SUMMARY:
The double calotte dome of the Treasure of St. Gennaro's Chapel in the Cathedral of Naples has been analyzed by means of an interdisciplinary approach regarding a detailed structural assessment. Sequential 3D analyses of the structural complex corresponding to subsequent historical construction and repairing stages (six phases in total) have been performed in the static and dynamic range to investigate the origin of the surveyed crack pattern. The reliability of the model was assessed delineating the areas where high tensile stresses concentrated, with peak values close to the strength of masonry, compared to monitored main damages both nowadays and reported in historical documents and treatises. Looking at the outcomes of various numerical phases it is clearly seen that the dome underwent effective consolidation interventions. It is remarked the skill and cleverness of the ancient technicians who designed proper and accurate interventions, leading to reduced tensile stresses today.

Keywords: Double Calotte Dome, Historical Phases, FEM

1. INTRODUCTION

The paper focuses the attention on an interdisciplinary approach developed within the case study of the double calotte dome of the Treasure of St. Gennaro’s Chapel in the Cathedral of Naples, Italy. This contribution wants to demonstrate the need of a methodological and interdisciplinary experience to analyze and assess the stability of monumental buildings. It is needed a correlation of archival and iconographical documentation with an accurate and exact geometrical survey and a numerical analysis.

In the case of study, the complex variability of factors that are involved in the structural analysis of the dome did not allow rapid schematizations but it requested a careful and deep analysis of the complex structure. The still existing historical documentation, relative to the building site and to multiple, successive consolidation and restoration, provided extremely detailed information about the static history of the building. The history of settlements, cracking and proposals regarding the structural interventions of the double calotte dome of the Treasure of St. Gennaro’s Chapel was presented by De Martino and Russo, 2004.

The knowledge of historical strengthening and repairing interventions and, moreover, an accurate knowledge of the geometry have constituted the first steps towards the structural assessment of the dome in each phase of its history. In particular, a LIDAR technique, integrated by classic topographic methodologies, was adopted to understand the complex shape of the dome. Details on the adopted methodologies and on the storage of geometric information were presented by Russo et al., 2006.

The deep knowledge of the geometry and materials in each phase of dome history allows to perform the final step of the approach involving seismic analysis of the dome structure towards finite element models able to provide a clear knowledge of its vicissitudes and to check the actual structural behaviour. The capability of FE models, presented in this study, to capture the behaviour of the dome during each phase of its history have been assessed by a preliminary investigation presented by Lignola and Cosenza, 2009.
2. THE DOME’S CONSTRUCTION AND ITS HISTORICAL STRENGTHENINGS

The Treasure of St. Gennaro’s dome, erected between 1608 and 1618, is composed of two groined masonry vaults with an internal diameter of about 14 meters (Figure 2.1). The structure is comparable to a long and narrow solid whose total height from ground is about 60 meters; it loads up four tuff masonry pillars connected by rounded arches. The external masonries of the Chapel, inscribable into a square, were made of tuff. The internal lantern was made of two rows of columns connecting internal and external domes and the external coating was made of lead. The external lantern, designed by Dionisio Lazzari in 1618, was made of piperno, a volcanic material widely used in Neapolitan monumental buildings. A first iron chain was already placed at the base of the second row of windows in 1612.

![Figure 2.1. The Treasure of St. Gennaro's Chapel in Naples: (a) external view; (b) internal view; (c) plane section; (d) cross section.](image)

During centuries the dome structure presented extremely different architectural configurations and materials compared to the initial ones, because subsequent and significant strengthenings alternated: in some cases, they have been only planned while, in other cases, they have been effectively carried out.

**First strengthening:** after the earthquakes of 1626 and 1627, a second coupled chain, designed by Giovan Giacomo Conforto, was inserted close to the third row of windows.

**Second strengthening:** after the 1688's earthquake, the external lantern made of piperno stone was substituted with one made of wood, a significant reduction in the applied load at the top of the structure was achieved.

**Third strengthening (hypothesis A):** in 1707 Giuseppe Lucchese designed a third chain close to the first one surmounting the second windows row in order to correct the weakness of the masonry of the
drum.

**Third strengthening (hypothesis B):** in 1708 Ferdinando Sanfelice planned to remove the external pillars’ row in the internal lantern; the idea was to reduce the load on the internal structure but keeping the connection between the two domes and avoiding the dangerous embedment of a chain in an ancient masonry.

