Application of a Newly Fiber Model for Load Bearing Masonry Members

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SUMMARY:
In the last few years, the possibility of being able to control the damage based on the probability of occurrence of an earthquake and designing on the basis of different performance levels, has arose. Masonry is still a widespread construction system for low-rise residential buildings even for countries prone to seismic risk, hence masonry needs to develop these design concepts.

Experimental tests were performed in recent years at the University of Padova on different masonry systems, both reinforced, and unreinforced with different joints types. The tests were aimed at characterizing the masonry behaviour under combined in-plane cyclic loading, and they were used to develop an analytical model that reproduce and extend the experimental results using parametric analyses.

This model is a formulation of a fiber element and is cast in the general framework of the mixed method. It includes effects of shear deformation, diagonal shear failure mechanism and it is able follow response in post-peak phase. The model is able to interpret the performances of masonry panels linking them with limit states resulting from integration of cross-section equilibrium equations.

The experimental results were extended throughout parametrical analyses using the analytical model and, finally, generalized proposing design equations directly related to performance levels and both geometrical and mechanical properties of URM panels.

Keywords: unreinforced and reinforced masonry, analytical modeling, displacement capacity, seismic design

1. INTRODUCTION

Finite Element modelling can provide powerful analysis tools for masonry structures; however, they are computationally demanding and a large number of user-specified parameters are needed for the constitutive behaviour definition. There are different approaches for modelling nonlinear behaviour of masonry structures: limit analysis, or models based on so-called storey-mechanism approach like the POR method or the so-called macro-element discretization (SAM method).

A different approach is proposed by (Benedetti and Steli, 2008). With a simple no-tension material and simple elastic perfectly-plastic rule for masonry and the usual hypothesis of cross section remain straight, these authors present an explicit formula for the shear-displacement curve of a URM pier, by integration of the curvature diagram. Also extend to the case of masonry with Fiber Reinforced Polymer (FRP) by introducing the hypothesis of an elastic–plastic resisting force (equal to the debonding value) of the FRP reinforcement up to the crushing of the masonry in compression. This is a very simple model includes only an elastic shear deformation and able to reproduce the behaviour until the ultimate ductility only accounting a flexure failure. Anyhow this work suggested that a cross sectional analysis approach can be developed also for masonry walls.

From the last consideration (Guidi, 2011) has developed a fiber model for masonry walls both unreinforced and reinforced. The model is a formulation of a fiber element and is cast in the general framework of the mixed method. It is based on a non-linear algorithm that always maintains static equilibrium within the element and converges to a state that satisfies the element constitutive relation within a specified tolerance. The proposed solution algorithm is suitable for the analysis of the highly
non-linear behaviour of softening members, such as reinforced and unreinforced masonry piers under varying axial load. The formulation of the wall element is based on the assumption of linear geometry. The element is subdivided into a discrete number of cross sections. Plane sections remain plane and normal to the longitudinal axis during the element deformation history. This hypothesis is acceptable for small deformations of elements composed of homogeneous materials, but it is also applied in studies which focus on hysteretic behaviour under large inelastic deformation reversals (Spacone et al., 1996). The developed model starts from Moment-Curvature analysis of panel with isostatic boundary conditions (fixed to the base) and considers masonry wall as an isotropic homogeneous material. Then, from pure flexural analysis, considering the contribution of vertical reinforcement bars at wall ends to the flexural strength and displacement (when they are present), the model adds shear deformation contribution. The model follows the wall behaviour in the post-peak branch of capacity curve, takes into account possible indirect tensile diagonal shear failure of panel and the strength contribution of horizontal reinforcement bars (when they are present), and approximates shear strength decay with increasing displacements.

From the suggestion, provided by Moehle (1992), Priestley (1993) among the others, that deformations can control damage induced by earthquake design procedures, the so-called Displacement-Based Design (DBD) methods, were developed. A comprehensive summary on these topics can be found in the report of FIB Bulletin 25 Calvi (2003) and Sullivan et al. (2003).

One of the most promising DBD methods available in literature is the so-called Direct Displacement-Based Design (DDBD) by Priestley et al. (2007). Its main idea is to identify, at the beginning of the design process, the design displacement that ensures an acceptable performance under the considered earthquake intensity and limit state taken into account. An advantage of this method is that it allows designing structure directly for different seismic intensity levels. In Italy, one topic (Linea 4) of the recent RELUIS project further developed DDBD methods. The results for the various structural typologies reached heterogeneous levels, due to different available state-of-the-art. Indeed, DDBD had been already developed in detail for some structural typologies (e.g. reinforced concrete structures), whereas for others, e.g. for masonry and retaining structures, it was at a first application level. The results are available in the RELUIS project final report Calvi and Sullivan (2009).

