Observed Effects of Footing Shape on Settlements Caused by Foundation Rocking

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SUMMARY:
Cumulative settlement is one of the primary potential detrimental aspects of rocking foundations. It is well known that permanent deformations, especially settlement, can accumulate with every cycle of rocking of a shallow foundation. It has been well documented that if the factor of safety against bearing failure is not very large (e.g., less than five or so), settlement can become significant. It has not been understood, however, that settlements are also quite sensitive to the shape of the rocking shallow foundation. In the present paper, we characterize the shape by the ratio \( B/L_c \), where \( B \) represents the width of the foundation (perpendicular to the rocking direction) and \( L_c \) is the minimum footing length required to support the vertical load on the foundation when the soil’s ultimate bearing capacity is fully mobilized. Recent analysis of settlement-rotation data from previous experiments clearly shows a significant influence of footing shape on settlement behavior.

Keywords: Rocking Foundations, Centrifuge, Soil-Structure Interaction, Physical Modelling, Settlement

1. INTRODUCTION

Gajan and Kutter (2008), Deng et al. (2011), Hakhamaneshi et al. (2012), Liu et al. (2012), Anastasopoulos et al. (2010), Pecker and Chatzigogos (2010) have indicated that adoption of shallow rocking foundations on competent soils can absorb seismic ductility demand and reduce demand on columns and thereby improve the seismic performance of soil-foundation-building systems. If the moment capacity of the footing is designed to be smaller than the moment capacity of the hinging column, footing rocking around the base can be initiated. In the conventional fix-based structural configuration, almost all energy will be dissipated through inelastic flexural behavior of the superstructure components, i.e. hinging columns. Foundation rocking behavior, however effectively transmits a ductility demand transfer from the hinging column level to the footing-soil interface. This reduces the magnitude of differential displacements imposed along the column for an otherwise fixed foundation. On the other hand, over-conservatively strong foundations, which usually preclude energy dissipation through foundation rocking, may place extra ductility demand on columns. The California Department of Transportation seismic design criteria (Caltrans 2010) states that "foundation components shall be designed to remain essentially elastic when resisting the plastic hinging moments". New Zealand Structure Design Actions (2004) states that “rocking components may be included in the structures, but special studies on the superstructure behaviors shall be mandatorily performed”.

1.1 Previous Research

The early work of Housner (1963) theoretically illustrated the beneficial self-centering mechanism of a rocking block. Researchers have since incorporated this concept into rigid shear-walls founded on shallow footings, while also foundation capacity mobilization. Bartlett (1976) and Wiessing (1979) conducted a series of 1-g tests and studied the behavior of rocking shallow foundations on clay and sand correspondingly. Bartlett found that rocking foundations reduce the seismic force on the structure.
by lengthening its natural period. Wiessing reported a continuous settlement of the foundations during rocking and a progressive rounding of the soil interface with large amplitudes of footing rotation, which led to loss of the contact area due to the softening of the system. Large amount of energy dissipation through the soil was noted.

Rosebrook (2001a&b) and Gajan (2003) conducted numerous experiments on shear wall structures supported by rocking shallow foundation and studied the nonlinear load-deformation behavior of the foundation during cyclic and dynamic loading. They reported that the moment and shear capacity of a rocking shallow footing depends on its geometry, initial vertical static factor of safety and the moment to shear ratio of the applied load. They saw similarities in the load-deformation behavior of rocking shallow footings when subjected to lateral cyclic loading and dynamic base shaking. However, the capacities were up to 30% smaller observed during dynamic base shaking. It was also noted that the moment and shear capacities are reached at different strains, however, the capacities did not degrade significantly with increasing amplitudes of strain. The energy being dissipated through the soil was calculated by integrating the moment-rotation hysteretic behavior of the foundation. It was found that footings with larger initial vertical static factor of safety dissipate less energy but will show better recentering mechanism.

