# Prediction of Higher-Mode Response of Tall RC Wall Buildings

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**SUMMARY**
Recent years have seen a great expansion of the tall building industry, even in areas characterised by high seismicity. With the increase of structural height, the importance of higher-modes in the dynamic response of buildings becomes predominant. However, the effects that ductility has on the response of higher-modes is not yet well understood, and traditional design procedures, based on response spectrum analysis, do not allow for an accurate estimation of higher-mode response of ductile systems. In this study a new approach for estimating the ductile higher-mode response is presented. The procedure consists of a modal analysis combination based on the so-called "substitute structure method", in which structural properties are suitably modified to account for the effects of ductility development. The method requires only elastic analysis and can be easily implemented at early stages of design. The proposed approach is validated against the results of an extensive non-linear time history investigation in which several RC wall systems are analysed. It is shown how the proposed methodology gives an improved prediction over the most traditional response spectrum approach for a wide range of different structural parameters and without a significant increase in computational requirements.

*Keywords: Higher-Mode Response, Performance-Based Design, Tall Buildings*

## 1. INTRODUCTION

The estimation of the maximum forces and deformations experienced by a structure during a seismic event is at the basis of structural design. The dynamic response of a tall building is influenced by multiple modes of vibration more than for low-rise buildings (Maffei and Yuen, 2007), and a rational design procedure should consider this phenomenon. Moreover, the contribution of higher modes is further modified by ductility development within the structure. As a consequence, the dynamic amplification of higher mode inelastic response can be difficult to estimate.

Traditional seismic design is based on the "reduced response spectrum analysis" (RRSA), which estimates the design seismic forces reducing all modal contributions by the same force reduction factor, R, proportionally to the ductility capacity of the system. However, when the contribution of higher modes is significant, the RRSA results in an underestimation of shear and moment demands (Eibl and Keintzel, 1988; Priestley et al., 2007; Sullivan et al., 2008).

The scope of this study is to gain a deeper understanding of higher mode effects on the seismic response of tall RC wall buildings, with regards to the influence of ductility on strength demands up the height of the structure. The aim is achieved through the interpretation of a large numerical investigation of tall cantilever and coupled RC walls subjected to different levels of ductility demand. A methodology for higher mode demand estimation, based on the substitute structure elastic approach (Shibata and Sozen, 1976), is also proposed and its accuracy evaluated against the numerical results.

## 2. NUMERICAL INVESTIGATION

The numerical investigation presented in this study is composed of a large campaign of 2D non-linear dynamic analyses of RC wall buildings. Several parameters have been varied, such as structural period, degree of coupling between walls and ductility demands, for a total of almost 4,000 non linear dynamic analyses. All analyses have been conducted using the commercial software Ruaumoko (Carr, 2009) and the general modelling approach and assumptions are described in the next subsections.

### 2.1. Structural Characteristics and Geometries

The structural typology analysed in this work consists of tall RC buildings, the structural characteristics of which have been varied to study their influence on higher mode response. In particular, the following parameters have been selected for investigation:

- Coupling ratio;
- Structural period;
- Ductility demand.

Structural walls can be cantilevered, when the walls are isolated and the overturning resistance is offered entirely by bending at the base, or coupled, when the walls are connected by coupling beams and the overturning resistance is offered by bending of the individual walls and push-pull between the walls (see Figure 1).