In this context, the work presented in this paper is a contribute to the development of DDBD method for masonry buildings. In particular, some suggestions about displacement capacity of masonry members, directly related with performance levels, are presented.

Experimental tests recently performed at the University of Padua on various masonry systems, were aimed at characterizing the behaviour of both reinforced and unreinforced masonry walls, with different horizontal and vertical joints types, under combined vertical and in-plane cyclic horizontal loads. The test results were used to validate of an analytical model which was then developed Guidi (2011). This model can reproduce the envelope curves of cyclic shear-compression tests and interpret the performances of walls relating them with limit states resulting from integration of cross-section equilibrium equations. The experimental results were extended throughout parametrical analyses using the analytical model and, finally, generalized proposing design equations directly related to performance levels and both geometrical and mechanical properties of URM panels.

2. USED MASONRY SYSTEMS

Masonry systems were built with traditional materials but using innovative building solutions. Unreinforced masonry was made with Thin layer joints (TM); with ordinary bed joint and interlocking system (Tongue and Groove) on the head joints (TG); and with ordinary bed joint and units with pocket for mortar infill (Po) (see Figure 1). The experimental and numerical results obtained on these masonry types are thoroughly described in da Porto et al. (2009), (2010).

The reinforced masonry system is based on the use of concentrated vertical reinforcement. Vertically perforated units are used for the reinforced confining columns, and special clay units, with horizontal holes and recesses for horizontal reinforcement, are used for the main portions of the masonry walls (see Figure 2). This system was developed within the European Project DISWall (2008). The construction system and the experimental and numerical results obtained on these masonry types are thoroughly described da Porto et al. (2011), Guidi et al. (2010).
3. VALIDATION OF PROPOSED MODEL AND PARAMETRICAL ANALYSES

The required inputs to build the moment and curvature functions are geometry, material properties and boundary conditions. These are respectively L, H, t for length, height and thickness of wall (geometry), $f_m$, $E_m$ and $G$ which are compressive strength, elastic and shear modulus of masonry (material properties, see also Figure 3), and $\sigma_0$ that is vertical compression stress (boundary condition). Obviously, in case that reinforced masonry is modelled, more input data are required, such as area of vertical and horizontal re-bars ($A_v$ and $A_{vh}$ respectively); for vertical re-bars, position with respect to the wall edge ($d'$); spacing between horizontal re-bars ($s$), and steel material properties, such elastic moduli ($E_s$ and $E_{sh}$ respectively) and steel yield strengths ($f_{ys}$ and $f_{ysh}$). Shear failure is evaluated considering masonry piers subjected to global mechanisms. Shear strength ($V_s$) is a sum of two contributions: masonry shear strength as a tensile-induced failure that takes into account vertical load (Turnšek and Čačovič, 1971, Tomaževič and Lutman, 1988) ($V_{m}$) and horizontal reinforcement ($V_{sh}$, for reinforced masonry only).

To validate the analytical model, a series of analyses to reproduce the experimentally observed data were done. The calibration process was quite easy thanks to the simplified hypotheses used in the model, and the resulting limited number of parameters needed. The model uses, obviously, the same geometrical and boundary conditions of experimental test set-up (namely L, H, t, $\sigma_0$). The mechanical parameters result from experimental tests done. Starting from the experimental results, $G$ was the only value to be calibrated, to catch the initial stiffness of the capacity curves. In general, modelled walls are greatly influenced by shear deformation and, obviously, shear deformation is controlled to a great extent by shear modulus ($G$). Despite this sensitivity, model was in good agreement with experimental
G obtained from shear-compression tests and with lower bound of values provided by Circolare 2/02/2009 n. 617 C.S.LL.PP. (2009). Shear strength formulation adopted was able to correctly forecast experimental walls subjected to shear failure and also, with acceptable approximation, their loads and displacements.

![Figure 3. Masonry stress-strain relation.](image)

As an example, Figure 4 shows the model versus experimental load-displacement capacity curves of two reinforced masonry walls. These are both under a vertical compression load of $\sigma_0 = 0.6$ N/mm$^2$. The diagram on the left shows a wall characterized by shear failure (with slenderness ratio $H/L=1.09$) and the diagram on right shows a flexural failure condition (with the slenderness ratio $H/L=1.63$).

