A numerical model for push-over analysis of RC structures with plate roofs.

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SUMMARY:  
The Lorca earthquake of May 11, 2011 which caused significant damage to many buildings, both in structures and perimeter walls has highlighted the fragility of reinforcement concrete flat-roofed structures. These ones, from many years ago, are still being built in Spain. The basic scheme is solid plate, or lightweight plate, and columns of reinforced concrete. Due to architectural requirements, often column plant positioning is random, i.e. it fails stiffness distribution criteria in plant. This paper presents the study of the seismic vulnerability of these buildings by pushover analysis. The modeling of the columns and the slabs is made with frame elements, which allow a good approximation to describe the mechanical behavior of reinforced concrete structures. Finally, it has been shown as the ductility factor of the structure behavior proposed by the Spanish seismic code in these cases underestimates the seismic action or, in other words, overestimates the ductility of these structures.

Keywords: Ductility, Capacity design, Pushover analysis, flat-roofed structure.

1. INTRODUCTION

The flat plate system has been adopted in many buildings constructed recently taking advantage of the reduced floor height to meet the economical and architectural demands. The basic scheme is solid plate, or lightweight plate, and columns of reinforced concrete. Often column plant positioning is random, i.e. it fails stiffness distribution criteria in plant.

Moreover, the flat roofed structures, unlike framed structures, decrease the response effectiveness of the "frames" to the horizontal translation and indeed, many standard codes recommend using unitary ductility factor of the structure in the calculation of the seismic action.

To set a real case, it is very common to see joint connections between plate and column where the plate thickness is greater than the column one. Therefore, this practice adopts geometric relationships of plate and columns, far removed from seismic design criteria that nowadays many standard codes recommend. If we consider that the perimeter walls are often very heavy as they are partially constructed with solid brick, it is clear that the structures of most of these buildings have a high seismic vulnerability.

The approach to the problem of the study of the seismic vulnerability of these buildings looks to implement a numerical model for computational analysis. It is important to model the plates and the columns by means of finite element able to take into appropriate account the non-linear behavior of the sections. The push-over analysis allows us to assess the displacement capacity of structures in different directions.

Analysis of the push-over curves allows us to individuate critical directions, i.e. directions with potential brittle fracture. In other words, the cross reading of the push-over results coupled with the
results of classical modal analysis, which would check that the direction considered is interesting to mobilize a large percentage of excited mass, provides engineers with a powerful and at the same time rapid tool, to evaluate the structural response in many different directions, like in the case of an earthquake.

2. THE PUSHOVER ANALYSIS

The pushover analysis consists of a incremental analysis set-by-step with constant vertical loads and horizontal seismic force which increases gradually until the formation of a mechanism that represents the collapse of the structure. The analysis allows to define a force-displacement curve and derive, for each direction of analysis, the overstrength relationship that expresses the ratio between the maximum acceleration and the acceleration achieved at the elastic limit.

![Figure 1. Pushover analysis scheme](image)

The scheme of a pushover analysis is developed in the following way:

1. Choosing a force distribution or a deformation form.
2. The load is applied vertically and then horizontally chosen as the basis for the pushover analysis; we draw the force-displacement curve of the MDOF system.
3. It uses the elastic response spectrum to obtain the displacement demand.
4. At the demanded displacement, to check the damage state of the structure and compared with that of limit states imposed.

Point 3 represents the general way proposed for the verification of pushover results. The process uses a one degree of freedom system which is elastic perfectly plastic equivalent to the structure under examination. The method is easy to apply but it will lose the rigor typical of the non-linear analysis.

3. FINITE ELEMENT MODELLING

The discrete model of the building structure is obtained by modeling beams, columns and floors by using one-dimensional elements like frames. In particular, the slabs are modeled by means of an equivalent trellis of one-dimensional elements, so as shown in Fig. 2.
The choice of using frame elements to describe the floors is due to the possibility to check the mechanical behavior of the elements in non-linear range, from which depends strongly the structure response under the action of the loads. The error introduced by the proposal model have been estimate in several studies and found acceptable by a technical point of view (see Sawczuk).

The mechanical behavior of the sections, described by the moment-rotation relationship, was obtained according to the guidelines FEMA-356, using the stress-strain curves for concrete suggested by Kent and Park.

It is assumed for both materials the bonds suggested in Euro-code 2 to the case of non-linear analysis.

With the proposed model has been made non-linear analysis of a circular plate reinforced concrete with a thickness of 30cm, resting on the edge and subjected to a uniformly distributed load. The armature of the lower and upper plate is made with bars with diameter of 10mm arranged in a distance of 10 cm in both directions. It is assumed for the concrete a maximum resistance fcm = 27.6 MPa and for the steel a yield stress fyk = 455.0 MPa. Then, section of the plate presents an ultimate moment capacity Mu = 97.7 KNm/m.