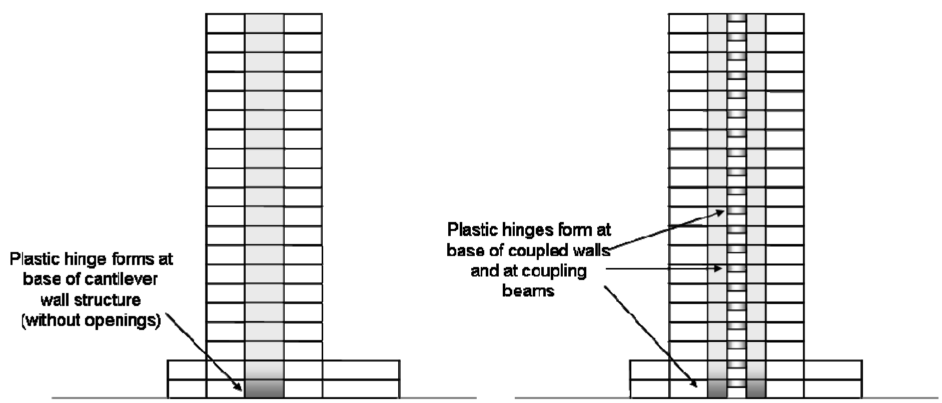



**Figure 1.** Plastic hinge locations in cantilever and couple walls.

In cantilever walls, ductility is mainly developed at the base through the formation of a single large plastic hinge. In coupled walls, instead, the ductility demand is more distributed as plastic hinges are formed at the base of the walls and at coupling beams.

The degree of coupling between walls can be measured by the coupling ratio, $\beta$, defined as the ratio between the overturning base resistance offered by the push-pull axial force in the wall and the total overturning base resistance. A fairly wide variation in the degree of coupling can be found in tall buildings, mainly dictated by architectural needs and organization of the central RC core in relation to the vertical transportation strategy (lift typology and distribution) of the structure. Therefore, four different coupling ratios have been considered in this study: 0 (cantilever walls), 0.25, 0.5 and 0.75 (extremely coupled walls).

The investigation focuses on tall buildings, the number of storeys of which is sufficiently large so that the mass and the stiffness provided by the coupling beams is equivalent to a uniform distribution. Pennucci et al. (2011) have shown that the variation of the modal response of wall buildings with more than 20 storeys for a given structural period does not depend on the specific number of storeys. Consequently the analysed structures have all been characterised with the same number of storeys (40) and the same height (120m), while the period has been varied between 1s and 10s (with 1s period increase), to simulate different flexibility levels which would be typical of buildings from medium rise to super high-rise.

To investigate the effect of ductility on the response of higher modes, the bending strength of the structure has been progressively reduced from elastic demand (that is strength reduction factor, R, equal to 1) to a level equal to $1/5^{th}$ of the elastic demand (that is R=5). The strength reduction has been applied to the base of the walls and in the coupling beams, with the formation of plasticity prevented anywhere else along the height of the walls.

The structures have been modelled using 1D elements in which the non-linear response is represented using the Takeda hysteresis loop, able to represent the stiffness degradation typical of reinforced concrete structures. The $\beta$ and $\alpha$ parameters of the Takeda loop have been set equal to 0.5 and 0 respectively, in line with the recommendations of Priestley et al. (2007). To reduce to a minimum the required computational effort, the vertical load carrying structure has not been modelled explicitly, but has been considered as seismic mass at each storey level. As a result the numerical models consisted of a single stick RC wall in the case of cantilever buildings and two coupled stick RC walls in the case of coupled buildings.

A traditional 5% Rayleigh damping model has been used in the analysis, with the stiffness contribution proportional to the tangent stiffness, as suggested by previous studies on the subject (see Priestley and Grant, 2005), which results in a substantially lower damping force when the elastic limit of the structure is surpassed.

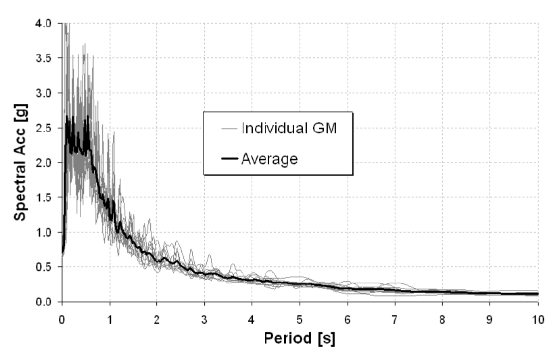
## 2.2 Ground Motions

For the non-linear dynamic analyses a collection of ground motions have been used as seismic input. To simplify result interpretation, the input records have been modified and made spectrum compatible. Spectrum compatibility has been achieved using time domain modification using the software RSP Match (Hancock et al., 2006). This approach overcomes the shortcoming of frequency domain spectral matching, which are similar to the shortcomings of artificial ground motions (Naeim and Lew, 1995).

