Analyzing the Effect of Moving Resonance on Seismic Response of Structures Using Wavelet Transforms

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SUMMARY:
When the dominant natural periods of a nonlinear structure elongate in such a way as to match with the energy in the ground motion, a phenomenon called moving resonance occurs. This paper investigates the strong effect that moving resonance can have on nonlinear structural response. Instances of moving resonance are identified using continuous wavelet transforms and compared to behavior of systems with similar parameters that do not experience significant moving resonance. It is demonstrated that moving resonance can contribute to significant increase in displacements. Also, the wavelet transform is shown to provide key information for analyzing moving resonance by revealing time-varying characteristics of the ground motions that are not available from traditional tools such as time histories, Fourier transforms, and response spectra. A method for quantifying the effect of moving resonance is proposed and used to examine the ground motion characteristics that contribute to moving resonance.

Keywords: *Moving resonance, record-to-record variability, wavelet transforms, spectral nonstationarity.*

1. INTRODUCTION

Nonlinear structures, when subjected to multiple ground motion records that are scaled to consistent ground motion intensity show significant variation in their response. This effect of ground motion randomness on the variation of structural response is defined as record-to-record (RTR) variability (Ibarra and Krawinkler 2011). Because of record to record variability, seismic effects are often treated statistically. For a suite of ground motions scaled to match a given design response spectrum, the peak interstory drifts obtained from response history analyses have large dispersion. Typical values of dispersion as measured by the standard deviation of the natural logarithm of the interstory drifts may be 0.3 or more (Cornell and Vamvatsikos 2002). Computational studies have demonstrated that standard deviation of peak interstory drifts can be 70% larger than the mean for a suite of ground motions scaled to a hazard level with 2% probability of exceedance in 50 years (Luco and Cornell 2000). This implies that the precision with which the structural response due to a given seismic hazard can be predicted is poor. Furthermore, because of this wide dispersion in response history results, the accuracy of the structural response predictions can be poor if a small number of ground motions are used such as allowed by current building codes (ASCE 2010). Finally, based on the current state-of-the-art it is impossible to predict whether a given building on a particular site is subject to the conditions that will lead to extreme values of interstory drift as large as two or three times the mean values.

Although there have been multiple studies characterizing the contribution of record to record variability on dispersion of response history results, there have been significantly less attempts to analyze the sources of this variability and to predict conditions that lead to unusually large response. Based on prior studies it is expected that the interaction of time-varying dominant structural periods in a nonlinear system and the time-varying periods containing dominant energy in the ground motion is the cause for much of the record to record variability. However, the tools used by the earthquake engineering profession to characterize ground motions, such as response and Fourier spectra and parameters derived from these spectra, do not give any information about the time-varying nature of the ground motion's frequency content. Advances such as wavelet transforms reveal considerably more information about the time-varying nature of earthquake related signals.

Typical forms of time-dependency in ground motion signals include variation of the signal's frequency content through time, also known as spectral nonstationarity, and the time dependent variation of a signal`s magnitude, also known as amplitude (or temporal) nonstationarity. An understanding of the spectral nonstationarity in ground motions and its effect on structural response is required to analyze or predict the behavior of buildings and bridges during earthquakes.

For instance, when the frequency content of the ground motion shifts in a similar manner as the natural frequencies of the structural response, a phenomenon referred to as moving resonance occurs (Beck and Papadimitriou 1993). Moving resonance can have a strong effect on the magnitude of structural response. Resonance occurs when a structure is subjected to harmonic or periodic loading with forcing frequency equal to the structures natural frequency. In elastic systems with low damping ratios, even small periodic driving forces can produce large amplitude oscillations when applied in resonance. For nonlinear systems, the natural frequency is shifting. Moving resonance is characterized by a small set of cycles in which the dominant ground motion oscillation frequency is resonant with the nonlinear system natural frequency. Even though the duration or resonant loading is not long enough to produce steady state resonant behavior, the short time period of resonant loading can cause significant increase in system response.

This paper investigates the phenomenon of moving resonance from multiple directions. First, the wavelet transform is identified as an important tool for investigating the time varying nature of ground motions and structural response. Next, an instance of moving resonance is explored to demonstrate the use of wavelet transforms in understanding the phenomenon and the effect that moving resonance can have on structural response. A method is then proposed for quantifying the effect of moving resonance. Finally, the proposed method is applied to a set of twenty-two ground motions as an example of how the method could be used to investigate the characteristics of ground motions that lead to moving resonance. It is concluded that a tool like the proposed method could be used to develop a ground motion characterization that accounts for the potential to cause moving resonance.

