SUMMARY:
This study presents the vulnerability analysis of a masterpiece of the architectural heritage of the ancient city of Pompeii in Italy. Pompeii is a partially buried Roman town-city, it was destroyed during a long catastrophic eruption of the Vesuvius volcano. Nowadays the ruins of the ancient town show many partially collapsed buildings, in effect, during its life the city underwent many earthquakes and “innovative solutions” in the building practice at that time were conceived to improve the seismic performance of structures. Mainly, temples and public places present slender columns and typical shapes of discrete marble overlaying blocks. The connection between these blocks was provided in many ways, e.g. by means of metallic dowels, hidden connections,… however in the case of the colonnade of the forum, the main square of the town, an “innovative solution” was adopted for the trabeation. To avoid long span beams over the columns, short segments were built up providing opposing inclined patterned edges. 

Keywords: Archaeological ruins, Blocks, FEM, Parametric analysis, vulnerability

1. MARBLE BLOCKS COLONNADE

According to all authors the civil forum in Pompeii responds to the prescriptions of Vitruvius about the correct place for civil administration place. For the seaside cities it had to be right by the sea and opened to the port, for the inland ones it was in the heart of the urban fabric. Pompeii site is, of course, a pre–roman city, and his forum is certainly one of the most ancient places, used to justice administration, trade, politic activity and also social activities for the population, including fighting shows and religious ceremonies. All around the place there were public buildings and temples to respond to all these functions. Some of them were built after the Roman victory of the so called social war (80 b. C.) when Pompeii become colony. Before that, the forum was the result of a large square in which converged the main streets with an orientation almost north-south. Not all the sides were orthogonal, so when the main temple (Jupiter) has been built, it stopped the development of the square in the north direction. The Capitolium remind that a new status is acquired, probably a municipality due to an alliance with Rome. Some of the existing buildings and temples (Apollo) had a different orientation, and although it was only few degrees, the forum needed to be extended to achieve a more “roman” shape and a better perspective toward the temple of Jupiter. To obtain the best result both in architectonical and in economic aspects, they built a two storey colonnade. The colonnade runs on three sides of the forum and there are different types of columns in style and in materials. While on the south side there are Doric columns in tuff, and on the western side it is still visible the so called colonnade of Popidius from the name of the quaestor that erected it. After this period and maybe for renovation needing, a new colonnade in limestone “travertino” began to substitute the old one. The Forum was closed to carts and horse carriages and was paved in travertine too. The dimension of the newest colonnade is according to the old one but the material and the style is different: it is still a Doric column and capital but somehow is less elegant and lacks in details in the decorative parts. Historical studies assert that the colonnade and the entablature collapsed during the earthquake of 62 B.C. and they were only partially reconstructed at the time of the great eruption. Maybe this colonnade
is another proof that a large amount of workers were engaged in repairing earthquake damages. The seismic history of southern Italy had already required that they developed advanced constructive techniques. Every reconstruction in Pompeii between the 62 A.D. earthquake and the volcanic eruption testifies how much the consolidation expertise was employed with familiarity in the situations that demanded this type of intervention; it’s often possible to find cases of reconstruction realized with techniques that already show a greater attention to the details in an earthquake prone zone. In the building history of the Forum it is possible to recognize constructive skill already in the phase of the construction. The dimensions and the proportions of the two storeys structure are considerable and the realization of the entablature in pieces has been interpreted like the necessity of execution rapidity with particular attention to the overall economy of the whole work. 

The absence of fluted columns has been explained considering that such a valuable material was set off already in the years immediately after the eruption of the Vesuvius. Magistrates were sent from Rome in order to make a report about the conditions of the cities hit from the eruption. For Ercolano they could only certify the total loss of the city, while for Pompeii, also irrecoverable, still could catch a glimpse of some higher buildings, right nearby the Forum (de Martino et al. 2006). Carving the higher part of the shaft such as the capitals and the metopes and the triglyphs on the entablature is the last part of the building, so this part of the work was not yet done. Some authors noticed that the whole colonnade on the west side was ready to be mounted but still in pieces on the ground with coal sign on the pavement to fit the columns in the right place.