The maximum effective period of the very tall case study structures is likely to be above 15s and therefore only recent digital ground motions with maximum usable period in excess of 15s have been considered. Records that showed important velocity pulses and forward directivity effects were disregarded. The best match with the design spectrum was observed in 11 ground motions, the details of which have been reported in Figure 2, together with the acceleration spectra of the matched records.

| Earthquake | PEER ID | Direction | Magnitude | Distance $[km]$ | Duration $[s]$ |
|---|---|---|---|---|---|
| Chichi | 1546 | E | 7.62 | 21.8 | 90 |
| Chichi | 1494 | E | 7.62 | 37.6 | 90 |
| Hector | 1762 | 360 | 7.13 | 48.0 | 60 |
| Landers | 900 | 270 | 7.28 | 86.0 | 44 |
| Landers | 900 | 360 | 7.28 | 86.0 | 44 |
| Kokaeli | 1147 | 90 | 7.51 | 112.0 | 150 |
| Tabas | 143 | TR | 7.35 | 55.0 | 33 |
| Denali | 2114 | 47 | 7.90 | 84.3 | 90 |
| Denali | 2114 | 317 | 7.90 | 84.3 | 90 |
| Loma Prieta | 801 | 225 | 6.93 | 26.7 | 50 |
| Duzce | 1062 | N | 6.69 | 55.6 | 43 |

a) Parameters of input ground motions




b) Acceleration spectra

**Figure 2.** Input ground motion characteristics

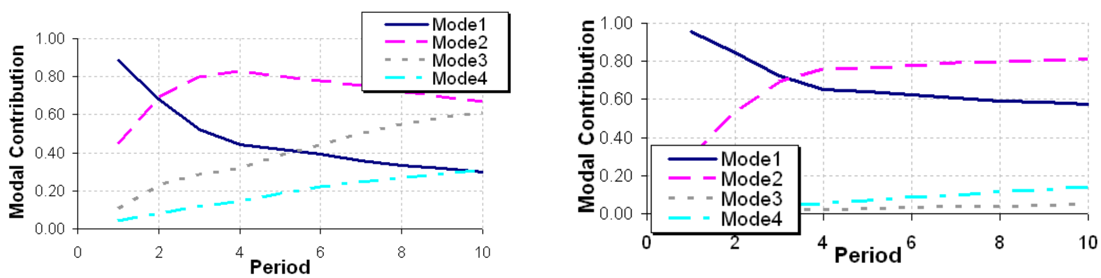
### 2.3 Elastic Modal Contributions

Before presenting the results of the inelastic time history analyses, interesting insights to the dynamic response of a tall structure can be gained using modal analysis theory. Modal analysis forms the basis of response spectrum analysis (RSA) approach, which computes the seismic demand of each mode separately and then combines them with appropriate modal combination rules. Assuming that periods are sufficiently spaced so that their response in time is statistically independent, the total response can be computed using the SRSS modal combination rule. The modal contribution of each mode can therefore be defined as:

$$\rho_i = \frac{F_i}{\sqrt{F_1^2 + F_2^2 + F_3^2 + \cdots}} \tag{2.1}$$

where $F_i$ can represent any modal parameter of interest of the i-th mode (e.g. inertia force, shear, moment, storey drift etc.).

The variation of the modal contribution factors with the fundamental period of a cantilever wall building can be observed in figure 3, where the ρ factor for the base shear and the mid-height moment is plotted versus the fundamental period of the structure.




a) Base shear modal contributions

b) Mid-height moment modal contribution

**Figure 3.** Shear and moment modal contributions for the first 4 modes in a cantilever wall

Figure 3.a shows that the base shear dynamic response of a cantilever wall building with a period greater than 2s is governed by the second mode rather than the first. For periods greater than 6s even the third mode is more important than the first. A similar but less extreme trend can be observed in terms of mid height moment in figure 3.b. The second mode contribution overtakes the first mode contribution for periods greater than 3s, although not by a large margin.