2. WAVELET SELECTION AND BACKGROUND ON WAVELET TRANSFORMS

The wavelet transform is a mathematical tool that can be used to identify frequency components of a signal at discrete windows in time. Whereas Fourier transforms act to convert a signal into a set of stationary harmonic waves, wavelet transforms convert a signal into a set of nonstationary wavelets. Wavelet transforms provide considerably more information than traditional methods of representing ground motions such as Fourier transforms, power spectral density, or response spectra, because they provide information about the nonstationarity in the signal.

Wavelets are mathematical functions that have limited duration in time (compactly supported), occupy a limited frequency band (Fourier transform of the wavelet function is square integrable), and have zero mean. The continuous wavelet transform is given in Eqn (1) allowing the use of any admissible mother wavelet function, Ψ, with complex conjugate Ψ^*. The wavelet coefficient, $W_{s,\tau}$, represents the amount of energy in the signal, $f(t)$, for scale level, s, and time, τ. If the wavelet function is complex, these wavelet coefficients are vectors in the real-complex plane containing both magnitude and phase information. Each wavelet coefficient represents how well the wavelet matches the signal given a particular scale (period) and position in time. The equation describing the complex Morlet wavelet is given in Eqn (2).

$$W_{s,\tau} = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \Psi^* \left(\frac{t-\tau}{s}\right) dt \quad \textbf{(1)}$$

$$\Psi\,(t) = \pi^{-1/4} e^{i(2\pi f_c)t} e^{-t^2/2} \quad \textbf{(2)}$$

The complex Morlet wavelet is a logical choice for a wavelet function in that it consists of a harmonic function with frequency, f_c, that has been modulated by a Gaussian window so as to have compact support. A plot of the complex Morlet wavelet is shown in Figure 1. The key advantage of wavelet transforms over tools such as the short term Fourier transform (STFT) is the variable resolution of the wavelet transform at different frequencies. The wavelet transform draws data from long time windows for capturing low frequency content and narrow time windows for capturing high frequency content. This is demonstrated graphically in Figure 2. Figure 2a shows a resolution box with width equal to the resolution in time and height equal to resolution in period. The period resolution is calculated using the points where the Fourier transform is 1% of the peak value. Figure 2b shows the dimensions of the resolution boxes for the complex Morlet wavelet scaled to have periods between 0.5 sec and 4.0 sec. For the STFT, the width of resolution boxes for all periods would be identical. To capture long period energy, a large time window is required for the STFT leading to very poor time resolution (as compared to the period) for short period energy. The aspect ratio and size of the resolution boxes varies with the choice of wavelet.

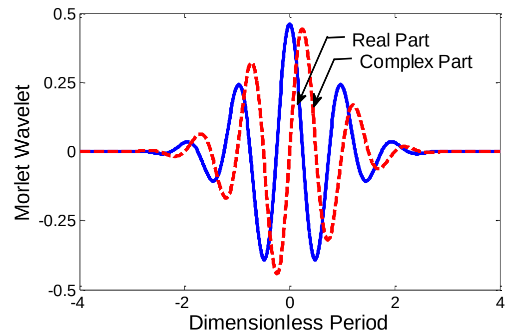
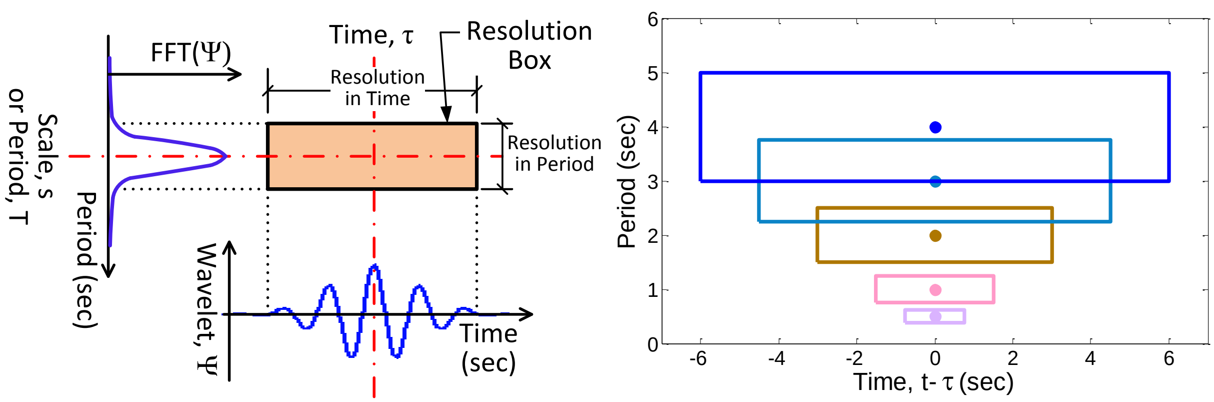


