Estimating Earthquake-induced Slope Displacements Using Vector Ground Motion Intensity Measures

Gang Wang & Wenqi Du
Hong Kong University of Science and Technology, Hong Kong SAR, China

SUMMARY:
Realistic prediction of earthquake-induced slope displacements is an important topic for evaluating and mitigating seismic hazards. Semiempirical predictive models are presented using multiple ground motion intensity measures (vector IMs) to improve the model performance. The predictive models are derived from statistical regression of empirical data using a fully-coupled equivalent-linear sliding mass model and the PEER-NGA strong motion database. The results show that using vector IMs can better characterize the ground motions and improve the efficiency of seismic displacement prediction for a wide range of slope conditions. Specifically, using PGA and the spectral acceleration value at 2 s can effectively improve the overall model performance in terms of efficiency, sufficiency and predictability. The model also has practical advantage since the vector-IMs are independent of the slope properties. An illustrative example is presented to compare the performance of the proposed models with a previous study by Bray and Travasarou (2007).

Keywords: Seismic slope displacements; Predictive model; Ground motions; Vector intensity measures

1. INTRODUCTION
Realistic prediction of permanent displacements of slopes under earthquake loading is an important topic for evaluating and mitigating seismic hazards. Since Newmark’s pioneering work on the rigid sliding block method (Newmark, 1965), extensive studies have been focused on developing empirical predictive models that can be reliably used to estimate the earthquake-induced displacements in earth dams, embankments and soil slopes (Jibson, 2011). The Newmark-type model assume that the slope behaves as a rigid-plastic material, and slope displacement is calculated by double integrating the part of the input acceleration that exceeds a critical value. The Newmark-type rigid sliding block model provides a simple index of dynamic slope performance. However, it is only appropriate for shallow landslides of stiff materials (e.g. rock blocks) that move along a well-defined slip surface. Prediction equations based on Newmark-type rigid sliding block often employs a single ground motion intensity measure (termed as a scalar IM), such as the peak ground acceleration (PGA) (Ambraseys and Menu, 1988; Jibson, 2007; Saygili and Rathje, 2008), the dominant period of the ground motion and Arias Intensity (Ia) (Jibson, 1993, 2007; Romeo, 2000) etc. There are also a few studies using multiple intensity measures (termed vector IMs) (Watson-Lamprey and Abrahamson, 2006).

Considering the deep sliding movement of earth slopes and earth embankments under earthquake loading, the simple equivalent-linear sliding mass model (Rathje and Bray, 2000; Bray and Travasarou, 2007) provides insights to account for the influence of soil nonlinearity on seismic slope response. In this model, the soil slope is simplified as a generalized single-degree-of-freedom system governed by the first modal shape of vibration. The dynamic characteristics of the slope are represented by the initial fundamental period of the slope ($T_s$, unit in s), and the nonlinear properties of soils are modeled using an equivalent-linear method similar to the well-known SHAKE analysis (Schnabel et al., 1972), such that the stiffness and damping ratio of the system is modified to be compatible with the induced strain level during shaking. Irreversible permanent displacements would occur if the base acceleration exceeds a prescribed yield acceleration ($K_y$, unit in g). Since soil nonlinearity causes elongation of the slope period during shaking, and the most efficient scalar IM for
the earthquake-induced displacements has been found to be the spectral acceleration at 1.5 times the initial period of the system, $Sa(1.5T_s)$. Therefore, Bray and Travasarou (2007) (referred to as BT07 model thereafter) employs $Sa(1.5T_s)$ as a scalar predictor in their predictive model. The standard deviation of the estimated displacement is chosen as 0.66 (in natural log units) for all slope conditions.