The trends reported in figure 3 do not have a general validity, because they have been generated with the specific spectral shape presented in section 2.2, and will vary appreciably if a different seismic input is used or a different structural system is analysed. However, the figure highlights the fact that in tall buildings higher modes are primary contributors to the dynamic response, while shorter structures are almost entirely characterized by first mode response. This concept is key to a correct interpretation of the results of the inelastic time history analyses and will help later in this paper when explaining the inapplicability of the RRSA approach for tall building seismic design.

### 2.4 Inelastic Response

The investigated structural systems are characterized by increasing structural period (1s to 10s), increasing coupling ratio (from 0 to 0.75) and increasing ductility demand (R=1 to R=5 at plastic hinge locations).

A detailed discussion of all the analyses conducted for this study would not be feasible, but two representative examples of the seismic demand on inelastic wall structures are reported in figure 4. For results of additional case study structures see Pennucci et al. (2011). The shear and moment graphs reported in figure 4 represent the envelope of the seismic demand of a coupled wall system characterized by a coupling ratio $\beta$=0.5, a fundamental period T=5s and a strength reduction factor R=5. On the same graph, for reference, are reported the seismic demand predicted by the elastic RSA and the elastic RSA reduced by the force reduction factor R.

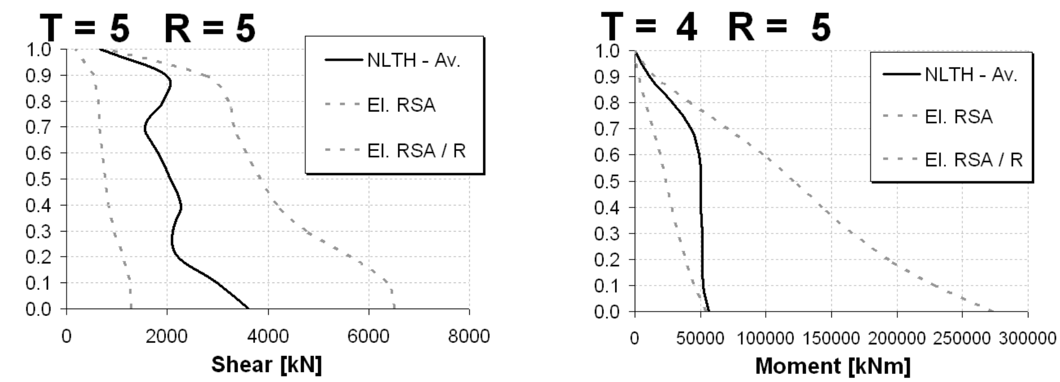
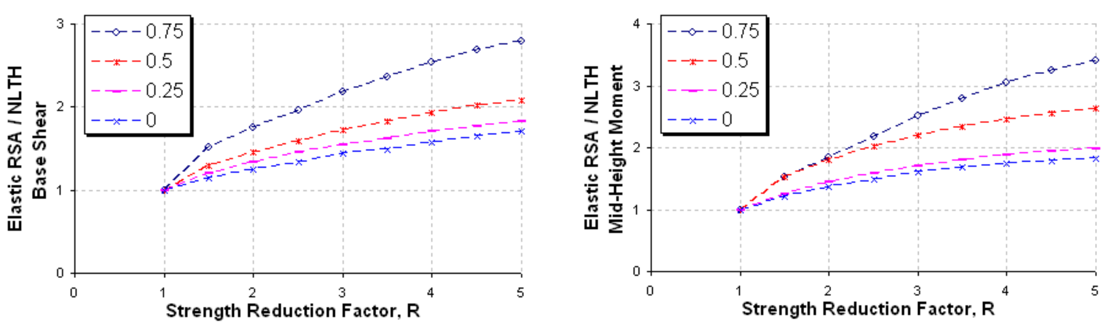



**Figure 4.** Comparison between shear and moment demand and predictions offered by elastic RSA and RRSA

Figure 4 shows that as expected, the ductility development affects the magnitude and the distribution of the inelastic seismic forces (the inelastic demands are significantly smaller than the elastic demand predicted by RSA). However, the reduction is not proportional to the strength reduction factor R, as commonly assumed by the reduced RSA. The examples reported in figure 4 show inelastic base shear and mid-height moment demands significantly larger than the seismic demand predicted by RRSA.