Figure 1 - Complex Morlet Wavelet



(a) Resolution Box for One Wavelet (b) Resolution Boxes for Several Periods

Figure 2 – Variable Resolution in Wavelet Transforms

Publications on wavelet analyses typically present the spectral value in terms of scale instead of frequency or period. Scale can be taken as inversely proportional to frequency. The relationship between scale and frequency is given by Eqn (3). The variables, s, Δt, f_c, T_a represent scale, sampling period, center frequency of the mother wavelet, and period corresponding to the given scale respectively.

$$T_a = 2\pi \frac{s * \Delta t}{f_c} \quad \textbf{(3)}$$

There are two main types of wavelet functions, namely continuous wavelets and discrete wavelets. Discrete wavelet transforms (DWT) use an orthogonal set of wavelet functions to produce a non-redundant set of wavelet coefficients. An inverse DWT can be performed to reassemble the original signal from the DWT wavelet coefficients. Continuous wavelet transforms (CWT) on the other hand produce a redundant set of wavelet coefficients, each representing overlapping information.

Continuous wavelets are further classified into real and complex depending on whether the function includes imaginary terms. Complex CWTs produce information about magnitude and phase. For the work described in this paper, complex continuous wavelet transforms were selected to visualize the frequency content of signals for several reasons. First, the visualization of frequency content will be smooth and continuous because of overlapping of wavelets. Second, although the DWT operates in frequency bands, the CWT operates at a specific frequency that can be adjusted. Similarly, the CWT can give be centered at any time. The result is a better visualization of the frequency content as compared to DWT.

A complex wavelet basis was chosen to capture the magnitude and phase of the energy content in the ground motion signal. CWT using real-valued wavelets are not capable of capturing frequency content when the wavelet is out of phase with the signal. The complex continuous wavelet rectifies this limitation as it includes a complex part. The magnitude of the resulting complex wavelet coefficients captures the frequency content in the signal regardless of phase. Furthermore, the phase of the signal at a given frequency can be obtained as the phase between the real and complex parts of the wavelet coefficient. The complex Morlet wavelet, presented in the previous section, is a complex continuous wavelets and was chosen for use in the studies described in this paper.

3. STUDY OF THE EFFECT OF MOVING RESONANCE ON STRUCTURAL RESPONSE

Prior research has demonstrated the significant effect that spectral nonstationarity can have on structural response (e.g. Beck and Papadimitriou 1993, Cao and Friswell 2009, Conte et al. 1992, Goggins et al. 2006). The effect of spectral nonstationarity is examined here in two steps: (1) Assessing the occurrence of moving resonance and (2) Quantifying its effect on the structural response.

3.1 Assessing the Occurrence of Moving Resonance

This section describes the details of a parametric study performed to identify the occurrence of moving resonance. It is expected that moving resonance may be related to initial structural period, the hysteretic shape for the given structural system, associated structural period elongation, phase between the structural response and the ground motion acceleration, sequence of ground motion dominant frequencies, duration of strong shaking, and possibly other factors. The parametric study is intended to isolate the effect of specific variables on moving resonance such as strength, hysteretic shape, and initial period. The study is also used to identify the combinations of ground motions and structural systems that lead to moving resonance.