As earthquake records are complex transient time series, different ground motion intensity measures (IMs) can only represent certain aspects of ground motion characteristics. Using vector IMs allows for a better representation of different aspects of ground motions and thus is promising to improve the performance of the empirical prediction. However, in developing predictive models based on vector-IMs, it remains to be a great challenge to identify the suitable vector IMs and proper functional forms. In principle, the intensity measures should satisfy efficiency (Shome and Cornell, 1999), sufficiency (Luco and Cornell 2007) and predictability (Krammer and Mitchell 2006) requirements. Efficiency requires the empirical prediction minimizes the errors of the prediction; Sufficiency requires the model will not significantly depend on other ground motion parameters such as magnitude and distance of the earthquake; Predictability requires the ground motion IMs can be effectively predicted by GMPEs given a causal earthquake event. The BT07 model has to introduce earthquake magnitude as one of the predictors in order to correct the model bias, so it can not satisfy the sufficiency requirement. Using $Sa(1.5T_s)$ as a scalar predictor in BT07 model is no doubt a sensible choice to optimize the overall efficiency of the model. On the other hand, it relies on a prior knowledge of the initial fundamental period of slope ($T_s$), which may be difficult to quantify accurately. The situation could result in increased uncertainty in seismic displacement prediction using these property-dependent IMs. The property-dependent IM is also not convenient in application if different types of slopes exist in a single portfolio, or the fundamental period of the slope itself is a design variable. It is therefore more desirable to have property-independent IMs so that they can be used for any slope condition. All these considerations prompt us to revisit the efficiency of scalar and vector IMs in predicting the seismic slope displacements. Based on this study, new predictive models is developed using vector IMs. The performance of the propose models are systematically evaluated and compared with BT07 model.

2. EFFICIENCY OF SCALAR AND VECTOR INTENSITY MEASURES

The input ground motion is the primary source of uncertainty in assessing the seismic performance of slopes. In this study, earthquake acceleration time histories are chosen from the PEER-NGA strong motion database (Chiou et al. 2008) (http://peer.berkeley.edu/nga/). The database contains a total of 3551 three-directional acceleration time histories from California, Japan, Taiwan and other seismic active regions. Only horizontal recordings from free-field conditions are used in the analysis, resulting in a total of 1560 pairs of ground motions of two horizontal directions (Campbell and Bozorgnia, 2008). The equivalent-linear sliding mass model (Rathje and Bray, 2000; Bray and Travasarou, 2007) is used in the present study. The permanent displacements of the sliding mass were computed using each of the as-recorded earthquake motions in the database. Two orthogonal recordings at the same station are treated as separate records. For each record, the permanent displacements were calculated by applying the record in the positive and the negative directions, and the maximum value of two directions was taken as the permanent displacement for that record. Figure 1(a) shows the computed permanent displacement using acceleration time history recorded during Superstition Hills Earthquake (1987) at USGS Station 286. The shear wave velocity of the soil is assumed to be 250 m/s. During shaking, the modulus reduction and damping ratio curves follow that for clays with plasticity index of 30 (Vucetic and Dobry, 1992), as is illustrated in Figs. 1(b). The yield acceleration $K_y$ is assumed to be 0.1g and the initial fundamental period of the slope, $T_s$, varies from 0–2 s. If the slope is rigid ($T_s=0$ s), the estimated median slope displacement is 46 cm. The slope displacement reaches approximately 90 cm if $T_s$ falls into the range of 0.1-0.4 s due to the resonance of the slope with the concentrated shaking energy at that period range (note the mean period of the record 0.38 s). As the slope becomes more flexible ($T_s$ increases), the permanent displacements decreases to negligible values if $T_s$ are longer than 1 s. For each ground motion in the strong motion database, the seismic slope displacement can be computed. Figs. 2 and 3 present the scatter plots of computed seismic slope displacements against PGA, $Sa(1.5T_s)$ and $I_a$ for a stiff ($T_s=0.1$ s) and a flexible ($T_s=1$ s) slope. The slope resistance
is assumed to be $K_y=0.1 \ g$ in this example. From these plots, it is evident that some IMs are better correlated with the displacements than the others. However, the predictive capacity of an IM changes with the properties of the slopes, i.e., an IM that is closely correlated to the seismic displacements of stiff slopes (e.g. $T_s=0.1 \ s$) may not be a good predictor for flexible slopes (e.g. $T_s=1 \ s$).

Figure 1. (a) Computed Permanent Displacements ($K_y=0.1 \ g$). Insert: Acceleration Time History; (b) Modulus Ratio and Damping Curve for Nonlinear Soils.