For a compact visualization of the trends identified by the 4,000 NLTH analysed (in Pennucci et al. 2011), the "effective Reduction Factor" parameter, eR, is introduced, defined as the ratio between the elastic demand and the inelastic demand in terms of base shear or mid height moment. An "effective Reduction factor", eR, equal to the strength reduction factor, R, at every location of the structure, implies that the seismic demand on the system has been reduced proportionally to the reduction in strength at plastic hinge locations. Such a response would justify the RRSA approach for tall buildings. On the contrary, an eR smaller than R implies that the seismic demand of higher modes has not been reduced proportionally to the strength reduction of the system, resulting in a seismic demand larger than the prediction of the RRSA.

The eR parameter has been computed in terms of base shear and mid height moment for every structure analysed for this study. Average trends have been computed and plotted in figure 5 versus the strength reduction factor of plastic hinges for every coupling ratio.




a) Base shear

b) Mid-height moment

**Figure 5.** Variation of the effective reduction factor, eR, in terms of base shear and mid-height moment with the strength reduction factor, R, and the coupling ratio $\beta$ (from 0 to 0.75).

Figure 5 shows that the effective reduction of the inelastic seismic demand in tall wall buildings is significantly less than the strength reduction factor the plastic hinges of the systems are designed for. This phenomenon is larger for systems with smaller coupling ratios, both in terms of shear and bending moments. This is related to the distribution of ductility within the system: in a cantilever wall system the ductility is developed only at the base, and the stiffness of the structure remains unchanged along the height of the building. In a coupled system, instead, the yielding of the coupling beams reduces the stiffness of the system also along the height of the structure, with a greater effect on the higher mode response.

The difference between the effective reduction of the seismic demand and the strength reduction of plastic hinges is larger in terms of base shear than mid-height moment. This finding is consistent with the finding in Figure 3 that the higher mode contribution is larger in terms of base shear rather than bending moment.

The results presented in figure 5 could be used to produce dynamic amplification factors to account for the different effect that ductility has on higher modes. This type of approach has already been attempted in the past (Eibl and Kentzel, 1988). However, it was previously explained that the contribution of higher modes is also a function of the spectral shape of the seismic input and therefore amplification factor methods should consider a lot of different factors, all influencing the inelastic higher mode response, such as seismic input characteristics (i.e. spectral shape), structural typology (e.g. frame or wall, which affects the spacing between periods), coupling ratio and building fundamental period.

Instead of focusing on the development of an amplification factor with limited applicability, this study investigates a more general approach based on a different modal analysis approach, capable of reflecting the effects of ductility on a tall structure better than the reduced response spectrum analysis.

## 3. DUCTILE HIGHER-MODE RESPONSE PREDICTION

Recognizing the limitations of RRSA when applied to tall buildings, a different methodology is sought. An alternative is represented by the so-called substitute structure method, proposed by Shibata and Sozen (1976) in the past.

The substitute structure method is based on the equivalent linearization concept, which implies that a non linear system can be represented by an equivalent linear system characterized by a stiffness equal to the secant stiffness of the structure at its peak inelastic response and by an equivalent viscous damping representing the energy dissipated through non-linear response. This concept is now successfully applied in displacement-based design methods, such as the Direct Displacement-Based Design, DDBD, by Priestley et al. (2007).

In the substitute structure method, the equivalent linearization concept is extended from single degree of freedom systems, SDOFs, to multi degree of freedom systems, MDOFs. The stiffness of the structural elements is reduced proportionally to the ductility demand they are subjected to, and a different equivalent elastic viscous damping is associated to each mode. A representation of the substitute structure defined by the method is reported in figure 6.