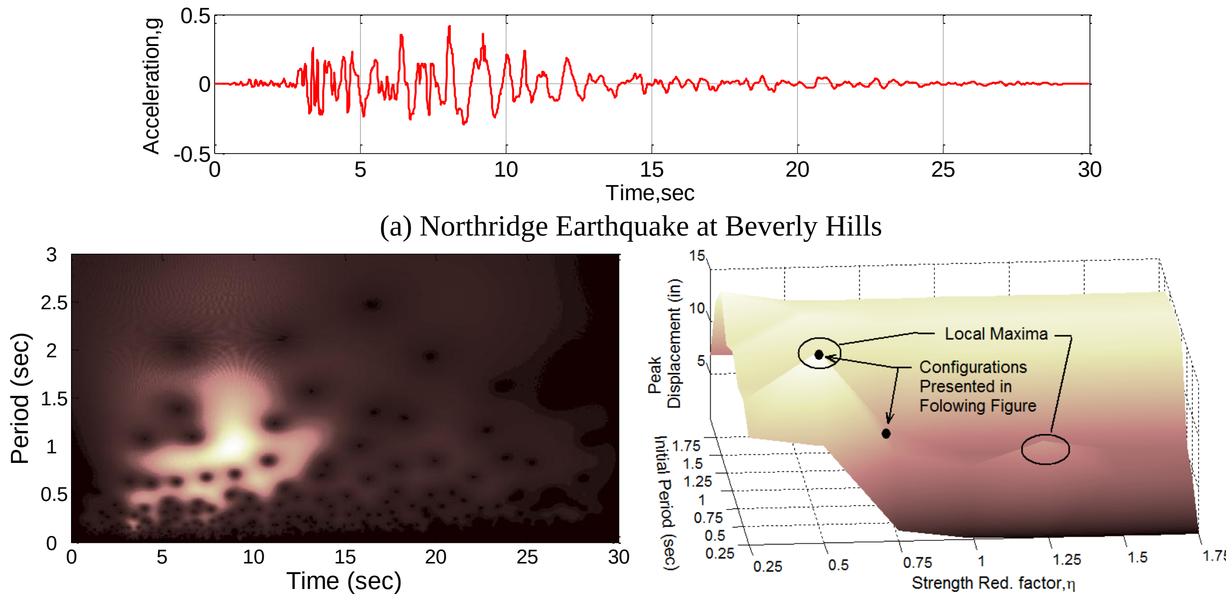
For simplicity and in order to maintain a non-specific approach, a generalized SDOF structure is considered. Two hysteretic models were considered in this study: bilinear elastic and the elastic-perfectly plastic. Elastic bilinear behavior was chosen because it isolates the effect of nonlinearity on period elongation by removing the effect of hysteretic damping and elastic perfectly plastic is used to represent a generic ductile structural system. The SDOF load-deformation behavior is generalized as a function of only two variables: initial fundamental period T_0 and strength ratio, η. The damping ratio ξ_0 is held constant. The equation of motion is given in Eqn (4) as a function of the nonlinear pseudo-restoring force of the system, $\bar{f}\,(u)$, given in Eqn (5) equal to the nonlinear restoring force, $F(u)$, divided by the initial stiffness, k_o. The restoring force is parameterized by the initial stiffness, k_o, and a strength reduction factor, η, that controls the yield force (force associated with change in stiffness), F_y, as given in Eqn (6). The secondary stiffness for the bilinear elastic system and the elastic-perfectly plastic system is set equal to zero.

$$\ddot{u} + 2\xi_0\left(\frac{2\pi}{T_0}\right)\dot{u} + \left(\frac{2\pi}{T_0}\right)^2 \bar{f}\,(u) = -\ddot{u}_g \quad \textbf{(4)}$$

$$\bar{f}\,(u) = \frac{F(u)}{k_0} \quad \textbf{(5)}$$

$$\eta = \frac{F_y}{mg} \quad \textbf{(6)}$$

Full details of the parametric study can be found in the report by Naga (2011). In this paper, one ground motion is analysed to demonstrate the related concepts. The ground motion chosen is the 1994 Northridge earthquake as measured at the Beverly Hills – Mulholland Drive recording station. The ground motion acceleration history is shown in Figure 3a and the spectrogram showing a visualization of the wavelet coefficients is given in Figure 3b.



(a) Northridge Earthquake at Beverly Hills

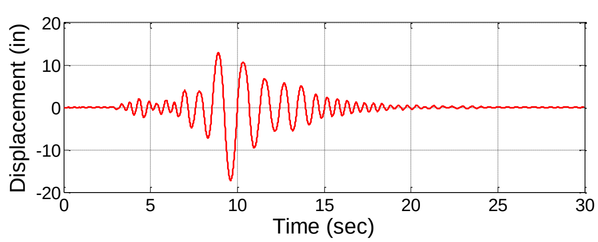
(b) (c)

Figure 3 – Investigation of One Ground Motion Including (a) Acceleration Record, (b) Associated Spectrogram, and (c) Bilinear Elastic System Peak Displacements for Varying Initial Period and Strength

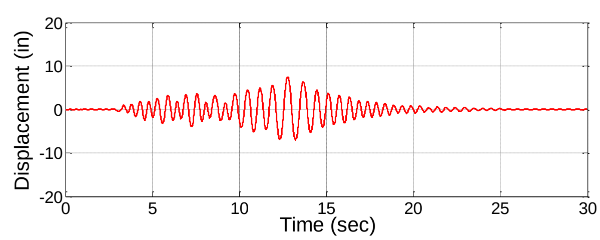
The spectrogram shown in Figure 3b was created using a continuous wavelet transform with the complex Morlet wavelet and the wavelet scale was converted to period using the relationship described in Eqn (3). Lighter colors in the spectrogram indicate larger magnitude wavelet coefficients and thus greater energy in the ground motion associated with that time and period. The spectrogram shows energy in the ground motion that starts at a dominant period of 0.5 sec and shifts to strong energy at a period of approximately 1 sec at time equal to 9 seconds.