Figure 2. Seismic displacements vs. IMs for a stiff sliding mass ($K_y=0.1 \ g$, $T_s=0.1 \ s$)

Figure 3. Seismic displacements vs. IMs for a flexible sliding mass ($K_y=0.1 \ g$, $T_s=1 \ s$)

The efficiency of scalar and vector IMs can be systematically evaluated under various slope conditions. Using 1560 pairs of ground motions, earthquake-induced displacements are calculated for $T_s$ ranging from 0.01 to 2 s, and $K_y$, ranging from 0.01 to 0.5g. It should be noted that $T_s$ usually fall between 0.2–0.7 s, so there is little practical importance to consider $T_s$ greater than 2 s. For each ($T_s$, $K_y$),...
Ky) case, regression analysis was performed by assuming that the seismic displacements $D$ (in natural log unit) are related to the scalar or vector IMs (in natural log unit) in a quadratic form as follows:

$$\ln D = a + \sum_{i=1}^{n} b_i \ln IM_i + \sum_{i=1}^{n} c_i (\ln IM_i)^2 + \varepsilon \cdot \sigma_{\ln D} \quad (2.1)$$

where $IM_i (i=1,2,\ldots,n)$ represent a particular scalar IM that may be combined to form vector IMs, and $n$ is the total number of IMs used (note $n=1$ for the special case when a scalar IM is used). Parameters $a, b_i, c_i$ are fitting parameters to be determined for each ($T_s, Ky$) case. The standard deviation of residuals is $\sigma_{\ln D}$, and $\varepsilon$ is a random variable following the standard normal distribution. The adjusted coefficients of determination ($R^2_{adj}$) is used to evaluate the efficiency of IMs. $R^2_{adj}$ increases only if new term improves the model more than that would be expected by chance. In general, a larger value of $R^2_{adj}$ implies a smaller standard deviation of residuals, thus higher efficiency of the prediction.

**Figure 4.** Contours of adjusted coefficients of determination for scalar and vector IMs

Using selected scalar and vector IMs as predictors, contours of $R^2_{adj}$ values were computed for slopes of different $T_s$ and $Ky$ are shown in Figure 4. It is worth mentioning that the analysis is based only on seismic displacements that are greater than 1 cm. Displacements smaller than 1 cm usually causes little engineering consequence. However, the scatter of these data in log space is much larger than these larger values. Therefore, it is necessary to only use data larger than 1 cm in regression analysis such that the result will not be biased by these small-valued data. Furthermore, $R^2_{adj}$ will not be calculated if the number of data is less than 30 to guarantee that the sample size is big enough for statistical analysis. The main observation is summarized as follows:

1. Due to elongation of slope period during shaking, the spectral acceleration at a degraded period,
is the overall most efficient scalar IM as was also pointed out by Travasarou and Bray (2003). Illustrated in Fig. 4, Sa(1.5Ts) is efficient ($R_{adj}^2 > 0.7$) particularly for flexible slopes with $T_s > 0.2$ s. However, the efficiency of Sa(1.5Ts) greatly decreases ($R_{adj}^2 < 0.5$) if the slope becomes stiffer ($T_s < 0.2$ s).

(2) Compared with the spectral acceleration, Ia is derived from time integration of the square of the entire acceleration time history (Arias, 1970), and it implicitly consider the amplitude and the duration of the ground motion. Among all scalar IMs that have been considered in Wang (2012), Ia is the most efficient IM for stiff slope cases ($T_s < 0.2$ s).

(3) Since Sa(1.5Ts) and Ia are the most efficient scalar IMs for flexible ($T_s > 0.2$ s) and stiff slopes ($T_s < 0.2$ s) respectively, the vector IM that incorporates both of them (termed as “Sa(1.5Ts) + Ia”) can improve the overall efficiency for all slope conditions. As is shown in Fig. 4, $R_{adj}^2 > 0.8$ is achieved for slope conditions with $K_y < 0.1$ g and $T_s < 2$ s. It should be noted that Bray and Travasarou (2007) recommended using Sa(1.5Ts) for $T_s > 0.05$ s and PGA for $T_s < 0.05$ s in their predictive model. However, their approach is slightly less efficient than using the “Sa(1.5Ts) + Ia” scheme.

(4) Considering the limitation of using property-dependent IMs, it is therefore more desirable to have property-independent IMs. A promising option is to use multiple spectral accelerations at short to long periods to form the vector IMs, which samples a range of frequency content of the ground motions and thus can better describe their overall characteristics. As a special case, the vector IM using PGA and Sa(2s) (termed as “PGA+Sa(2s)” scheme in Fig. 4) is the most efficient one if only two IMs should be used. Of course, the efficiency can always be improved by using more spectral acceleration ordinates.