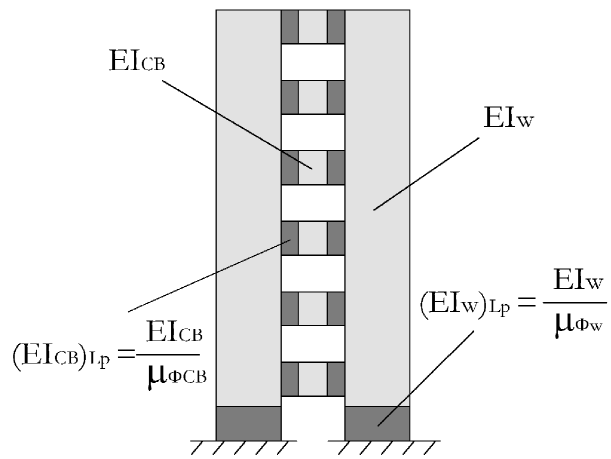


**Figure 6.** Stiffness reduction at plastic hinge locations according to the substitute structure method.

The substitute structure method has been given little attention in the past though, probably because of its slightly more complicated procedure, and the RRSA has become a standard in the estimation of inelastic seismic design instead. However, because of the limitations of the RRSA when higher mode response becomes predominant, new methodologies based on simplified versions of the equivalent substitute structure method for specific structural systems have been proposed recently (e.g. MMS for cantilever wall buildings by Priestley and Amaris, 2002, EMS for continuous concrete bridges by Alvarez, 2004, and TIMS for dual systems by Sullivan et al., 2008). A detailed comparison of the application of the MMS, EMS and TIMS approaches to tall wall buildings can be found in Pennucci et al. (2011). Pennucci et al. (2011) have shown that the most general method for predicting inelastic seismic demand in tall RC wall buildings is the EMS approach, when suitably adapted to reflect the characteristics of such systems.

Similarly to the substitute structure method, the EMS approach, as adopted by Pennucci et al. (2011) in their work, is based on the following assumptions:

- The inelastic demand of the first mode can be computed as the modal elastic demand divided by the strength reduction factor of the system;
- The inelastic demand of higher modes shall be computed as the modal elastic demand of the substitute structure.

In other words, the EMS requires that the seismic demand of the higher modes must not be reduced by the force reduction factor, R, but the stiffness of the inelastic elements shall be reduced to represent the ductility development within the structure. This way of estimating higher mode response is equivalent to the substitute structure method, assuming that the equivalent viscous damping generated by higher modes is so small that it can be neglected.

The key step of this methodology is the estimation of the curvature ductility demand of the yielding elements within the structure. Curvature ductilities can be related to the global displacement ductility if the yield mechanism is known, as it should be in a structure newly designed according to capacity design criteria.

The yield mechanism of a cantilever wall consists in the formation of a single plastic hinge at the base of the walls. Assuming a linear elastic curvature distribution and a concentration of plastic curvature within the plastic hinge region as shown in Figure 7, the curvature ductility can be expressed as a function of the displacement ductility as follows (ref. Paulay and Priestley, 1992, for the entire derivation):

