Seismic Performance of Very Short Period Buildings

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SUMMARY:
Nonlinear dynamic analysis of very short period buildings indicates that these structures have probabilities of collapse that are much higher than that of longer period buildings, even though the short and long period buildings are designed under the same design rules. The higher probabilities of collapse for the shorter period buildings are related to the extremely high ductility demands that are computed for these systems. Observation of the performance of short period buildings that have been subjected to actual ground shaking does not indicate an unusual propensity of collapse for these systems. Potential reasons for the difference in computed and observed behaviour are explored in this paper, and recommendations are given for improved analysis and design of these systems.

Keywords: Collapse, Ductility, Nonlinear Analysis, Reinforced Masonry

1. INTRODUCTION

FEMA P-695 (FEMA, 2009) provides a computational methodology for determining the Seismic Performance Factors, R, C_d, and Ω_o, for buildings designed in conformance with the ASCE 7-10 (ASCE, 2010) minimum loads standard. In this methodology, a "Performance Group" of several mathematical "archetypes" of a given design procedure are systematically re-analyzed for 44 different ground motion records with increasing intensity. The ground motion intensity at which 22 out of the 44 archetypes collapse produces a dimensionless quantity called the Collapse Margin Ratio (CMR). The CMR is mathematically equal to the ratio of the ground motion intensity that caused the 22nd collapse, to the Maximum Considered Earthquake (MCE) ground motion intensity. The CMR is then adjusted to account for the spectral shape, producing the Adjusted Collapse Margin Ratio (ACMR). Finally, the ACMR, together with information related to uncertainty in the testing, analysis and design process, can be used to determine the conditional probability of collapse. An acceptable structural design is achieved when the probability of collapse, conditional on the occurrence of MCE ground motions, is no larger than 20% for individual archetypes, nor greater than an average 10 percent for all of the archetypes within the performance group. The MCE motion has a 2% probability of being exceeded in 50 years (or a 2475 year mean recurrence interval).

Initial analytical studies using the FEMA P-695 Methodology found that there was a general trend that structures with shorter fundamental periods of vibration ($T \leq 0.6$ sec.) had a higher probability of collapse and were more likely to fail the acceptance criteria than longer period structures ($T > 0.6$ sec.) of the same basic structural material and configuration. In the FEMA P-695 report, none of the shorter period systems actually failed the acceptance criteria, but these systems tended to have the lowest ratio ACMR to the acceptable ACMR for individual archetypes of a given performance group. Problems with short-period archetypes were particularly evident in subsequent exercising of the P-695 methodology, wherein several special reinforced masonry and special reinforced concrete shear wall systems were evaluated and reported in NIST Report GCR 10-917-8 (NIST, 2010). Several of the one and two-story systems with very short periods failed to meet the collapse related performance criteria or came very close to failing the acceptance criterion. All of the taller archetypes, with longer periods of vibration, passed the acceptance criterion.

The trends in the P-695 analyses indicate that it might be advisable to reformulate the ASCE 7 seismic provisions such that *R* becomes period dependent. This approach was suggested by authors of relevant sections of GCR 10-917-8, in which the results of P-695 analyses of reinforced masonry shear wall systems were investigated. The authors stated the following:

> "Current code provisions do not adequately distinguish between the wide range of performance characteristics of different masonry wall systems for which the use of the same *R* factor might not be appropriate. In particular, current codes do not account for the fact that the ductility demand induced by an earthquake ground motion on low-rise walls and high-rise walls can be very different. Their ductility capacities can be very different as well, so that different *R* factors may be needed for low-rise and high-rise walls."

As shown later in this paper, the design *R* value would need to trend towards 1.0 for very short period systems ($T < 0.2$ seconds) to eliminate the unacceptable collapse performance. This approach seems extreme given the lack of physical evidence (behavior of low period systems in real earthquakes) that there are a disproportionate number of collapses among shorter period systems.

The discrepancy between analysis that indicates poor performance and field observations that do not indicate poor performance may be related to assumptions made in the modeling of the structure and in how collapse was defined in the analysis. The authors of the masonry section of GCR 10-917-8 made this point, as follows:

> "Observed results were sensitive to assumptions made about the collapse behavior of reinforced masonry walls and decisions made in nonlinear modeling. Because of difficulties in quantifying collapse for low-rise walls, it was decided that collapse would be defined as excessive crushing of the masonry cross section or rupture of a significant percentage of the cross section. Neither of these conditions would necessarily lead to collapse in the low-rise shear walls system. Rather, collapse would more likely be expected to occur when drifts are so large that other gravity-load carrying systems lose their ability to carry vertical loads."