3. PREDICTIVE MODELS USING SCALAR AND VECTOR INTENSITY MEASURES

3.1. Bray & Travasarou (BT07) model

For comparison, Bray & Travasarou (2007) predictive model (BT07) is briefly summarized. The model consists of two steps: (1) estimate the probability of negligible “zero” displacement (smaller than 1 cm), and (2) estimate the median nonzero displacement (greater than 1 cm). By doing so, these very low values of displacements that are of no engineering significance do not bias the results.

Using probit regression analysis, the probability of “zero” displacement can be expressed as a function of IMs, $K_y$ and $T_s$ as follows:

$$P(D = 0) = 1 - \Phi(-1.76 - 3.22\ln(K_y) - 0.484T_s\ln(K_y) + 3.52\ln Sa(1.5T_s))$$

(3.1)

where $\Phi$ is the standard normal cumulative distribution function. The nonzero displacement can be estimated by regression analysis using non-zero displacement data (greater than 1 cm),

$$\ln D = b_0 - 2.83\ln K_y - 0.333(\ln K_y)^2 + 0.566\ln K_y\ln Sa(1.5T_s) + 3.04\ln Sa(1.5T_s) - 0.244(\ln Sa(1.5T_s))^2 + 0.278(M - 7)$$

(3.2)

where $b_0 = \begin{cases} -0.22 & \text{if } T_s < 0.05s \\ -1.10 + 1.5T_s & \text{if } T_s \geq 0.05s \end{cases}$; $D$ is the predicted median nonzero displacement in cm; $M$ is moment magnitude of earthquakes; $K_y$ is the yield acceleration in g; $T_s$ is the initial period of the sliding mass in s. When $T_s < 0.05s$, lnSa(1.5Ts) will be replaced by lnPGA in Eq.(3.2). Note that IMs in the above equations are in natural logarithmic scale, since modern seismology found that IMs usually follow lognormal distributions. Consequently, the residual (error) of Eq. (3.2) can be evaluated as:
\[
    r = \ln \hat{D} - \ln D
\]  
(3.3)

where \( \hat{D} \) is empirical data obtained from the fully-coupled sliding mass model, \( D \) is the predicted value, \( r \) is error (residual) of the model prediction, which usually follows a normal distribution. BT07 model provides the standard deviation of residuals \( \sigma = 0.66 \).

In summary, BT07 model specifies the probability of zero displacement \( P(D = 0) \) via Eq. (3.1), and the nonzero displacement \( D \) is assumed to follow a lognormal distribution with the median value provided in Eq. (3.2) and the standard deviation \( \sigma \) (in natural log unit). The model results in a full probabilistic description of the slope displacements suitable for \( T_s = 0 - 2s \) and \( K_y = 0.02g - 0.5g \). Accordingly, the predicted displacement according to a specified percentile \( p \) (in decimal form, i.e. \( p = 0.25 \) for the 25th percentile) can be determined as

\[
    \ln D_p = \ln D + \sigma \cdot \Phi^{-1} \left[ \frac{p - P(D = 0)}{1 - P(D = 0)} \right]
\]  
(3.4)

\[\text{Figure 5. The distribution of (a) median and (b) standard deviation of residuals of Eq. (3.2)}\]

It is worth noticing that, the predictive equation Eq. (3.2) is derived from regression analysis of all nonzero displacement data. The model performance should be checked for each individual case of \( K_y \) and \( T_s \). In this study, the medians and standard deviations of residuals in Eq.(3.3) is computed using the strong motion database for a large number of \( T_s \) and \( K_y \) combinations (a total of around 600 combinations for \( 0.01s \leq T_s \leq 2s \) and \( 0.01g \leq K_y \leq 0.5g \)). As is plotted in Fig. 5(a), BT07 model has considerable negative bias in the median residuals when \( T_s \) is from 0.1 to 0.3s. The bias can reach as much as -0.5, indicating the predicted displacements via BT07 model can be overestimated as much as 60% for these cases. Although BT07 model specifies a standard deviation of 0.66, the actual distribution is in fact dependent on the range of \( T_s \). Shown in Fig. 5(b), the standard deviation falls between 0.7–1.1 when \( T_s < 0.2s \), and it is reduced to 0.5–0.7 when \( T_s > 0.2s \).