Figure 3c shows the peak displacements of a bilinear elastic system characterized by an initial period, T_o, and nondimensional strength ratio, η, as given in Eqn (5) and Eqn (6). Figure 3c is essentially a threedimensional nonlinear displacement response spectrum. Although large values of strength reduction factors (values greater than 1) might be unusual in seismic design, they represent more elastic behavior such as might be expected for systems with large overstrength or systems subjected to smaller earthquakes than the design level. The right side of the plot approaches the elastic displacement response spectrum. This nonlinear response spectrum encompasses a large amount of information that facilitates quick assessment of potential cases of moving resonance. For instance, it can be seen that specific combinations of initial period and strength ratio (T_o, η) result in local maxima in peak displacements. These specific structural configurations are prone to large displacements when subjected to the chosen Northridge ground motion whereas slight changes in structural characteristics result in reduced displacements and ductility demand.

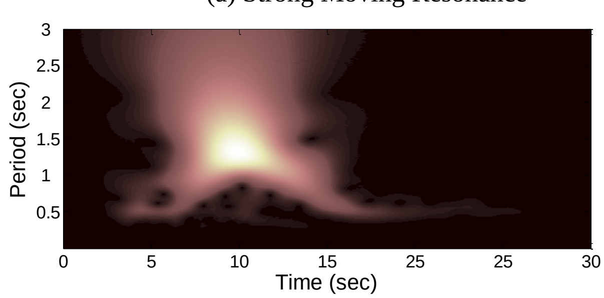
To further demonstrate the reasons why some configurations produce larger drifts than others, the structural response for configurations represented by the (T_o, η) pairs equal to (0.5,0.5) and (0.5,0.75) are analyzed further. Figure 4a shows the displacement history of the (T_o=0.5, η=0.5) system and Figure 4c shows the associated spectrogram. The trend in the structural period is shown to start at 0.5 sec, elongate to a value between 1 sec and 1.5 sec at a time approximately equal to 10 seconds, and then return to the elastic period of 0.5 sec at approximately 15 seconds. The trend in the structural period correlates quite well with the dominant periods in the ground motion shown in Figure 3b as described above. This correlation indicates the occurrence of moving resonance. On the other hand, the displacement history for the (T_o=0.5, η=0.75) system is shown in Figure 4b with associated spectrogram shown in Figure 4d. In this case, the larger strength of the system causes less period elongation. The structural period does not match with the large amount of energy in the ground motion located at a period of 1 sec and time equal to 9 seconds. Thus, moving resonance does not occur and the peak displacement is approximately half of the case shown in Figure 4a in which moving resonance is demonstrated. To summarize, it is shown for this ground motion that the system with T_o=0.5 and η=0.5 experiences the right amount of period elongation to cause the system period to match the ground motion energy (moving resonance), while systems with larger η do not experience enough period elongation to cause moving resonance. Similarly, plots like Figure 4, that are not included here, show that smaller η produces too much period elongation, causing less moving resonance which leads to reduction in peak displacements as compared to η=0.5 as shown in Figure 3c.



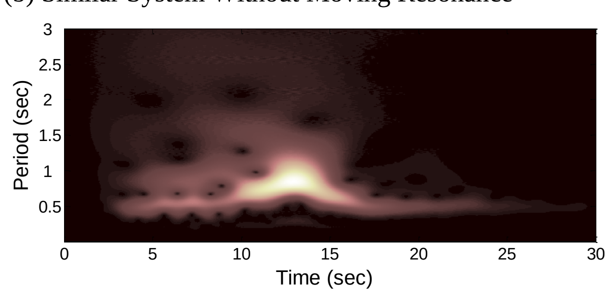
(a) Strong Moving Resonance



(b) Similar System Without Moving Resonance



(c) Spectrogram for (a)



(d) Spectrogram for (b)

Figure 4 – Demonstration of Moving Resonance Including (a) Acceleration Record for Northridge Earthquake Measured at Beverly Hills, (b) Wavelet Coefficients

3.2 Quantifying the Effect of Moving Resonance

The objective of this section is to develop a measure for quantifying the effect of moving resonance. Several potential measures for moving resonance were investigated by Naga (2011). In that report, the correlation between the dominant periods in the structural response and the dominant periods of the ground motion were measured. The phase between the structural response and the ground motion acceleration was also analyzed. Further, a metric was proposed for quantifying moving resonance which will be described in this section.