In the first case, the sequence of limit states was: horizontal cracking at base cross-section, first non-linearity in masonry ($F_e$), achievement of masonry shear strength ($V_m$), masonry yielding in compression ($F_{ym}$), maximum horizontal capacity ($F_{max}$) and maximum displacement ($d_{max}$). In the second case the, sequence was similar but, before reaching $F_{max}$, yielding of vertical reinforcements in tension ($F_{yt}$) also occurred. In Figure 4 the red dots represents the limit states. For a complete

<table>
<thead>
<tr>
<th>Wall</th>
<th>$f_m$ [N/mm$^2$]</th>
<th>$E_m$ [N/mm$^2$]</th>
<th>$G$ [N/mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM</td>
<td>6.95</td>
<td>4497</td>
<td>700</td>
</tr>
<tr>
<td>TG</td>
<td>5.67</td>
<td>4983</td>
<td>600</td>
</tr>
<tr>
<td>Po</td>
<td>5.34</td>
<td>5113</td>
<td>550</td>
</tr>
<tr>
<td>RM</td>
<td>4.00</td>
<td>5835</td>
<td>650</td>
</tr>
</tbody>
</table>

![Table 1. Mechanical parameters used modeling masonry](image)

**Figure 4.** Model (red line) versus Experimental (blue lines) load-displacement capacity curves (the positive and negative displacement envelopes for $\sigma_0=0.6$ N/mm$^2$). Reinforced masonry $H/L=1.09$ (left) and $H/L=1.63$ (right).
description of model and its validation see Guidi (2011), it comprises both reinforced and unreinforced masonries with different joint arrangements.

After model validation, a series of parametrical analyses aimed at extending the experimental results and investigating the influence of different parameters on masonry behaviour was carried out, on both unreinforced and reinforced masonry systems. For unreinforced masonry system these analyses included the variation of unit compressive strength (from 5 to 20 N/mm²) with 2 different vertical loading combinations: constant vertical load equal to experimental values; constant vertical stress ratio with masonry compressive strength. The wall slenderness (H/L ratio), adding 0.95 and 1.7 (L = 1.24 and 0.74m) ratios to the experimental condition (see Figure 5) was also varied. For reinforced masonry system, the parametric analyses included the variation of axial load, from 0.2 to 2.0 N/mm²; the wall H/L ratio from 0.65 to 2.19 (L = 2.60÷0.75m); the vertical reinforcement percentage, from 0 to 0.27%; the horizontal reinforcement percentage (no reinforcement, any other joint and all joints, see Figure 6). All analyses were repeated using two different vertical reinforcement ratios: the experimental one and the minimum required as prescribed in Italian Seismic Code (DM 14/01/2008, 2008).

The parametrical analyses carried out with the model presented in this paper, were also carried out using a FEM modelling approach, calibrated on the experimental data. The good correlation that was found between the two approaches (FEM and the analytical approach presented here) and it further validates the results of the parametrical analyses. Details of the FEM analyses of unreinforced and reinforced masonry can be respectively found in da Porto et al. (2010) and Guidi et al. (2010).

In this section an example of parametrical analyses results in terms of drift (ψ=δ/H), at each main limit state, is presented. In any case, the work shown here was repeated for all combinations of different parameters, which were varied in order to see the influence of mechanical and geometrical conditions on displacement capacity of masonry walls under in-plane actions Guidi (2011).

Figure 7 shows vertical stress variation in reinforced masonry walls when critical drift is achieved (above on the left), at maximum horizontal capacity (above on the right) and at maximum displacement (below). The results can be grouped by slenderness (green and yellow for slender walls, and blue and red for squat walls) or by vertical reinforcement ratio (green and blue for minimum reinforcement required by Italian code, and yellow and red for experimental reinforcement).