Deng et al. (2009a,b and 2010) investigated innovative foundation designs that would improve the seismic performance of bridges while being less sensitive to site conditions. Three series of centrifuge tests were performed on shallow spread foundations and the behavior of a rocking foundation on sand was studied. They reported the beneficial effects of rocking on the seismic performance and stability of bridges. It is concluded that, the ratio of the footing length (in the direction of rocking) to the critical contact length required to support the vertical load ($L/L_c$), is a controlling parameter which will affect the dynamic performance of rocking foundations.

1.2 Current Research

Previous research efforts have contributed significantly to the development of the idea of nonlinear analysis in performance-based evaluation. One of the limitations of the work done by Bartlett and Wiessing, however, is that the experiments were conducted at smaller confining stresses and the reduced scale 1-g models tested seem unrealistic and unable to fully capture the nonlinear prototype footing behavior. Previous centrifuge tests have characterized the behavior of rocking foundations mainly on sand, but fewer results are available for clayey soil. In responding to the research gap, a centrifuge test named MAH01 was performed to examine the footing rocking behavior on clayey soil. Three different types of tests were:

1. Lateral slow cyclic test on two different rigid shear-wall structures of different factors of safety,

2. Plate bearing tests to provide us with an accurate measure of the undrained shear strength ; and

3. Dynamic shaking of 2 single-degree-of freedom (SDOF) structures with different factors of safety.

Figure 1 shows the configuration of these types of tests that were conducted sequentially and its instrumentation plan.
Hakhamaneshi et al. (2011 & 2012) explain the design procedure, soil preparation, model construction and the details of this experiment. The footings were placed at the surface of a medium strength clay (Yolo Loam, LL=29.1, PL=20.8). Based on the 1st centrifuge test results:

- The theoretical rocking moment capacity of the footings previously defined by Gajan & Kutter (2008) holds for footings placed on clayey ground also.
- The data from the slow cyclic tests showed how moment capacity of the footing, settlement and energy dissipated relate to different rocking factor of safety \( \frac{A}{A_c} \).
- The data from Bearing Failure tests enabled us characterize the clay and to improve our design by obtaining the actual undrained shear strength of the soil and how it affects the \( \frac{A}{A_c} \) ratio.
- The data from the dynamic tests allowed us to compare the moment capacity, energy dissipation and recentering tendencies of footings with the same \( \frac{A}{A_c} \) ratio with the slow cyclic experiments.
- The settlement data allowed us compare the results with those reported previously by Deng et al. (2012). These results are shown and analyzed in section 3 (Footing Settlement Correlations).

2. CONCEPT OF ROCKING FOUNDATIONS

Consider a vertical load \( V \) applied to the center of gravity of the structure as shown in Figure 2. A horizontal load \( H \) is also applied at a height \( h \) above the base of the footing with length of \( L \) (in the plane of rocking), width of \( B \) (perpendicular to the plane of rocking), and area of \( A = B \cdot L \). These loads can cause the footing to rotate \( \theta \), slide horizontally \( u \) and settle \( s \). The soil exerts a resultant force on the footing, which consists of a sliding resistance force \( F \), and a normal force \( R \).

For rigid soil, the sliding is expected to occur first if the applied horizontal force is equal to the frictional resistance of the soil-footing interface and the applied moment about the base of the footing \( M = H \cdot h \) is smaller than the resisting moment \( V \cdot L / 2 \). On the other hand, if the horizontal load is applied at a height greater than \( L / (2 \mu) \) (where \( \mu \) is the coefficient of friction), the footing will tip about its corner.
For non-rigid soil, as the footing rocks, it does not bear on a sharp corner of the footing. Instead, a minimum contact area, $A_c$, is required to support the vertical loads. The moving of the contacting area results in a curve interface, with localized bearing failure apparent near the edges in Figure 3. Rosebrook and Kutter (2001a, b), Gajan and Kutter (2008), Deng et al. (2012) and Hakhamaneshi et al. (2012) showed that the amount of settlement during rocking can be correlated to the contact area, the amplitude of footing rotation and the footing factor of safety ($A/A_c$). For rectangular footings, when it is loaded along the length of the footing, the critical contact length, $L_c$, is directly related to the critical contact area: $L_c = A_c/B$. The value of $L_c$ represents the minimum length of the footing required to support the vertical load when the soil bearing capacity is fully mobilized on the contact area. With the knowledge of ultimate bearing capacity of the clay ($q_{ult}$), one can analytically determine $L_c$ by using the conventional equation of $L_c = V/(q_{ult} B)$.