$$\mu_{\phi} \approx 1 + (\mu_{\Delta} - 1)\frac{L}{L_p} \tag{3.1}$$

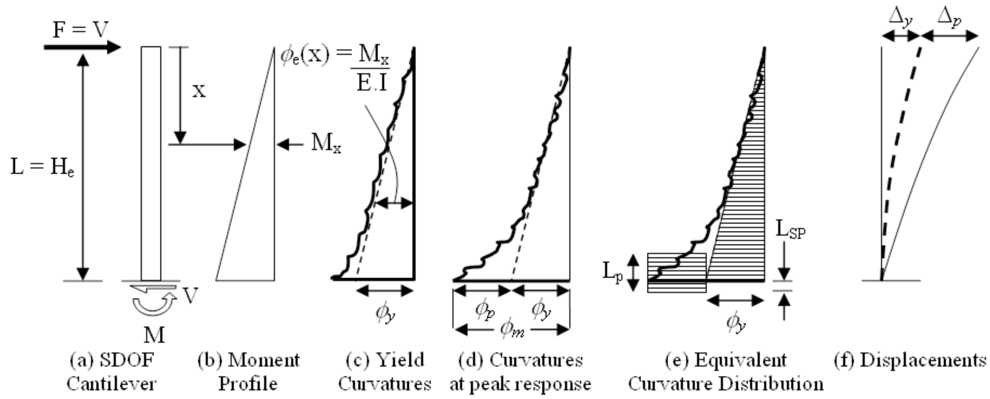



**Figure 7.** Curvature distribution in an inelastic cantilever (adapted from Paulay and Priestley, 1992)

The assumption of a linear curvature distribution might not be appropriate for coupled walls. However, it should be noted that the importance of an accurate curvature ductility estimation within the plastic hinge at the base of the wall decreases with increasing wall coupling ratio, as for larger coupling ratios the ductility is mainly developed in the beams. Therefore, it is suggested that Eqn. 3.1 is used to estimate wall curvature ductility for the conceptual design of any coupled wall configuration.

The estimation of force distribution within coupling beams is not relevant to the design, as rebar quantities are usually kept constant along the length of the element. Therefore the bending stiffness of the entire beam can be reduced instead of reducing the stiffness within plastic regions only, according to the Eqn. 3.2.

$$EI_{CB,eff} = \frac{EI_{CB}}{\mu_\theta} \tag{3.2}$$

The coupling beam rotation ductility is the ratio between the maximum coupling beam rotation and the yield coupling beam rotation. The maximum coupling beam rotation is equal to the wall rotation demand amplified by the rigid body motion of the wall, while the yield rotation can be calculated knowing the rebar detailing of the coupling beam. In the numerical investigation presented in this study, however, the same strength reduction factor has been used for both walls and coupling beams. Therefore, the rotation ductility of the coupling beams can be approximated with sufficient accuracy as being equal to the displacement ductility of the system.

The effective stiffness of ductile elements in the EMS method for tall RC wall buildings can therefore be easily correlated to the overall displacement ductility demand as shown above. Displacement-based design procedures use the displacement ductility demand as input parameter, and no further calculations are needed. Force-based design procedures, however, use the strength reduction factor, R, as a design parameter and therefore the ductility demand must be evaluated as a function of R.

The equal displacement rule, at the base of traditional seismic design, implies that the displacement ductility demand is equal to the strength reduction factor. However, other studies (Grant et al., 2005) seem to indicate that the $\mu$ and R are not always equal. Specific studies on the ground motions adopted in this work (see Pennucci at al., 2011) find that the best correlation between $\mu$ and T is $\mu = R^{1.3}$, and such a formulation is therefore used to estimate displacement ductility demand from the strength reduction factor in this study. It is foreseen, however, that the adoption of different relations between R and $\mu$ will not significantly affect the quality of the wall shear and moment prediction offered by the EMS method.

In line with the above, the EMS method has been used to estimate the response of the structures analysed in the numerical investigation. A complete presentation of the results obtained from the EMS method cannot be included in this text, but two representative examples are offered in figure 8. Further results are reported in Pennucci et al. (2011). From inspection of Figure 8, one notes that the EMS prediction matches the shear and moment profiles obtained from NLTHAs well.