The moving resonance (MR) amplification ratio is developed here as the ratio of the peak structural displacement due to a ground motion divided by an estimate of the structural displacement associated with no moving resonance. Estimating the structural displacement that might occur in the absence of moving resonance is accomplished by subjecting the structure to components of the ground motion with limited frequency band. The procedure for computing the MR amplification ratio is summarized as follows:

1. The structure is subjected to the ground motion and the peak displacement is recorded as D_{peak}.
2. The peak displacement in the absence of moving resonance, D_{noMR} is estimated.
    a. The ground motion is decomposed into component signals with limited frequency band using discrete wavelet transforms.
    b. The nonlinear structural response to each ground motion component is computed using response history analyses.
    c. The components of structural response are added together and the peak displacement is recorded as D_{noMR}.
3. The MR amplification ratio is computed as the ratio, D_{peak} / D_{noMR}

The total input energy from the ground motion components (step 2b) is identical to the input energy from the original ground motion (step 1), but the structure is not allowed to interact with more than one frequency band in each response history analysis removing any possibility for moving resonance. However, since the principle of superposition is not applicable for nonlinear systems, the resulting MR amplification ratio cannot be taken as an absolute measure of the effect of moving resonance and instead should be used as a relative measure.

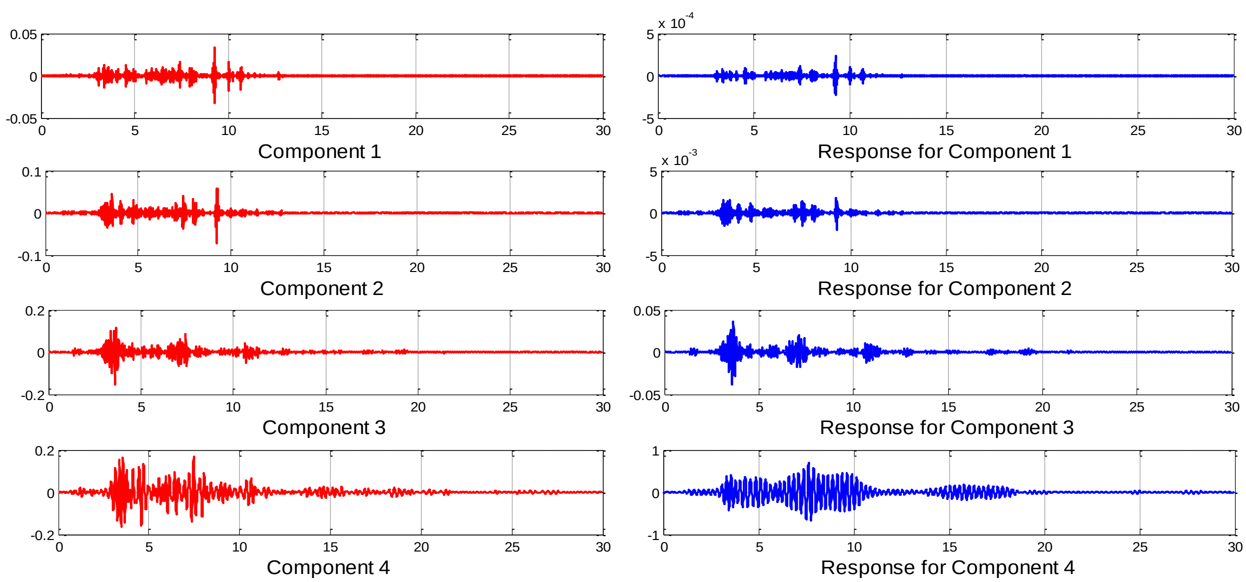
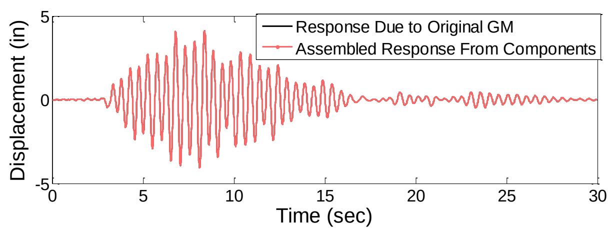