3.2. “Sa(1.5Ts)+Ia” model

Based on the efficiency study of the vector IMs in the previous section, a predictive model is developed based on \( Sa(1.5T_s) \) and \( Ia \) following similar analytical framework of BT07 model. The probability of “zero” displacement is derived as:

\[
    P(D = 0) = 1 - \Phi(-1.861 - 2.782 \ln K_y + 2.05 \ln Sa(1.5T_s) + 1.127T_s \ln Sa(1.5T_s) + 0.703 \ln Ia - 1.312T_s \ln Ia)
\]  
(3.5)

or, we can simply use Eq.(3.1). The estimated median nonzero displacement can be expressed as:
\[ \ln D = b_0(T_s, K_y) - 1.457\ln K_y + 0.856\ln Sa(1.5T_s) - 0.024(\ln Sa(1.5T_s))^2 + 0.607\ln Ia + 0.339(M - 7) \quad (3.6) \]

\[ b_0(T_s, K_y) = \begin{cases} 
1.062 - 0.789K_y - 4.478T_s & \text{if } T_s \leq 0.1s \\
2.021 + 0.864K_y + (0.611 + 0.718K_y)\ln(T_s) & \text{if } 0.1s \leq T_s \leq 0.3s \\
1.285 & \text{if } 0.3s \leq T_s \leq 0.4s \\
1.285 - 0.148\ln(K_y)(T_s - 0.4) & \text{if } T_s \geq 0.4s 
\end{cases} \]

where \( Ia \) is in the unit of g\cdot s. Considerable efforts have been spent to select a suitable function form for a wide range of \( T_s \) and \( K_y \) conditions. The above equation is a quadratic function of \( \ln Sa(1.5T_s) \) and a linear function of \( \ln Ia \), respectively. A piecewise continuous function \( b_0(T_s, K_y) \) is adopted to minimize the model bias instead of using a complicated higher order function of \( T_s \) and \( K_y \). All the constant coefficients in Eqs. (3.5) (3.6) are obtained from nonlinear regression analysis of empirical data, and they are all statistically significant. Again, the distribution of medians and standard deviations of residuals for Eq. (3.6) is plotted in Fig. 6. Compared with Fig. 5, the “Sa(1.5Ts)+Ia” model has more uniform distribution of the median errors. The standard deviations (in log scale) of residuals is also more uniformly distributed and slightly reduced. It is proposed a constant \( \sigma = 0.64 \) should be used for all cases. Like BT07, dependency of model prediction on earthquake magnitude is still observed in this model. Therefore, a magnitude term is incorporated in the predictive equation.

![Figure 6](image_url)

**Figure 6.** The distribution of (a) median and (b) standard deviation of residuals of Eq. (3.5)

### 3.3. “PGA+Sa(2s)” model

To eliminate the limitation of property-dependent IMs, two property-independent IMs are used to develop the predictive equation. Using PGA and the spectral acceleration value at 2s, the probability of “zero” displacement can be evaluated as:

\[ P(D = 0) = 1 - \Phi(3.224 - 2.454\ln(K_y) - 4.41K_yT_s - 2.415T_s \]

\[ + 1.356\ln(PGA) + 1.591\ln Sa(2s) - 0.345\ln(T_s)\ln Sa(2s)) \quad (3.7) \]

And the median predicted nonzero displacement is:

\[ \ln D = b_0(T_s, K_y) - 1.668\ln K_y + (1.727 + 0.277\ln K_y)\ln PGA \\
+ (0.517 - 0.343\ln K_y)\ln Sa(2s) + 0.134(\ln Sa(2s))^2 \quad (3.8) \]
\[ b_3(T_s, K_y) = \begin{cases} 
1.583 - 1.696K_y & \text{if } T_s \leq 0.05s \\
2.638 + 1.141K_y + (0.352 + 0.947K_y)\ln(T_s) & \text{if } 0.05s \leq T_s \leq 0.3s \\
2.214 & \text{if } 0.3s \leq T_s \leq 0.5s \\
1.866 - 0.502\ln(T_s) & \text{if } T_s \geq 0.5s 
\end{cases} \]