The authors of the section on reinforced concrete shear walls in GCR 10-917-8 had similar observations:

> "One and 2-Story archetypes failed to achieve the acceptable collapse margin ratios primarily because shear failures used as a proxy for collapse occurred at relatively low drift levels (below 1.5 %). In general, collapse of low-rise shear wall buildings has not been observed in an earthquake except in cases where the floor system failed (e.g. precast parking structures). This suggests that findings related to low-rise walls were biased by the modeling assumptions and potentially conservative criteria used to assess collapse. At this time, however, insufficient information exists to establish more liberal failure criteria."

Subsequent to the GCR 10-917-8 report, NIST supported an additional project, ATC 84, which dealt primarily with possible reformulation of the basic design values R, C_d, and Ω_o (NIST, 2012) in ASCE 7. A significant part of this project specifically addressed Short-Period systems. In this report, the following possible approaches for dealing with the "Short Period Problem" were forwarded:

1. Make no modifications to current specifications (and therefore accept a higher computed probability of collapse), justified on the basis that there is a not a substantial body of field evidence that short-period buildings utilizing a given lateral load resisting system have a greater tendency to collapse than do longer period buildings of the same system type.

2. Make modifications to detailing requirements such that the modes of failure of the lateral load resisting system (e.g., rocking or sliding in the reinforced masonry shear wall systems) are accommodated or controlled in such a way that the integrity of building structure is not jeopardized after the ductility limit has been reached.
3. Make modifications to the design specifications such that the probability of collapse of the short period systems is the same as that of longer period buildings using the same system. Such modifications would require a substantial reduction in the value of R used to establish the required strength of the system. For the purpose of predicting inelastic displacements, it would also be necessary to introduce a short period deflection multiplication factor, $C_{ds.}$ This factor is needed because the computed ratio of inelastic displacement to elastic displacement increases well beyond that predicted by the "equal displacement" or "equal energy" concepts when the period of vibration falls below about 0.4 seconds.

Ultimately, none of the above recommendations were made because it was felt that the mathematical models and the definitions of collapse used in the prior analysis were insufficient to properly determine if the Short-Period problem is real or is merely a relic of the analysis. These notions, as well as the preceding points, are further discussed in the remainder of this paper.

2. BACKGROUND

The fundamental problem with the very short-period systems is that such systems do not adhere to the "equal displacement" rule, which is the underlying principle for the R and C_d factors used for design (Newmark and Hall, 1982). This rule, based on observation rather than theory, indicates that displacements for elastic-plastic inelastic systems are approximately equal to the displacements computed for the same system responding elastically. However, it is well known that the rule is applicable only to systems with periods greater than about 1.0 second. As the period decreases below 1.0 second, an "equal energy" design basis might be more appropriate, and for very short period systems, with periods less than about 0.2 seconds, there is no observational relationship between elastic and inelastic displacements.

2.1. The sliding block analogy

For these very short period systems, ductility demands become very large and are impossible to accommodate with traditional detailing. This can be shown by use of a simple physical model, illustrated in Fig. 2.1. In the figure, an elastic-plastic system is represented as a block resting on a frictional interface. When subjected to ground shaking, the total displacement of the block (relative to its initial position) is equal to the elastic shear deformation in the block plus the "inelastic" sliding deformation. As the stiffness of the block increases (and the period decreases), the elastic deformation in the block decreases, while the sliding deformation stays relatively constant. Thus, it is impossible for the inelastic and elastic displacements to be similar. The ductility demand of the system will approach infinity as the block becomes stiffer, because the yield deformation approaches zero, while the total displacement demand stays relatively constant (and is nonzero).

For the sliding block of Fig. 2.1 there is virtually no recoverable elastic deformation in the block, thus it is likely that there will be significant residual deformation at the end of the event. Such behavior was reported in the dissertation by Jennings (1963) but is rarely mentioned in other studies dealing with computation of R.