Gajan and Kutter (2008) examined the relations between the moment capacity of the footing, net settlement and energy dissipated during footing rocking with various ratios of $A/A_c$ ($\approx FS_v$). Gajan and Kutter (2008) and Deng et al. (2012) observed that when the external applied moment approaches the resisting moment capacity of the footing (denoted as $M_{c, foot}$) in Equation 2.1, the footing rocking is about to occur.

$$M_{c, foot} = \frac{V \cdot L}{2} \left(1 - \frac{A_c}{A}\right)$$  \hspace{1cm} (2.1)

### 3. FOOTING SETTLEMENT CORRELATIONS

Gajan and Kutter (2008) focus on the data from centrifuge tests mainly on sand, and correlate the settlement of the foundation per cycle of footing rotation to the rotation amplitude. The results were grouped in categories based on the $A/A_c$ ratio. Deng et al. (2012) and Hakhamaneshi et al. (2012) further extend data on sand and clay, and correlated the cumulative footing rotation ($\theta_{cum}$) to the
normalized footing settlement ($s/L$) instead of considering settlement on a cycle by cycle basis. The footing’s cumulative rotation ($\theta_{cum}$) is obtained by summing up the absolute value of all the local maxima and minima ($extrema_i$) for which the amplitude exceeds a threshold rotation ($\theta_t$ is set empirically at 1 mrad in this study). Rotations smaller than the threshold value are assumed to cause an elastic response to the rocking without leading to any residual settlement or uplift. In this study, the procedure proposed by (Hakhamaneshi et al. 2012) and Deng et al. (2012) has been modified by subtracting the elastic rotation for each cycle by introducing the $-N \theta_t$ in equation 3.1. Figure 4 illustrates the determination of extrema and the rotation threshold used to determine footing cumulative rotation.

$$\theta_{cum} = \sum_{i=1}^{N} |extrema_i| - (N \cdot 0.001) \text{ for } |extrema_i - extrema_{i-1}| \geq 0.002 \text{ rad.}$$

(3.1)

where $N$ is the number of extrema with peak amplitude greater than the threshold rotation.

The data from the present MAH01 test series on clay are added to the aforementioned datasets on sand (Gajan and Kutter 2008 and Deng et al. 2012) and clay (Rosebrook and Kutter 2001a&b). Table 3.1 summarizes the scope of parameters varied during these experiments and provides us the ranges of the footing properties being tested. In this table, "Test ID" specifies the name of each experiment and the specific event name; "Loading" differentiates whether the event was a Slow Cyclic (SC) test or Dynamic shaking (Dyn) test. As illustrated in Figure 5, "B" is the width of the footing parallel to the axis of rocking and "L" is the length of the footing in the plane of rocking. More importantly, the aspect ratio ($B/L_c$) is introduced to characterize the shape of the critical contact area. A footing with high aspect ratio would have a strip-shape critical contact area (Figure 5, Short Wide Footing). While for lower aspect ratio case, one may expect a more square shaped contact area if $L_c$ approaches B (Long Narrow Footing). For a short-wide footing, the soil deformations associated with the localized bearing failure would be expected to be primarily in the plane of rotation as indicated by the arrows in Figure 5; for this case, perhaps about half of the soil that is displaced due to bearing failure of the strip-shaped loaded area gets pushed back under the footing, and the other half gets pushed away from the footing, accounting for settlement. As the footing becomes long and narrow, the contact shape could become square, at which point out of plane soil displacements associated with bearing failure become significant and a smaller fraction of the displaced soil would be pushed back into the gap under the footing. If other factors (e.g., $A/A_r$) are held constant while varying footing shape, due to a greater proportion of soil being pushed away from the footing, one may expect greater settlements for rocking of long narrow footings than for short wide footings.
Normalized Cumulative Settlement is being calculated by normalizing the settlement obtained in each event by the length of the footing. Because the footing width appears in the $N_q$ term of the standard bearing capacity equation, it is apparent that the bearing capacity of sand is more sensitive to the shape of the loaded area than is the bearing capacity of clay. The value of $L_c$ can be determined through an iteration process using the standard bearing capacity equation as described by (Deng et al. 2012); the vertical load on the footing is treated as a known, a value of $L_c$ is assumed, and adjusted by trial and error until the calculated bearing capacity of the shape $B \times L_c$ matches the known bearing load (Deng et al. 2012).