**Fourth strengthening:** between 1724 and 1726 Donato Gallerano designed to substitute the internal lantern with a timber and iron made truss; this brave intervention allowed to virtually separate the domes and to concentrate the weight of the upper vault directly on the drum. It is representative of the actual state of the dome.

Table 2.1 summarizes the geometrical active structural sets in each historical phase as described previously that have been analyzed in the following sections.

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<th>Table 2.1. Phased analysis: active geometrical sets</th>
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<tbody>
<tr>
<td>1st chain</td>
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<tr>
<td>Phase I (1608-1618)</td>
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<tr>
<td>Phase II (1628)</td>
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<td>Phase III (1688)</td>
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<td>Phase IVa (1707)</td>
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<td>Phase IVb (1708)</td>
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<td>Phase V (1724-1726)</td>
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3. F.E. MODEL DEFINITION

Complete three-dimensional FE models of the dome were developed to simulate both static and seismic behaviour of the structure in each phase of dome history and to better understand the crack pattern noticed during its life and the many retrofit proposals.

Starting from detailed geometric and structural surveys, obtained using sophisticated laser scanner and the correlation of bibliographical, archival and iconographical documentation, the Chapel’s dome was modeled using the commercial computer code SAP 2000 (Figure 3.1).

![Figure 3.1. FE models development: (a) scanner digital image; (b) plane and cross sections of the dome in Cad environment; (c) 3D views of SAP FE model.](image)

The FE modelling strategy is based on the concepts of homogenized material. In particular the masonry structures have been modeled by means of three-dimensional solid brick elements, while frame elements have been used to model the chains, the timber and the iron truss. The dome rises up over the roof of the Chapel. To avoid modeling all the complex structures below the dome, elastic...
springs have been modeled at the base nodes of the dome. The total stiffness of the Chapel structures was assigned proportionally at each spring.

The dome has been considered as composed of six different materials (Table 3.1) for the structural and ornamental elements: old tuff masonry for the main structure, low modulus iron for the chains, timber for the truss (in the last phase), lead for the top coating, piperno for the former lantern and wood for the rebuilt one. Material properties assigned to the structural elements have been defined only in the elastic range, whereas the mechanical properties of non-structural elements have been neglected.

<table>
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<th>Table 3.1. Materials properties</th>
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<tr>
<td>Young Modulus</td>
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<tr>
<td>[GPa]</td>
</tr>
<tr>
<td>Old tuff masonry</td>
</tr>
<tr>
<td>Iron for chains</td>
</tr>
<tr>
<td>Timber for truss</td>
</tr>
<tr>
<td>Lead</td>
</tr>
<tr>
<td>Piperno stone</td>
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<td>Wood</td>
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The FE models (Figure 3.2) have been refined for each phase to account for openings, ribs on the internal calotte, buttress at the bottom drum, steel chains and timber truss when the lantern was removed.

**Figure 3.2.** FE models and material schematizations for each phase.
The analyses have been performed for both dead and seismic loads. In particular, dynamic simulations of the dome are presented considering both the response spectrum registered by National Accelerometric Network at Torre del Greco, that is the closest station to the church area, during the Irpinia earthquake of the 1980 and the inelastic design spectrum provided by Italian Code (NTC ’08) for the site of interest. The inelastic spectrum is obtained from the elastic one through a reduction factor equal to 2, which accounts for the limited ductility and deformation capacity of masonry structures. (Figure 3.3)

4. NUMERICAL RESULTS AND DISCUSSION

In the present section, the numerical results have been presented for each phase of the dome history. Contour maps of principal stresses have been shown looking both the intrados surface of the domes from inside the Chapel and across of the walls. For seismic analysis the results have been reported only for the heaviest load combination. In the following maps, negative and positive values are referred to compressive and tensile stresses respectively.

4.1. Phase I (1608–1618). Building site

Figures 4.1a and b report, for the building site, the maps of the compressive and tensile principal stresses, respectively. As it can be seen, the maximum tensile stress zones, that are cracking prone, are located on the inner dome over the windows and these are reduced where groins increase the cross section. These high tensile stresses cross the thickness of the wall rising also close to the third row of windows. The maximum value in the dome is higher than 0.1 MPa also for the static case; this value is higher than a reasonable tensile strength of ancient tuff masonry, thus resulting in crack pattern evolution.