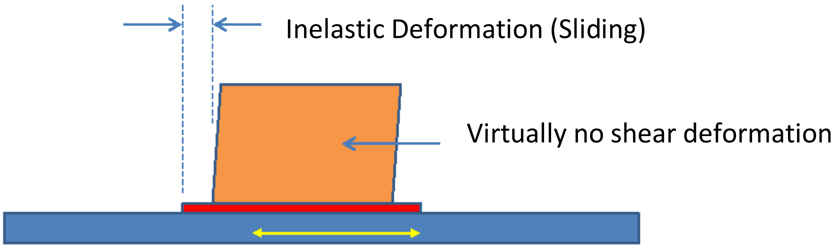


Figure 2.1. Rigid Block Sliding on a Frictional Plane

2.2 Response history analysis of a simple bilinear system

The trends indicated in the sliding block are manifested when a simple bilinear system is analyzed. Table 2.1 shows the results that were obtained when a structure of constant strength, but increasing initial stiffness is subjected to a recorded component of the Loma Prieta ground motion. In the analysis the system stiffness was adjusted to produce periods from 1.0 second to 0.1 seconds, in 0.1 second increments. For each system, an elastic analysis was run, followed by an inelastic analysis assuming elastic-plastic properties. As the period decreased from 1.0 seconds to 0.1 seconds, the ratio of the inelastic displacement to the elastic displacement increased from an average of about 1.0 for the first three systems (*T*=1.0, 0.9, and 0.8 sec) to a high of 45.6 for the system with *T*=0.1 seconds. The ductility demand increased from an average of about 5.0 for the first three systems to a high of nearly 150 for the system with T=0.1 seconds.

Table 2.1. Response of Elastic-Plastic System to the Loma Prieta Ground Motion when F_y= 40 kips

T (sec)	K (k/in.)	Elastic Disp. (in.)	Inelastic Disp. (in.)	Residual Disp. (in.)	Ductility Demand	Yield Excursions	Inelastic to Elastic Disp. Ratio
1.0	39.5	4.42	5.92	3.84	5.84	8	1.34
0.9	48.7	3.61	4.27	1.68	5.20	9	1.18
0.8	61.7	5.18	3.10	0.79	4.79	7	0.60
0.7	80.6	5.54	2.97	0.56	5.99	11	0.54
0.6	108.7	4.20	2.66	0.52	7.28	24	0.63
0.5	159.7	1.56	2.43	1.57	9.58	21	1.55
0.4	246.7	1.12	3.07	2.27	18.9	23	2.73
0.3	438.6	0.60	2.18	1.26	23.9	39	3.63
0.2	987.0	0.23	1.83	1.31	45.2	39	7.97
0.1	3984.8	0.03	1.51	0.96	148.6	13	45.6

Additionally, the response of the short period systems is dominated by residual deformations. This can be seen from Fig. 2.2, which shows the elastic and inelastic displacement histories for the system with T=0.8 seconds (top), and for the system with T=0.2 seconds (bottom). The response for the elastic and inelastic system with T=1.0 seconds is mostly transient in nature, whereas the behavior of the system with T=0.2 seconds is impulsive. This again illustrates that it is impossible to predict the displacement history of a very short period system using elastic analysis, because the elastic analysis cannot represent the impulsive behavior and the dominating residual deformation.