Table 3.1 shows the results obtained with two discrete models and compared with the reference value derived from Sawczuk.

<table>
<thead>
<tr>
<th>Frame mesh</th>
<th>$q_u \times r^2/M_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame mesh 10 × 10</td>
<td>6.38</td>
</tr>
<tr>
<td>Frame mesh 20 × 20</td>
<td>6.50</td>
</tr>
<tr>
<td>[Sawczuk]</td>
<td>6.51</td>
</tr>
</tbody>
</table>

The values obtained show that an approximate modeling of the plates with an equivalent trellis technically provides reliable results even in the plastic range, offering the advantage of proper control of the mechanical behavior of the finite element frame. Fig. 4 shows the load curve of the plate and the evolution of plasticity.
4. PUSHOVER ANALYSIS OF RC STRUCTURES

The building analyzed in this section is representative of the typical construction method of RC flat-roofed building. It presents dimensions of 12x18m and four storey. The vertical elements are constituted by columns which dimensions are 30x30cm$^2$, while the horizontal elements are made with a lightened plate.

For the push-over analysis of the building creates a finite element model. The columns and floors are modeled with frame elements, using for floor the equivalent trellis modeling. In order to model the non-linear behavior, it has been assigned to the each element of the model the corresponding hinge with non-linear behavior. Model approach is made following assumptions of Mehmet-Hayri Baytan article.

SAP2000 implements the plastic hinge properties described in FEMA-356 (or ATC-40). As shown in Fig. 6, five points labeled A, B, C, D, and E define the force–deformation behavior of a plastic hinge.
The values assigned to each of these points vary depending on the type of element, material properties, longitudinal and transverse steel content, and the axial load level on the element. SAP2000 provides default-hinge properties and recommends PMM hinges for columns and M3 hinges for beams. Once the structure is modeled with section properties, steel content and the loads on it, default hinges are assigned to the elements (PMM for columns and M3 for beams). There is no extensive calculation for each member. The definition of user-defined hinge properties requires moment–curvature analysis of each element.

![Figure 6. Force–deformation relationship of a typical plastic hinge](image)

The mechanical behavior of the frame element sections, described by the moment-rotation relationship, was obtained according to the guidelines ATC-40 or FEMA-356, using the modified Kent and Park model for confined concrete. Therefore, the typical steel stress–strain model with strain hardening for steel are implemented in moment–curvature analyses.

The push-over analysis was performed using constant and lineal distribution of load.

![Figure 7. FEM model and load distribution in principal direction](image)

As a result of the pushover analysis are obtained capacity curves in the two principal directions and for both load distributions. See figure 8.
The analysis of the results shows the behavior slightly ductile typical of flat-roofed structures, especially for the constant distribution load for which plasticize quickly the columns of the ground floor. These ones are highly stressed for the gravity actions and for the seismic actions both. The linear distribution load shows an increase of ductility due to increasing of the horizontal action on the upper floors where the columns are less stressed, so it reduces the negative effect on the lower floors. The deformation of the structure shown in the graphs is inherent to the constant distribution load and it presents the typical behavior with a mechanism on the ground floor (soft storey).

So as a further result, we can derive the overstrength ratio between the maximum acceleration and the corresponding to the first yielding one (elastic limit). It quantifies the ductile behavior of the structure. A comparison between the ductility factor proposed by the Spanish seismic code shows how it underestimates the seismic action for this type of structures or, in other words, it overestimates their ductility. In the following table have shown the overstrength ratio carried out from no linear analysis.

**Table 4.1.** Overstrength ratio for the example building (in brackets linear distribution of load).

<table>
<thead>
<tr>
<th>Direction</th>
<th>Ductility factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0º</td>
<td>1.69 (1.95)</td>
</tr>
<tr>
<td>45º</td>
<td>1.25 (1.75)</td>
</tr>
<tr>
<td>90º</td>
<td>1.69 (2.07)</td>
</tr>
<tr>
<td>[Spanish seismic code]</td>
<td>2</td>
</tr>
</tbody>
</table>

However it is important to note that engineers use in calculations the unitary factor ductility (without ductility) and that provides an extra degree of safety.

6. CONCLUSION

The model studied is an example of how to analyze and evaluate the nonlinear response of flat-roofed structures under horizontal actions. The results obtained are fully consistent with the damages observed in this type of structures in the recent Lorca earthquake, in May 2011. This type of analysis is easy and fast with the engineering tools available today. On the other hand remains to be investigated, for this type structures, the behavior of different types of plastic hinge.
**Figure 9.** Collapse mechanism on the ground floor (soft storey and brittle column).

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**REFERENCES**