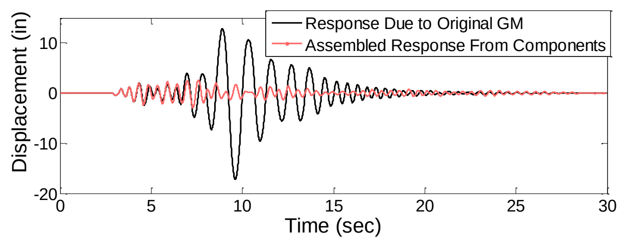
Figure 5 - Example of the first 4 Components of Ground Motion and their responses

The development of the MR amplification ratio is demonstrated in Figure 5 in which the first four limited frequency band components of the Northridge at Beverly Hills ground motion are shown as obtained using discrete wavelet transform. The displacement histories for an elastic system are also given in Figure 5. For this particular ground motion, it is shown that structural response is dominated by component 4 because it is the frequency band that includes the natural period of this elastic system.

Figure 6a shows a comparison between the response built from adding up the individual response components (step 2c) and the response history due to the original ground motion (step 1). For elastic systems, the two are identical. For nonlinear systems experiencing moving resonance, such as the bilinear elastic system discussed in the previous section with (T_o=0.5, η=0.5), the difference between the two is significant as shown in Figure 6b. For this case the MR amplification ratio is approximately six.



(a) Elastic Behavior



(b) Large Amplification Due to Moving Resonance

Figure 6– Comparison of Displacement History Due to Original Ground Motions and Displacement History Assembled from Response Histories Due to Ground Motion Components, Used in Calculation of the MR Amplification Ratio

The MR amplification ratios for a range of initial periods and strength ratios are shown graphically in Figure 7 for the bilinear elastic system subjected to the Northridge ground motion given in Figure 7. Although the moving resonance amplification ratio is approximately unity for most configurations, there is a range of combinations of initial period and strength reduction factor that create significant moving resonance. This set of structural configurations is particularly susceptible to excessive displacements when subjected to this Northridge ground motion.

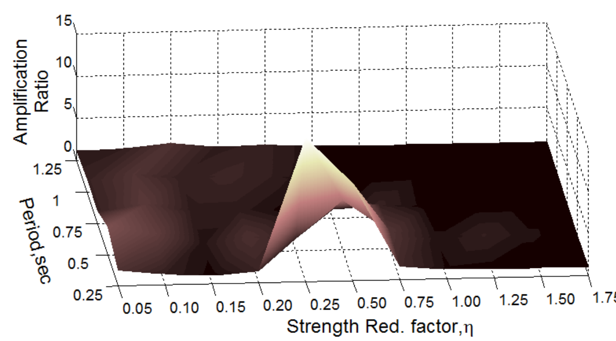


Figure 7 – Moving Resonance Amplification Ratios for Bilinear Elastic SDOF System Subjected to Northridge at Beverly Hills Ground Motion

4. INVESTIGATING THE CAUSES OF MOVING RESONANCE

The objective of this section is to demonstrate how the proposed MR amplification ratio can be used to identify ground motion characteristics that lead to moving resonance. It is hypothesized that the duration of strong shaking may affect the ability of the system to develop moving resonance. The MR amplification ratio is used to determine the effect.

For the purposes of this study, the bilinear elastic system was investigated along with a suite of ground motions. A total of 22 far field ground motion records were selected from the FEMA P695 far field set (FEMA 2009). These ground motions are chosen because the magnitude of record to record variability

(β_{RTR}=0.4) has been investigated (Ibarra and Krawinkler 2011). The interval between the times at which 5% and 95% of the total Arias intensity and the interval between the times at which 5% and 50% of the total Arias intensity are defined as the strong motion duration and the duration to peak shaking in the present study (Trifunac and Brady 1975). The Arias intensity plots for both definitions are shown in Figure 8 for the 1995 Kobe, Japan earthquake as measured at the Shin Osaka station.