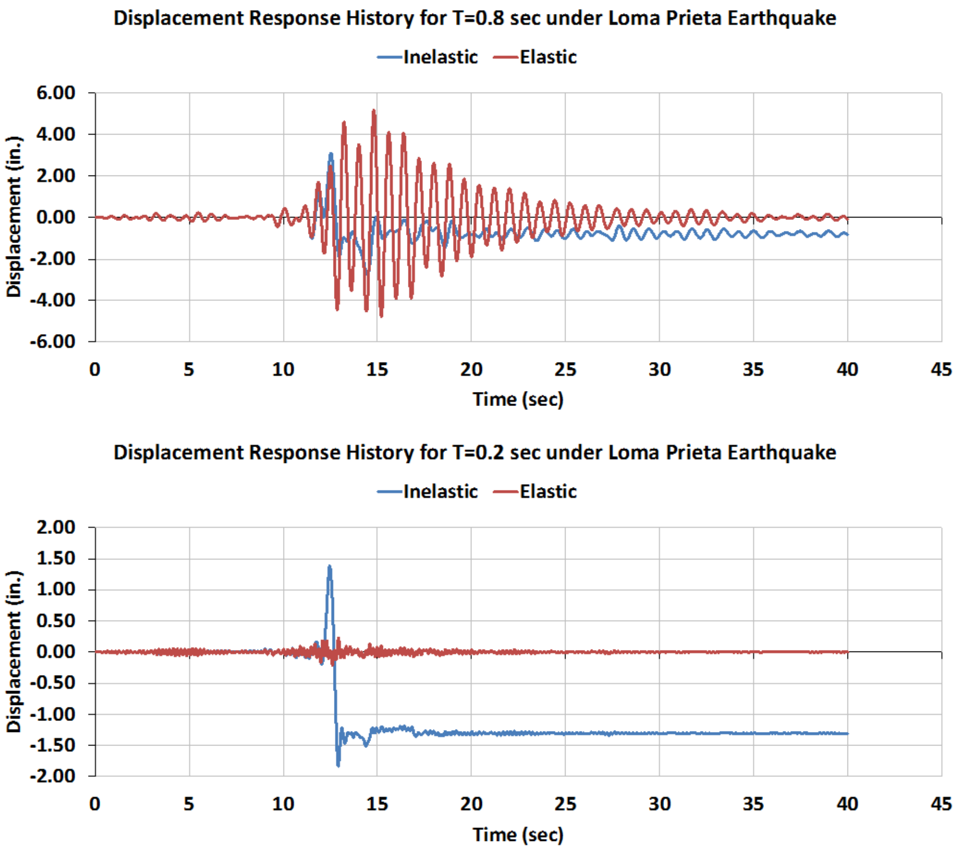


Figure 2.2. Response Histories for Systems with T=0.8 seconds (top) and T=0.2 seconds (bottom)

2.3. FEMA P-695 analysis of simple bilinear systems

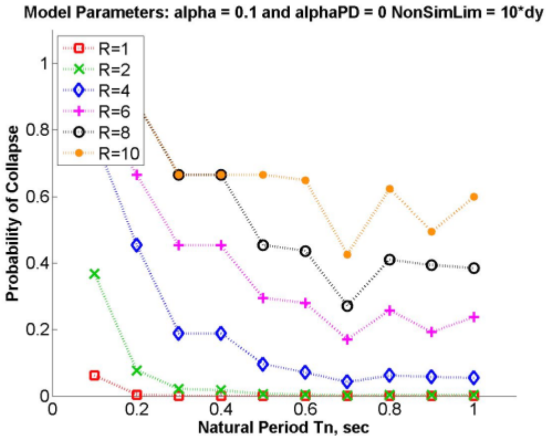
In order to obtain an understanding of the influence of short period on collapse probability, a "design space" of simple bilinear Single Degree of Freedom (SDOF) systems was designed with system R values ranging from 1 to 10 and with system periods ranging from 0.1 second to 1.0 second in 0.1 sec. intervals. Other variables included strain hardening stiffness (in terms of initial stiffness) and the ductility demand at which "collapse" was defined. Each model in the design space was analyzed using the P-695 methodology with Adjusted Collapse Margin Ratios (ACMRs) and probabilities of collapse reported for each model. Also reported in the analysis was the ratio of the median computed inelastic displacement to the median elastic displacement for each model.

A typical set of results from the SDOF study is shown in Fig. 2.3. For this set of results the system strain hardening (slope of second branch of bilinear response relative to initial branch) was 0.1, and "collapse" was assumed when the ductility demand reached 10.0. Part (a) of the figure shows the probability of collapse on the vertical axis, and the period of vibration on the horizontal axis. Results are collected in terms of R values, with one curve being plotted for each R value. As may be observed, only the R=1 system has a probability of collapse of less than 0.1 at all periods.

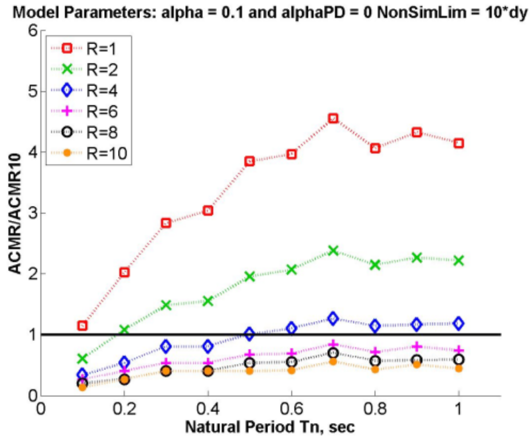
Part (b) of Fig. 2.3 shows the ratio of computed adjusted collapse margin ratio to the acceptable collapse margin ratio for 10% collapse probability, plotted against period, with one curve for each R value. Ratios less than 1.0 indicate a greater than 10% probability of collapse, and hence, a failure to meet the P-695 acceptance criterion for a given performance group. As may be observed, the R=1 system passes for all period values, and the R=6, 8, and 10 systems fail at all period values.