For both static and seismic load conditions the maps show maximum value of compressive stress lower than the minimum compressive strength equal to 1.4 MPa suggested by NTC’08 for tuff masonry.
4.2. Phase II (1628). Second coupled chain insertion

Figures 4.2a and b report, for the present case, the maps of the compressive and tensile principal stresses, respectively. As it can be seen, the second coupled chain, inserted close to the third row of windows where high stresses were previously found, provides a reduction of about 14% of the maximum tensile stress; reduced tensile stresses are also still present in the upper part of the internal drum. The present consolidation intervention, for the static case, allows to obtain tensile values lower than 0.1 MPa, except close to the openings. As in the previous case, for both static and seismic load conditions, compressive stress is lower than the minimum compressive strength equal to 1.4 MPa suggested by NTC’08 for tuff masonry.

Figure 4.1. Principal stress map of the dome in the Phase I: (a) compressive stresses; (b) tensile stresses.

Figure 4.2. Principal stress map of the dome in the Phase II: (a) compressive stresses; (b) tensile stresses.
4.3. Phase III (1688). Substitution of piperno stone with wood for the external lantern

Figures 4.3a and b report, for the present case, the maps of the compressive and tensile principal stresses, respectively. As it can be seen, a reduction of about 20% is found in principal compressive stresses at the top of the structure due to the significant reduction in the applied load achieved by the substitution of the external lantern from piperno stone to wood.

The careful graphic descriptions of the crack pattern made by Lucchese in 1707 (Figure 4.4a) and Sanfelice in 1708 (Figure 4.4b) are clearly compatible with the peak localizations in the tensile stress map for phase III.

![Figure 4.3. Principal stress map of the dome in the Phase III: (a) compressive stresses; (b) tensile stresses.](image)

![Figure 4.4. The 18th century descriptions of the crack pattern made by: (a) Lucchese; (b) Sanfelice.](image)

4.4. Phase IVa (1707). First consolidation hypothesis: the third chain

Figures 4.5a and b report, for the present case, the maps of the compressive and tensile principal stresses, respectively. As it can be seen, the third chain close to the first one surmounting the second windows row provides a reduction of the maximum tensile stress of about 10%, but it was never realized. As in the previous cases, no relevant peak compressive stresses were found in the dome.
4.5. Phase IVb (1708). Second hypothesis: external pillars’ row removal in the internal lantern

Figures 4.6a and b report, for the present case, the maps of the compressive and tensile principal stresses, respectively. As it can be seen, the reduction of the maximum tensile stress due to the present intervention is smaller than in previous case. As in the previous cases, no relevant peak compressive stresses were found in the dome.

4.6. Phase V (1724–1726). Present situation: substitution of the internal lantern with a timber and iron made truss.

Figures 4.7a and b report, for the present case, the maps of the compressive and tensile principal stresses, respectively. As it can be seen, the insertion of the timber and iron made truss provides a
reduction of principal tensile and compressive stresses in the inner dome and partially in the outer dome. Although some stress concentrations raised at the supports of timber truss, a favourable reduction of stresses is gained all across the thickness of the walls. However, a weak tensile zone is still present above the second windows row, where a third chain was planned only (phase IVa) in 1707.

Figure 4.7. Principal stress map of the dome in the Phase V: (a) compressive stresses; (b) tensile stresses.

5. CONCLUSIONS

Sequential numerical analyses corresponding to subsequent historical construction and repairing stages have been performed. The model was able to delineate the areas where high stresses concentrated, thus resulting in a powerful tool to interpret the historical phases, to have a clear knowledge of its vicissitudes and to check the dome behaviour under both static and seismic loads. The main results, summarized in Figure 5.1, show that the structural problem of the dome is related to the peak tensile stress (0.10 MPa is a reasonable tensile strength of ancient tuff masonry), that for seismic load conditions are about two times the static values. Whereas compressive stress values, also under seismic loads, are compatible with the ancient tuff masonry strength.

Figure 5.1. Comparison of peak stress value for each phase: (a) tensile stress in the inner vault over the windows; (b) compressive stress at the base of the dome.
Different phases highlight clear differences in terms of tensile stresses, while differences are almost negligible in the case of compressive stresses. Seismic analysis shows also that the stress fields in the dome are very similar for both Irpinia Earthquake and Italian code spectrums.

Looking at the outcomes of various numerical phases, it is clearly seen that the dome underwent effective consolidations: the chain positions were accurately identified, and the need to reduce weights was rightly specified. It is remarked the skill and cleverness of the ancient technicians who designed proper and accurate interventions, leading to reduced tensile stresses today.

Ongoing work, the developed finite element model will be used to verify and drive the designer and the selection of future consolidation interventions both in term of retrofitting techniques and materials.

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REFERENCES