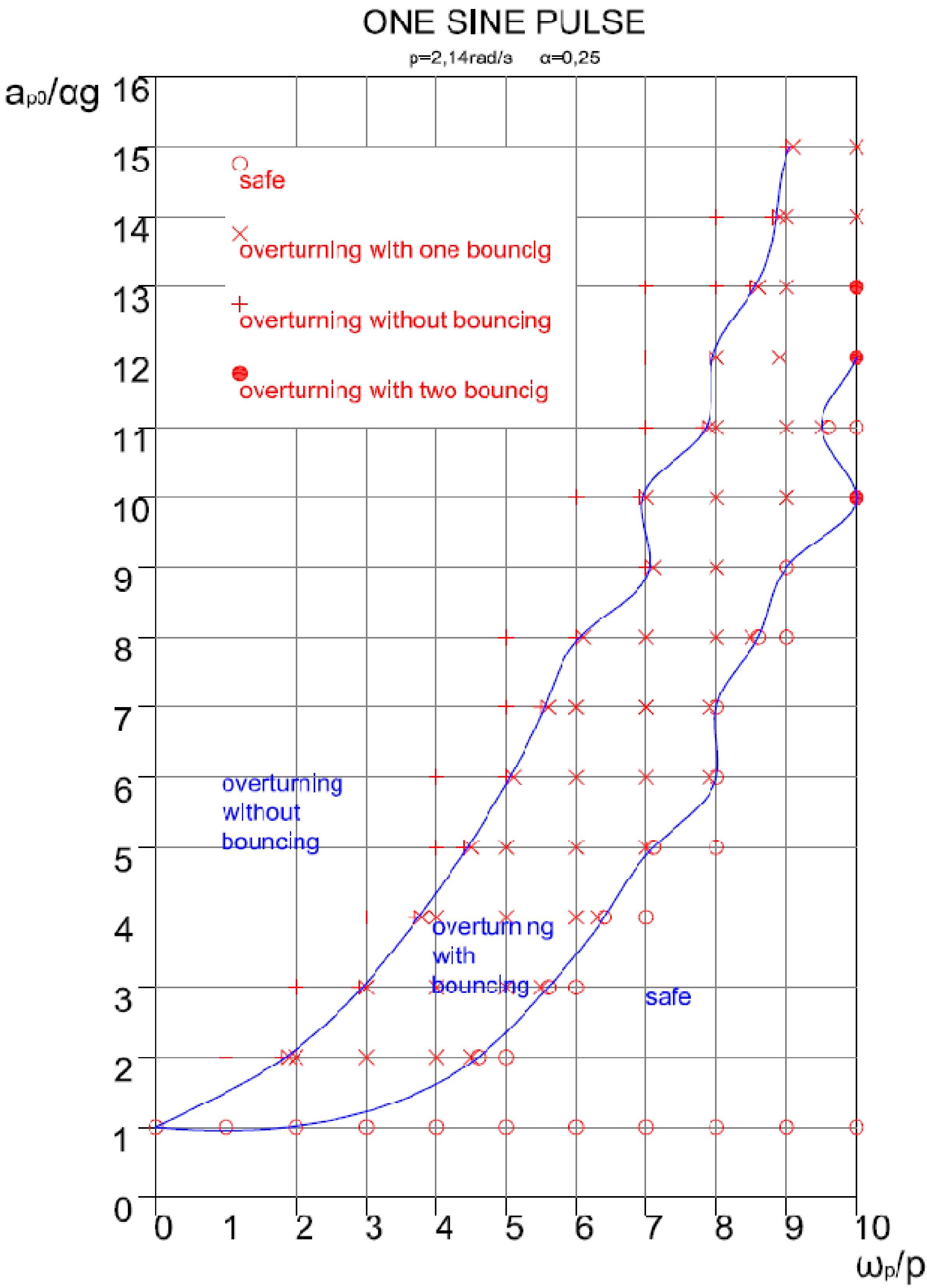

Figure 7. Acceleration-frequency plane.

5. CONCLUSION

The paper reviews the rocking motion and overturning behaviour of a rigid block subject to sinusoidal pulses. Furniture inside a building, an important museum art craft, or any object that can overturns during an earthquake can be assumed as a rigid block,therefore is very important to analyze the behaviour of this object under a dynamic force.
This study reveals that, under one-sine pulse, a free-standing block can overturn with two distinct modes, bouncing on the vertex or without bouncing; nevertheless it's easy to determinate a safe region in the domain of frequencies and acceleration causing only rocking motion.
An important development of this work can be the comparison between the results of different rigid body shapes by varying the slenderness α of the block; and using different waves input as the cosine function, or more realistic time history floor accelerations. Also further research should be done related to the interaction between the structural and non-structural components.

AKCNOWLEDGEMENT
The research leading to these results has also received funding from the European Community's Seventh Framework Programme - Marie Curie International Reintegration Actions - FP7/2007-2013 under the Grant Agreement n° PIRG06-GA-2009-256316 of the project ICRED - Integrated European Disaster Community Resilience.

REFERENCES

Ishiyama Y. (1982). Motions of Rigid Bodies and Criteria for Overturning by Earthquake Excitations. Earthquake Engineering and Structural Dinamics, Vol.10, 635-350.
Housner GW. (1963)The beaviour of inverted pendulum structures during earthquakes.*Bulletin of the Seismiological Society of America*, **53(2)**,403-417.
Shenton, H.W. (1995)"Criteria for Initiation of Slide, Rock, and Slide-Rock Rigid-Body Modes. *Journal of engineering mechanics, 690-693.*
Yim, Chopra, Penzien.(1980). Rocking response of rigid block to earthquake. Earthquake Engineering and Structural Dinamics, Vol. 8, 565-587.
Zhang Jian, Makris Nicos (2001). Rocking respone of free-standing blocks under cycloidal pulses.*Journal of engineering mechanics, 473-481.*
Makris Nicos, Yiannis Roussos (1998) Rocking response and overturning of equipment under horizontal pulse type motion.
Kaneko Mika, Hayashi Yasuhiro (2004). A proposal for simple equations to express a relation between overturning ratios of rigid bodies and input excitation
P. D. Spanos and A. S. Koh, "Rocking of rigid blocks due to harmonic shaking", Journ. Engrg.Mech. Div. ASCE 110, 1627-1642, (1984).
A. Pompei, A. Scalia and M. A. Sumbatyan, "Dynamics of rigid block due to horizontal ground motion", Journal of Engineering Mechanics, ASCE, Vol. 124, 713-717, (1998).
A. Scalia and M. A. Sumbatyan, "Slide rotation of rigid bodies subjected to a horizontal ground motion", Earthquake Eng. and Str. Dynamics, Vol. 25, 1139-1149, (1996).