The curves in Fig. 2.3(b) can be used to interpolate the value of R at which the ACMR ratio is exactly 1.0 for each period analyzed. The resulting interpolated curve, shown in Fig. 2.3(c), indicates that the value of R required to meet the P-695 criteria approaches 1.0 as the period decreases towards zero and is approximately 1.0 for systems with a period of 0.1 second. R could be as high as 5.0 for systems with periods greater than 0.6 seconds. If desired, a curve such as that shown in Fig. 2.3 (c) could be used to establish a period dependent formula for R.

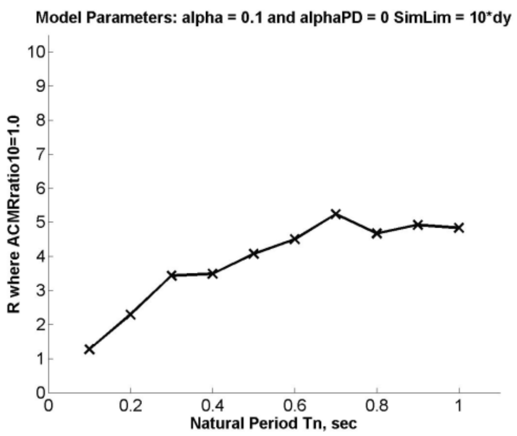
Part (d) of Fig. 2.3 shows the computed ratios of inelastic displacement to elastic displacement for the different systems. This ratio is approximately 1.0 for systems with periods greater than 0.6 seconds but increases exponentially as the period decreases from 0.6 seconds to 0.1 seconds. Note the very high ratios (greater than 8) for the larger R value systems with periods less than 0.2 seconds. Such results indicate that if the current R values are maintained (regardless of the computed probability of collapse), it might be necessary to make significant adjustments to C_d to accommodate the higher computed displacements. This is in stark contrast to the recommendation given in the P-695 report, where it is recommended that C_d be taken as equal to R for all systems.



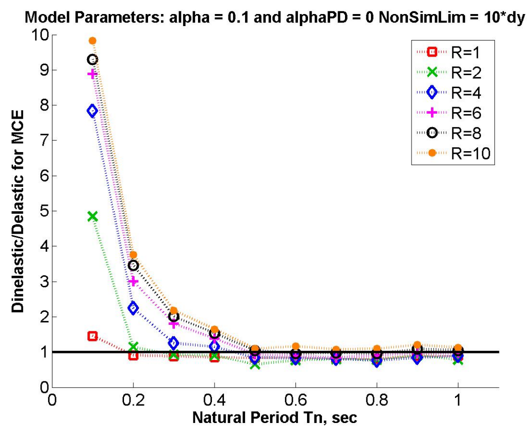
(a) Probability of Collapse



(b) ACMR Ratios for 10% Prob. of Collapse



(c) R Values for 10% Prob. of Collapse



(d) Inelastic to Elastic Drift Ratios

Figure 2.3. Results for SDOF systems with 10% hardening and a ductility of 10

2.4 Analysis of special reinforced masonry and steel buckling restrained brace archetypes

More traditional P-695 systematic studies of reinforced masonry archetypes were also run, and the results followed trends very similar to that shown above for the SDOF systems. In this study, 1-, 2-, and 4-story archetypes, subjected to heavy gravity load and Seismic Design Category (SDC) D_{min} and D_{max} shaking, were redesigned using R values of 1, 2, 4, 6, and 8 and analyzed using the FEMA P-695 methodology to determine how the collapse margin ratios and probability of collapse vary with design R values. Also computed in the analysis were the ratios of the peak computed inelastic displacement to the peak elastic displacement. A sample of the results of the analysis is shown graphically in Fig. 2.4. As may be observed form the graph on the left side of the figure, the probability of collapse exceeds 0.1 (10%) at periods less than about 0.25 seconds for the R=2, 4, 6, and 8 systems and is barely above 10% for the R=1 system with a period of 0.2 seconds. The right side of the figure shows greatly increased ratios of inelastic to elastic displacement, which is consistent with the SDOF results.