For clay, $L_c$ may be determined using the conventional bearing capacity equation and appropriate shape and depth factors, $s_c$, and $d_c$ for the loaded area.

$$L_c = V/(B \times s_c \times d_c \times N_c \times S_u)$$  \hspace{1cm} (3.2)

Table 3.1 Details of the data points gathered from the past centrifuge tests (in model scale)

<table>
<thead>
<tr>
<th>Test Series Name</th>
<th>Soil</th>
<th>Loading</th>
<th>B (mm)</th>
<th>L (mm)</th>
<th>$A/A_c$</th>
<th>$B/L_c$</th>
<th>Cum. Rotation ($\theta_{cum}$)</th>
<th>Norm. Cum. Settlement ($s/L$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KRR</td>
<td>Sand</td>
<td>SC</td>
<td>33-45</td>
<td>135-150</td>
<td>2-4</td>
<td>0.72-2</td>
<td>0.1-0.15</td>
<td>0.015-0.018</td>
</tr>
<tr>
<td>LJD</td>
<td>Sand</td>
<td>SC/Dyn</td>
<td>43-78</td>
<td>50-122</td>
<td>8-30</td>
<td>5-30</td>
<td>0.03-0.14</td>
<td>-0.0072-0.003</td>
</tr>
<tr>
<td>MAH</td>
<td>Sand</td>
<td>SC/Dyn</td>
<td>33-40</td>
<td>140-160</td>
<td>1.2-11</td>
<td>2.5-18</td>
<td>0.001-0.326</td>
<td>0.0003-0.02</td>
</tr>
<tr>
<td>SSG</td>
<td>Sand</td>
<td>SC/Dyn</td>
<td>111-176</td>
<td>107-111</td>
<td>2.2-15.9</td>
<td>0.5-4</td>
<td>0.005-0.101</td>
<td>0.007-0.024</td>
</tr>
</tbody>
</table>

Figure 6 plots the normalized cumulative settlement with respect to the updated footing cumulative rotation for many events of the test series described in Table 3.1. A conservative envelope, indicated by the straight lines in the graph encloses most of the data points from $A/A_c$ groups. These boundaries for sand (black data points) set previously by Deng et al. (2012) seem to be conservative for the data on clay (colored data points) as the settlements for the same $A/A_c$ ratio seem to be smaller on clay. It is promising that the data on clay do not exceed these boundaries and is consistent with the data on sand. The similarity in the relationships for sand and clay indicate that the effect of changing soil type can be reasonably accounted for through the effect on $A/A_c$.