Figure 7: The distribution of (a) median, (b) standard deviation of residuals of Eq. (3.8). (c) The residuals versus magnitude

Compared with previous two models, the “PGA+Sa(2s)” model has several distinctive advantages. Firstly, as shown in Fig.7(a), the medians of residuals of Eq. (3.8) are uniformly small for a wide range of \( K_y \) and \( T_s \). For most cases, the median values are between -0.1 and 0.1. Secondly, the standard deviations of the residuals are rather uniform for a wide range of \( K_y \) and \( T_s \). A constant standard deviation of residuals (in natural log unit) can be adopted as \( \sigma = 0.72 \), which is only slightly larger compared with the previous models. Finally, the residuals show very little bias when plotted against earthquake magnitude. As shown in Fig. 7(c), considerable negative bias only occurs when \( M \) is smaller than 5.5. It is not a critical issue since sliding displacements from a smaller earthquake is less significant, and the equation is conservative (overestimating the results) for these cases. Therefore, the model does not need to incorporate a magnitude term, and it satisfies the sufficiency requirement.

4. MODEL COMPARISON AND CONCLUSIONS

An illustrative example is presented in this section to compare the performance of three predictive models. Assuming an earthquake event of moment magnitude of 7 occurs on a strike-slip fault, and the slope under study is located at a rupture distance of 10 km. Based on recently developed ground motion prediction models for spectral accelerations (Campbell and Bozorgnia, 2008) and Ia (Campbell and Bozorgnia, 2010), the median predicted Arias intensity for this scenario earthquake is \( I_a = 0.123 \) g⋅s=1.2 m/s, and the median predicted spectral accelerations are shown in Fig. 8. Specifically, the median predicted PGA=0.26 g and Sa(2s)=0.2 g.

Figure 8. The predicted median spectral accelerations of the scenario earthquake
The predicted seismic displacements are plotted in Fig. 9 for $T_s$ from 0 – 2s and $K_y$ equals 0.05g, 0.1g, 0.2g, respectively. Fig. 9(a) compares the predicted median nonzero displacements computed via Eqs. (3.2) (3.6) and (3.8). The probabilities of nonzero displacements are shown in Fig. 9 (b) using Eqs. (3.1) (3.5) and (3.7). The median (50th percentile) displacements, computed via Eq. (3.4), are also plotted in Fig. 9(c). Furthermore, Fig. 9 (d) compares the 16th, 50th and 86th percentile displacements obtained from Eq. (3.4). In general, three predictive models result in rather consistent results for this case. Close inspection also reveals that the overestimation of BT07 model at long periods, as was reported in Bray and Travasarou (2007), are rectified in the proposed “Sa(1.5Ts)+Ia” and “PGA+Sa(2s)” model.

In conclusion, using the PEER-NGA strong motion database and an equivalent-linear sliding mass model, predictive equations are developed to estimate the seismic slope displacements using vector IMs. The new models yield comparable results with BT07 model, and are applicable for a wide range of $T_s$ and $K_y$ conditions. However, Sa(1.5Ts) used in BT07 and “Sa(1.5Ts)+Ia” model is dependent on the properties of the slopes. Sometimes, such a property-dependent IM may not be desirable in engineering application. Instead, “PGA+Sa(2s)” model employs property-independent vector IMs and the model demonstrates very good overall performance. The model has additional advantage in that it
is the only model that satisfies all the requirements of efficiency, sufficiency and predictability. Finally, it is worth mentioning that all of these predictive models are based on empirical data derived from simplified mathematical models, they are in fact “models of models”. The results should merely be considered as an index of the expected seismic performance of slopes. Eventually, comprehensive case studies are necessary steps to further validate these predictive models.

ACKNOWLEDGEMENT
The research is supported by Hong Kong Research Grants Council (grant No. 620311), which is greatly acknowledged.

REFERENCES
Campbell, K.W. and Bozorgnia, Y. (2008). NGA ground motion model for the geometric mean horizontal component of PGA, PGV, PGD and 5% damped linear elastic response spectra for periods ranging from 0.01 to 10 s. *Earthquake Spectra* **24**: 139–171.