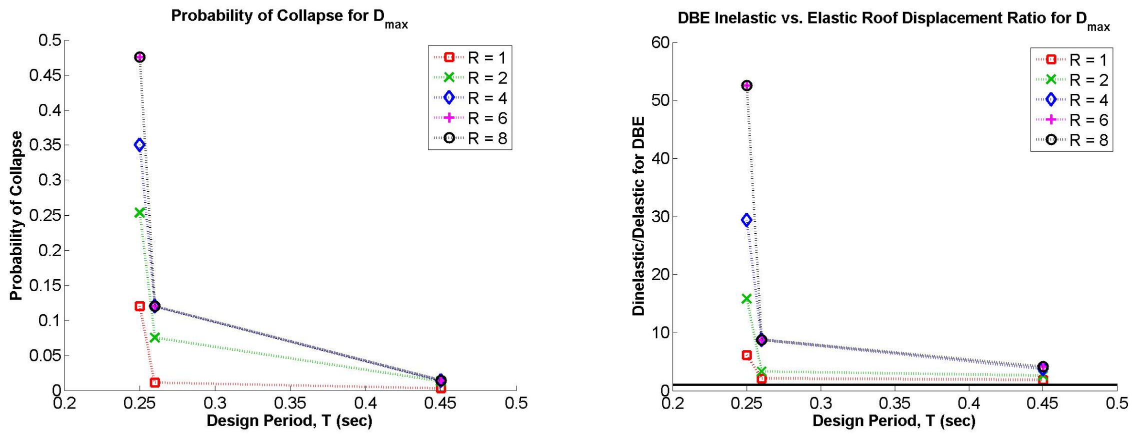


Figure 2.4. Results for Special Reinforced Masonry Wall Systems

It is important to note that the mathematical models for the masonry walls consisted of a simple cantilever representation, with a fixed base condition and did not include inelastic shear deformation or sliding at the wall foundation interface. The constitutive modeling provided restrained rocking of the wall. For these walls collapse was based on exceeding certain limiting strains in the concrete and reinforcement in the wall cross sections. This "nonsimulated" collapse often occurred at lateral drifts that were less than 1% of the story height. A detailed description of the modeling approach can be found in (NIST, 2010 and NIST, 2012).

Short period steel buckling restrained brace frames were also studied. The design space consisted of one- and two-story systems designed for SDC D_{max} and D_{min} ground motions and for R=2, 4, 8, and 8. Thus a total of sixteen systems were designed and evaluated. The buckling-restrained brace systems had a clear trend of increased probability of collapse as the system period decreases, and there was some tendency for the ratios of inelastic to elastic displacement to increase as the design period decreased. It is noted, however, that none of the BRB systems failed the FEMA P-695 acceptance criteria, and the ratios of inelastic to elastic displacement never exceeded 1.1, even for the shortest period system. Detailed descriptions of the modeling approach and the results of these studies are presented in (NIST, 2012).

3. SUMMARY OF KEY FINDINGS

The study of SDOF systems and reinforced masonry archetypes demonstrated consistent behavior as follows:

1. If it is desired to maintain a probability of collapse of 10% under MCE ground motions, the design *R* values should range from about 1 for systems with periods less than about 0.2 seconds and increase to about 5 for systems with periods of about 0.6 seconds. The probability of collapse is not strongly sensitive to the *R* value for periods from 0.6 seconds to 1.0 seconds.

2. The ratio of computed inelastic displacement to computed elastic displacement is about 1.0 for systems with periods ranging from 0.6 seconds to 1.0 seconds but increases exponentially as the period decreases from about 0.6 seconds to 0.1 seconds. In the period range of 0.1 to 0.2 seconds, the computed ratios of inelastic to elastic displacement far exceed those predicted using the "equal energy" concept.