From Figure 6, it is apparent that if $A/A_c > 8$, the settlement is always smaller than 1% of the length of the foundation even at cumulative footing rotations up to 0.3 radians. Furthermore, we suggest that such small settlements would likely be considered to be acceptable performance during a major seismic event. At this level of settlement, one may expect some damage but not collapse.
Figure 6. Cumulative normalized settlement-cumulative footing rotation relation of the footings, grouped into different \( \frac{A}{A_c} \) ratios (Hakhamaneshi et al. 2012)

To isolate the effect of footing shape, the data in Figures 7 (for clay soils) and 8 (for sand soils) have been grouped into two categories based on their factor of safety \( \frac{A}{A_c} \). For each value of \( A/A_c \), the data have been divided into different subcategories based on their aspect ratio \( \frac{B}{L_c} \). It is clear that there is a consistent trend apparent indicating that as \( B/L_c \) increases, normalized settlements \( \frac{s}{L} \) decrease. In some cases the importance of \( B/L_c \) is greater than what we might expect based on the articulated mechanism of sand being pushed back under the footing described in Figure 5 along previous discussion of Figure 5 presented earlier.

For each identified value of \( B/L_c \), a conservative envelope indicated by a straight line in the graph encloses most of the data points and would provide a conservative estimate of expected settlements for design purposes, perhaps. The lines ignore the extreme cases, but do cover the majority of the observed data points.

Figure 7. Normalized footing cumulative settlement-cumulative rotation for different \( B/L_c \) ratios on clay (note the scale difference)
Results verify that for $A/A_c > 8$, settlements were always less than 1% of the length of the foundation, regardless of the value of the aspect ratio. The settlements may be calculated from the equation 3.3, given by the constant values as the slopes of the conservative envelopes (Deng et al. 2012). For design purposes, Deng et al. (2012) proposed a procedure for estimating the footing's cumulative rotation of footings in bridge structure. They suggested that the design lateral displacement of bridge deck ($D_d$) may be obtained from the spectral displacement, given the period of the structure. The design amplitude of footing rotation ($d$) is estimated by dividing the design displacement of the deck by the column height ($H_c$). Depending on the magnitude of an earthquake, it may be reasonable to assume that foundation undergoes two full cycles of rotation with the design amplitude, so equation 3.4 is used to correlate the design rotation to the dynamic settlement. Note that during two full cycles of rotation, the cumulative rotation calculated using the equation 3.1 would be four times the amplitude of rotation. The constant “const” in equations 3.3 and 3.4 is the slopes of the appropriate line such as those indicated in Figures 7 and 8, depending on the static factor of safety with respect to bearing capacity (expressed in terms of footing areas $A/A_c$) and the footing shape (accounted for by the ratio $B/L_c$).

$$s = \text{const} \cdot L \cdot \theta_{\text{cum}}$$  \hspace{1cm} (3.3)

$$s = 4 \cdot \text{const} \cdot L \cdot d$$  \hspace{1cm} (3.4)

4. CONCLUSIONS

It is found that under the same factor of safety, wide rectangular rocking footings (with higher $B/L_c$) settle less than narrow rocking footings. As a footing rocks, the soil near the loaded edge yields in bearing and some fraction of the displaced soil is pushed under the footing, ameliorating settlement, while another fraction of the displaced soil is pushed away from the footing, causing irrecoverable footing settlement. Due to less lateral confinement around the rocking edge for narrow footings and the three-dimensional failure mechanism, a larger fraction of soil is pushed out from under the footing, hence larger normalized settlements ($s/L$) are expected for narrow footings. For wide footings, lateral confinement reduces out-of-plane soil displacements and settlements. Depending on the footing shape, a significant fraction of the displaced soil finds its way back under the unloaded gap area between the soil and the footing. At large factors of safety, the footing seems to undergo uplift. This uplift could be attributed to the sand falling into the gap as the footing rocks. This mechanism would also be enhanced.
for narrow footings. Larger factors of safety imply smaller required critical contact area between the soil and footing, which increases the material falling into the gap and causing uplift.

It has been well known, for obvious reasons, that the settlements associated with rocking of shallow foundations increase as the factor of safety with respect to bearing capacity decreases. The contribution of this paper is to point out that the footing shape also has a very significant effect on residual settlements associated with rocking. The observed effect of footing shape on settlements is greater than we expected it to be. Further investigation of this issue is the topic of ongoing research.

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