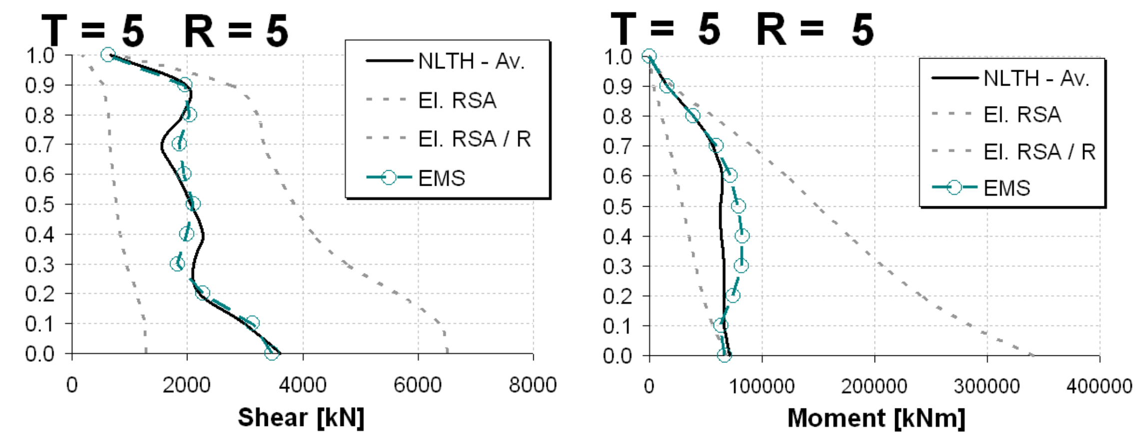
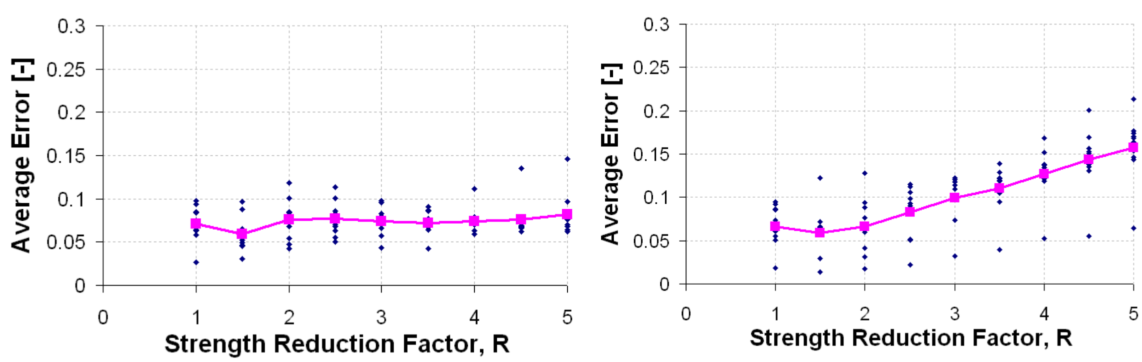



**Figure 8.** EMS prediction examples in terms of shear and bending moment

A more rigorous, compact way of assessing the quality of the prediction offered by the EMS method consists of the calculation of the average error, defined as the SRSS of the differences between analysis results and predictions in terms of bending moment and shear at a few discrete locations along the height of the building. Figure 9 plots the average error for shear and bending moment versus the strength reduction factor, R, considering all the performed analyses (each dot representing the error between the average results of 11 time histories and the EMS prediction) for a coupling ratio of $\beta =$ 0.5. Similar errors were found for the other coupling ratio values and are not reported here for brevity (see Pennucci et al., 2011). The average errors are generally very low, especially in terms of shear. Slightly larger errors can be observed in terms of bending moment for increasing strength reduction factor. The amount of error, though, is still well within acceptability limits for a linear calculation approximating the results of a series of dynamic non-linear time histories. These results indicate that the EMS approach could provide a very useful tool for the conceptual seismic design of tall buildings.




a) Shear prediction error

b) Moment prediction error

**Figure 9.** Average prediction error in terms of shear and bending moment

## 4. CONCLUSIONS

This paper has reviewed the applicability of reduced response spectrum analysis (RRSA) to the evaluation of seismic force demand on tall wall buildings. It is shown that the RRSA can largely underestimate seismic demand when the contribution to the inelastic dynamic response of the system is governed by higher modes. Moreover, it is shown that the effect of plastic deformations on the response of higher modes is greater in systems with a greater degree of coupling between walls.

An alternative methodology for the estimation of the inelastic response of RC wall buildings has then been investigated. The methodology is the adaptation to tall RC wall buildings of the effective modal superposition method, EMS. The EMS method consists of a linear elastic analysis in which the first mode response is reduced by the strength reduction factor, R, while higher mode response is computed using the elastic input spectrum and a substitute structure in which the stiffness of the plastic hinge locations is reduced proportionally to their curvature ductility demand. The methodology is tested against the results of an extensive non-linear time history investigation, in which a broad range of structural periods, strength reduction factors and coupling ratios have been adopted. The EMS method was found to perform well, providing excellent predictions of shear and moment demands.

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