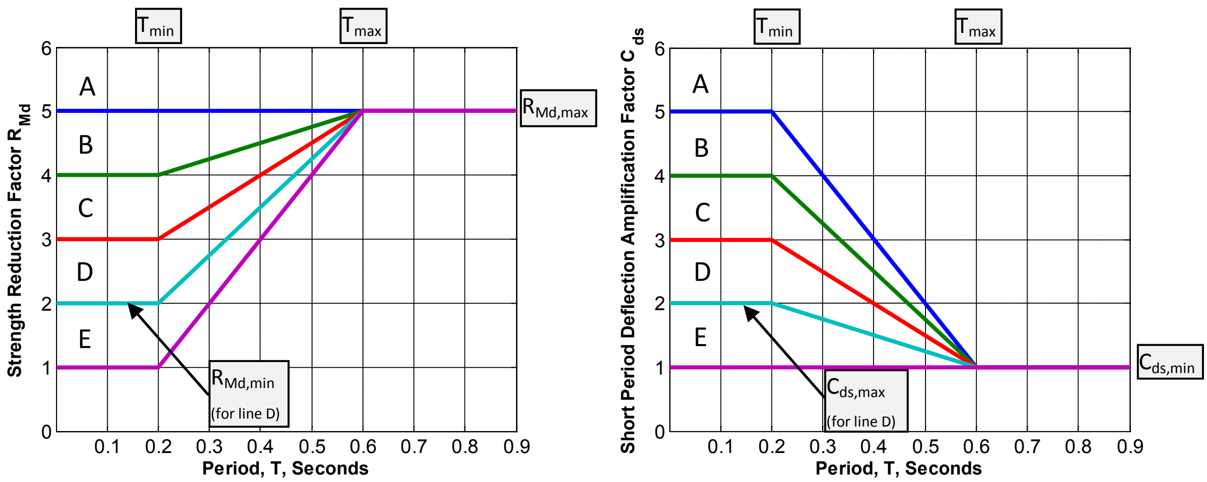
If it is desired to maintain a 10% probability of collapse under MCE shaking across all periods less than 0.6 seconds (given a constant collapse metric, such as a limiting strain in shear wall reinforcement), it is necessary to reduce the design *R* value as the period decreases, with a limiting value of *R*=1 being required when the period is less than about 0.2 seconds. It is very important to note, however, that the reduction in *R* towards the limiting value of 1.0 is needed because the ratio of inelastic displacement to elastic displacement increases exponentially as the period decreases. For very short systems it is impossible to provide sufficient ductility to accommodate this behavior, so a design *R* value of 1.0 is needed.

As noted in NIST GCR 10-917-8, it is doubtful that exceeding the collapse metric for the reinforced masonry and reinforced concrete shear walls systems would lead to a true collapse of the system, where collapse in this sense would include the loss of the gravity load resisting system. If, for example, the wall reaches its strain-based collapse at an interstory drift of 1.0 percent of the story height, and the system loses its gravity load resisting capacity at an interstory drift of 2.5 percent of the story height, there is a range of 1.5 percent drift in which the wall must be able to continuously rotate or slide after the strain limit is reached. If the wall is detailed to accommodate that additional deformation, the system will not collapse so long as the 2.5 percent drift limit is not exceeded. If the P-695 collapse metric is adjusted to represent the full system failure and not the wall failure, the probability of true collapse could be determined. It is recognized that this is not the intent of P-695, and thus the use of P-695 in this context is debatable.

3.1. Discussion of Results Pertinent to Reformulation of the ASCE 7 *R* and C_d Factors

A possible reformulation for *R* and C_d for short-period systems is proposed in Fig. 3.1. Fig. 3.1(a) shows a period-dependent *R* factor, and Fig. 3.1(b) shows a period dependent C_{ds} factor, where C_{ds} represents the "Short Period Displacement Multiplication Factor. C_d would be taken as C_{ds} times *R*. FEMA P-695 essentially uses C_{ds}=1, because that document recommends that C_d be taken as equal to *R*. In these figures a series of lines, labeled A through E, are provided that represent a range of approaches for varying the parameters.

If it is desired to have a uniform probability of collapse across all periods, the relationship between *R* and period would look like line "E" on Fig. 3.1(a). Line "A" on the same figure represents the current approach wherein the same reduction value is used for all periods and where the probability of collapse theoretically increases significantly at very short periods. Lines B, C, and D represent intermediate approaches.