The second, and fundamental, limit state refers to the attainment of masonry compressive strength (see Figure 3) and start of yielding branch in the compressed zone, i.e. at the wall toe. At this point masonry stress-strain curves enter a plastic phase and the walls leave the linear behaviour. In the case of reinforced masonry this limit state can be fixed when masonry contribution to shear strength (Vₘ) is reached if it develops before masonry start the yielding phase. This limit state was chosen as critical limit state which ductility parameter (µ) refers to. Usually, it can be easily noted that there is a discontinuity in the curves. This is referred to a change in failure mode between flexure and shear, the latter being associated with smaller displacement capacity. Similar curves were built for each parameter and also for unreinforced masonry systems. For a complete description refer to Guidi (2011).
4. DISPLACEMENT CONSIDERATIONS IN SEISMIC DESIGN OF MASONRY FOR S.D.O.F. STRUCTURES

The proposed model is able to interpret the performances of panels linking them with limit states resulting from integration of cross-section equilibrium equations. The model results were generalized proposing some design equations that relate geometrical and mechanical properties of URM panels to performance levels (similar developments are awaited for RM systems). Differently from what is proposed by RELUIS, but according to experimental evidences and Italian building code (DM 14/01/2008, 2008) the present work defines four different limit states for masonry piers. Two of them are related to serviceability limit states (immediate occupancy and damage control limit states) and other two are related to ultimate limit states (life safety and collapse prevention).

As an example, the drift achieved by model at maximum horizontal strength of walls is presented in a compact form in Table 2 for TG masonry. By doing so, it is possible to start a fitting process to provide some formulae, which give drift limit states without performing the whole model analysis, and of course, can make the design process faster.

Table 2. Drift at maximum strength (%) for TG masonry, under varying vertical compression and masonry compressive strength.

<table>
<thead>
<tr>
<th>TG</th>
<th>$\sigma_0$ [N/mm$^2$]</th>
<th>$f_m$ [N/mm$^2$]</th>
<th>1.37</th>
<th>2.74</th>
<th>4.12</th>
<th>5.49</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23</td>
<td>0.245</td>
<td>0.254</td>
<td>0.270</td>
<td>0.288</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>0.293</td>
<td>0.295</td>
<td>0.307</td>
<td>0.323</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.47</td>
<td>0.401</td>
<td>0.389</td>
<td>0.393</td>
<td>0.403</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.60</td>
<td>0.474</td>
<td>0.465</td>
<td>0.462</td>
<td>0.468</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>0.512</td>
<td>0.517</td>
<td>0.510</td>
<td>0.512</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.91</td>
<td>0.522</td>
<td>0.621</td>
<td>0.609</td>
<td>0.605</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.93</td>
<td>0.514</td>
<td>0.636</td>
<td>0.622</td>
<td>0.617</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.21</td>
<td>0.230</td>
<td>0.753</td>
<td>0.750</td>
<td>0.738</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Considering that new loadbearing unreinforced masonry systems are not generally used for building of high importance class related to the occupancy level, the limit states which govern the acceptable risk are damage control and life safety according to DM 14/01/2008 (2008). Hence, only these limit states were considered, but is possible extend the procedure even to serviceability limit states.

Two main failure mechanisms influenced the experimental specimens and were thus considered in the model: flexure and diagonal shear failure mechanism. These failure modes also influence the response in terms of drift showing different shape and slope in curves. For this reason, it was chosen to subdivide the fitting process for walls showing flexural failure and walls showing a diagonal shear failure. The smaller resulting drift form these two relations will be the drift for the specific limit state.

In the light of experimental tests and FEM and analytical modelling results, it was assumed that the response of unreinforced masonry walls, under combined vertical and horizontal in-plane forces, is influenced by three main parameters: masonry compressive strength, vertical compression stress (mechanical parameters) and the slenderness ratio (geometric parameter). The mechanical parameters were taken as separated as they have a non-linear relation due to non-linear stress-strain constitutive law adopted and, in general, for the non-linearity of response, hence, considering only their ratio cannot fully represent the wall response.

\[
\psi_{cv} = \left[-0.8 \cdot \frac{\sigma_0}{f_m}\right] + \left[(0.16 \cdot \sigma_0) \cdot \left(\frac{H/L}{1.2}\right)^{1.2}\right] + \left[(0.07 \cdot f_m^{0.4}) \cdot \left(\frac{H/L}{1.2}\right)^{0.6}\right] \leq \psi_{F_{max,sn}} \tag{10}
\]

\[
\psi_{F_{max,ft}} = \left[-0.6 \cdot \frac{\sigma_0}{f_m}\right] + \left[(0.55 \cdot \sigma_0) \cdot \left(\frac{1}{H/L}\right)\right] + \left[0.17 \cdot f_m^{0.1}\right] \tag{11}
\]