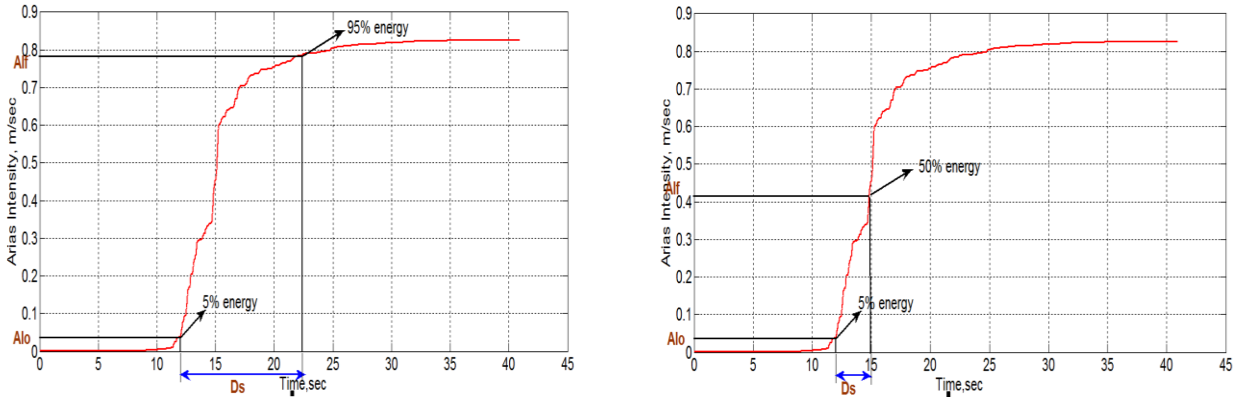


Figure 8 – Definition of Duration of Strong Shaking as (a) time interval at which 5% and 95% of total Arias intensity is attained (b) time interval at which 5% and 50% of total Arias intensity is attained

Figure 9 shows the average MR amplification ratios versus the duration of strong shaking for the bilinear elastic system subjected to the full set of ground motions used in the study. The average MR amplification ratio is calculated as the average of the MR amplification ratios for periods ranging from 0.25 sec to 1.25 sec and strength reduction factors between 0.05 and 2.5 Figure 9 shows that the very short duration events may not be long enough to develop moving resonance, very long duration events may be too long to capitalize on the period elongation in the structure, and medium length events may be most conducive to moving resonance. For more details on this study including results for the elastic perfectly plastic system see Naga (2011).

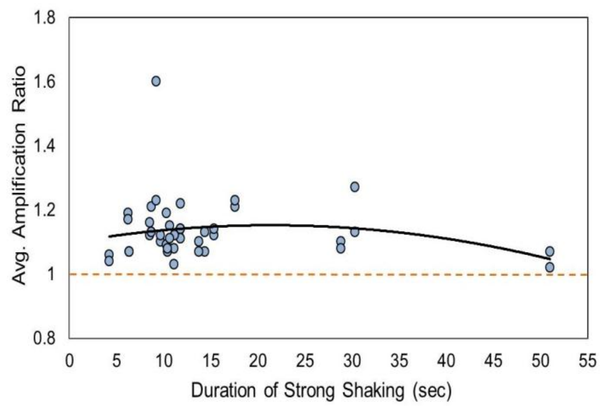


Figure 9 – Correlating Displacement Amplification Factor to Duration of Strong Shaking

5. CONCLUSIONS

Since common methods for characterizing ground motions such as response and Fourier spectra do not capture spectral nonstationarity, it is possible that buildings might be designed based on ground motions that don't represent the spectral nonstationarity for a given site and earthquake scenario. This can lead to the design of inadequate structural systems. This paper represents a first step toward the development of a ground motion characterization which includes the potential for moving resonance. Specific conclusions about the study described herein include the following.

- Wavelet transforms and the related Spectrograms which graphically show the wavelet coefficients are useful for examining spectral nonstationarity of ground motion signals. This information is lost in typical earthquake engineering tools such as Fourier spectra and response spectra.

- The phenomenon of moving resonance was investigated and a specific case of moving resonance was demonstrated in which time-varying structural period was found to correlate well with the energy in the ground motion leading to large displacements, as much as two times larger than systems with slightly altered characteristics.
- A Moving Resonance (MR) Amplification Ratio was proposed with the intent of assessing and quantifying moving resonance. The MR Amplification Ratio was found to be quite large for some configurations indicating strong effect of moving resonance on peak displacements.
- To demonstrate the potential use of the MR Amplification Ratio, the effect of strong shaking duration on the occurrence of moving resonance was studied. It is concluded that ground motions with medium duration of strong shaking create the most potential moving resonance.
- Using these types of tools, further research may allow the potential for moving resonance to be predicted based on the characteristics of the expected ground motions and the structure hysteretic behavior. Better understanding of this phenomenon will lead to the design of safer structures as the consideration of moving resonance can be included in ground motion selection or structural design.

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