(a) Variation in *R* with Period

(b) Variation in C_{ds} with Period

Figure 3.1 Empirical expressions for Period-Dependent Design Parameters

For a system designed using line "E" the value of C_{ds}, would be 1.0 for all periods. This is shown in Fig. 3.1(b). A constant value of C_{ds} is used because for low periods (T<0.2 sec) the system is responding elastically (C_d=R=1), and for periods greater than 0.6 seconds, the value of C_{ds} is also 1.0. There is no basis for varying the value of C_{ds} above 1.0 in the intermediate period range.

However, if the value for *R* is not period dependent (the probability of collapse of greater than 10% for a performance group is accepted), it is theoretically necessary to provide a short period displacement amplifier in the very short period range T < 0.2 seconds and to transition this multiplier to 1.0 for periods greater than 0.6 seconds. Such a transition is provided graphically by line "A" on Fig. 3.1(b). Here the maximum short-period deflection multiplier is 5.0 for very low periods. This value is in the general range of inelastic to elastic deflection ratios determined from the analysis of SDOF systems. Note that line "E" on Fig. 3.1(b) corresponds to the case where *R* is modified to produce a constant probability of collapse across all periods.

Lines "B", "C", or "D" on Figures 3.1 (a) and (b) could be used if an intermediate design was desired. Note, however, that the same alphabetically designated line would be used from each figure (for example, one would use lines D from each figure, and not line D from Fig. 3.1(a) and Line B from Fig. 3.1(b).

Clearly, the use of lines "E" would have a great impact on the required strength of structures and may not be economically justifiable given the scarcity of evidence that short period systems are indeed problematic. The use of Line "A", which increases displacements at short periods, but may not be as severe a penalty, because short period systems are rarely drift controlled. In all cases, significant research would be required to determine the appropriate period bounds and upper and lower limits on *R* and C_{ds} in Figures 3-1(a) and 3-1(b). Another factor in the research is whether the period limits should be based on empirical or computed periods of vibration. The use of expressions as illustrated in Fig. 3.1 would, of course, be system dependent, and in some cases (e.g. steel moment frames), the figures would probably not be needed, because even 1-story systems often have periods exceeding 0.4 seconds.

4. SUMMARY and CONCLUSIONS

Given the lack of clear evidence that short-period systems are problematic outside of the computational/theoretical arena, it seems unwise to proceed with a recommendation to make significant adjustments to *R* for short period systems. However, adjustments to the computed deflection of short period systems might be warranted. Thus, the principal preliminary recommendation is to make no modification to *R* but to further develop a period dependent relationship for C_{ds}.

It is essential to note that any final recommendation to provide period dependent expressions for C_{ds} (or for *R*) must come only if additional studies on short period systems indicate that this is necessary, and that such formulas represent the best approach for "solving" the short-period problem. Key features of such studies should include improved modeling of material behavior, improved component modeling, improved system modeling, refined definitions and metrics for collapse, and re-thinking of the pure ductility-based design paradigm for very short period structures (in which post-yielding loss of strength accompanied by limited sliding and rocking does not necessarily indicate collapse).

A second approach (not discussed herein but described in NIST, 2012) that has been recommended for short-period systems is to increase the ductility supply of the systems as the period reduces. This concept, adopted in Eurocode 8 (BSI, 2005), does not seem reasonable for very short period systems given the extremely high ductility demands.

In both of the above approaches, the solution attempts to force short-period systems to be designed on the basis of assumptions that perhaps can be successfully applied only to longer period systems, such as models that respond to masses lumped at floors levels, bases that are fixed against sliding and rotation, and rigid diaphragms. Thus, a third way to resolve the short-period problem is to recognize that the traditional approach of dissipating energy entirely through inelastic material behavior is not viable for systems with extremely short periods. Instead, these systems could be designed by a completely different set of rules, not yet developed.

A final possible approach is to make no modifications to design rules or system behavior and accept the increased probability of collapse (from the perspective of the analysis of FEMA P-695) on the basis that there is little experimental or post-earthquake evidence that the short-period problem exists outside of the theoretical arena. Additionally, the "make no modification" approach may be made on the basis of arguments that the nonsimulated collapse metrics used in FEMA P-695 and NIST GCR 10-917-8 analyses were not particularly realistic for these buildings because a total system collapse (complete loss of the structure, including the gravity system) would probably not occur as a result of the nonsimulated collapse parameter being exceeded.

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