\[
\psi_{F_{max,sh}} = \left[0.82 \cdot \left(\frac{6.9}{f_m}\right)^{1.125} \cdot \left(\frac{\sigma_0}{f_m}\right) \cdot \left(\frac{H}{L}\right)\right] + \left[\frac{4.4}{f_m} \cdot \left(\frac{6.9}{f_m}\right) \cdot e^{-\left[\frac{9.5}{H/L} - \left(\frac{f_m}{\sigma_0}\right)^{1.5}\right]} \leq \psi_{F_{max,sh}} \tag{12}
\]

\[
\psi_{d_{max,ft}} = \left[1 + 0.13 \cdot \left(\frac{f_m}{\sigma_0}\right)^{1.5}\right] \cdot \left[(-0.5 \cdot \frac{\sigma_0}{f_m}) + (0.5 \cdot \sigma_0) \cdot \left(\frac{1}{H/L}\right) + (0.17 \cdot f_m^{0.1})\right] \tag{13}
\]

\[
\psi_{d_{max,sh}} = \left[0.92 \cdot \left(\frac{6.9}{f_m}\right)^{1.125} \cdot \left(\frac{\sigma_0}{f_m}\right) \cdot \left(\frac{H}{L}\right)\right] + \left[\frac{5.9}{f_m} \cdot \left(\frac{6.9}{f_m}\right) \cdot e^{-\left[\frac{6.75}{H/L} - \left(\frac{f_m}{\sigma_0}\right)^{1.5}\right]} \leq \psi_{d_{max,sh}} \tag{14}
\]

Figure 8. Graphical view of different drift limit-states (black dots) for slenderness variation (TM masonry). Surfaces deriving from EQ: maximum drift (blue), drift at F\textsubscript{max} (red) and critical drift (green).
The fitting was performed with a trial and error process, concurrently taking into account the consistence of each limit state function with the others. For example, the first limit state curves were constrained so as their drift is smaller than that at maximum strength limit state, and so on. The fitting procedure in first few trials fixed all parameters except one in order to simplify the understanding of global shape of otherwise complex iper-surface (drift depends on three variables). Following, two parameters were released and one fixed in order to catch the cross-correlation between them (and in a broad sense the covariance).

The resulting formulae are reported in the following Eq. 10 to 14, where $\psi_{cr}$ is the critical drift and $\psi_{f_{\text{max}}}$ and $\psi_{d_{\text{max}}}$ are respectively the drift at maximum strength and at maximum displacement. The suffix “fl” refers to flexural failure, whereas the suffix “sh” refers to diagonal shear failure relations. Obviously drift, that has to be considered as a limit, is the lesser between flexural and diagonal shear failure drift at the corresponding limit state.

Figure 8 represents the curves generated by the proposed functions fixing masonry compressive strength to the value of TM masonry. In this graphs, the parametrical results are presented with black dots and the different limit state curves using different colors.

5. CONCLUSIONS

A newly model for masonry walls (S.D.o.F. systems) under in plane vertical and horizontal forces, capable of reproducing experimental load-displacement capacity curves, considering non-linear shear deformations and taking into account both flexural and shear failures, was used. The model catches the achievement of various limit states, which represent the performances of masonry walls related to cross-sectional behaviour (e.g., when masonry piers reach compressive strength at the base section) or to the global wall behaviour (e.g., when walls reach their shear strength).

Reinforced masonry system and three types of load-bearing unreinforced masonry walls, made with perforated clay units and differing types of head and bed joints were modelled under in-plane cyclic loads. Parametric analyses showed how the wall performance under combined shear and compression depends on different parameters (masonry compressive strength, vertical compression stress, slenderness, vertical and horizontal reinforcement ratio).

In addition, some formulations aimed to provide drift limits for unreinforced masonry walls, and directly related to the achievement of their limit states, were proposed. These formulations use simple geometrical and mechanical properties of URM panels to find drifts at different performance levels. They can catch both flexural and shear failure drift limits. Similar further developments are awaited for RM systems.

The proposed formulae have a complex form, however, they only depend on a limited number of engineering parameters (such as masonry compressive strength, vertical compression stress and wall slenderness). Therefore, although they need to be simplified in the light of practical use, and further validated with other experimental tests, it appears that they follow the engineering criteria generally adopted in DDB design methods. They can be thus regarded as a first indication in the case of unreinforced masonry systems.

REFERENCES