Figure 1.1. Félix-Emmanuel Callet, Comparison between ruins and reconstructive hypotese of the Forum, 1823.

Then after the erection of the colonnade, the stonemasons realized the flutes and the decorative details on the entablature. The Forum area has been one of the first discovered in the “modern era” of the excavation; between the 1812 and 1823 it was almost free from the debris and ashes so that the pensionnaires of the Academy of France in Rome sent the first graphic reconstruction of this part of the city (VV.AA., 1981, see Fig. 1.1). Previously the hypotheses about this area reported it as a single storey portico for soldiers barracks, but then the great amount of fragment and pieces of two orders columns showed clearly the sequence Doric – Ionic for the two levels of a public structure.

1.1. Present situation.

From the very first moment from the excavation of the Forum, the meaning of this piece of the city has been clear to the archaeologists in charge to the site. During the Second World War the Forum area was stroke by a bomb suffering severe damage. In the second half of the XXth century the second storey of Ionic order columns was re-erected using metal elements to assure the safety of the intervention. Only a small part of the colonnade was erected
for educational and tourist purpose and resulted useful to show the real proportion of the gallery around the place. The earthquake in 1980 produced damages in the whole archaeological site and for the columns it made necessary to remove the entablature over the second storey (Fig. 1.2), even if it was assured in place with metallic elements. In the late ‘90th of the past century, the colonnade has been object of a new intervention of restoration.

![Image of the southern part of the Forum](image)

**Figure 1.2.** The southern part of the Forum (Guadagno et al., 1973)

The shape adopted to use limestone blocks over the capitols is the smart answer to the unavoidable fracture of the stone itself when used in long span (Di Pasquale, 1996). In this way instead the break is designed in the construction phase, forcing the entablature to work as a “flat arc”, to achieve the most useful performance by the stone and in the same time preserving the *venustas* (beauty) of the architecture facing the Forum.

### 2. STRUCTURAL EVALUATION OF “INNOVATIVE SOLUTION” FOR TRABEATION

To avoid long span beams over the columns, short segments were built up providing opposing inclined patterned edges. This solution was conceived to simplify construction phases; however the blocks mutually supporting each other over inclined surfaces induce horizontal thrust in the structure to carry loads without any tensile capacity. In a fully functioning structure each block pushes over the other two contiguous and this load is counteracted. The static problem may arise at the corners of the colonnade where there is no symmetric, mutual interaction. To solve this static issue the builders at that time avoided the reduced size blocks at the end of the colonnade, instead of placing over each column a block, they placed a longer block on the first two extremity columns. This solution provided a main advantage; the horizontal thrust, not counteracted by the contiguous block, missing, was counteracted by two columns, so halving the horizontal thrust. This thrust potentially could overturn the extremity column.

A further issue raised due to the unstable scheme: in fact the first block, supported over two columns can be considered as fixed, then a portion of this first block overhangs and acts as a support for the contiguous block. The block in between two columns is simply supported, but the block on the subsequent column is not fixed as the first one, because it is simply supported on a single column, so potentially it can swing. To avoid this, a second row of smaller, but longer blocks was simply based on
the main blocks with inclined sides. Their joints were staggered with respect to the other blocks to avoid any weak plane, and in particular to reduce the rotation of the “suspended” blocks in between two columns.

However this solution was not able to avoid tractions in the inclined joints, but, even if joints open and tend to slide, the geometrical configuration guarantees a certain degree of stability. This mechanism has been studied in details numerically by means of Finite Element Modelling (FEM). It is noted that, even if under gravity loads the stability is granted despite the opening of inclined joints, the cracking of the joints is considered negative because it may lead to dangerous water leakage and lubrication of the potentially sliding planes. This cultural heritage material needs preservation and this condition can be considered as unfavourable for conservation of the archaeological ruins. Numerical analyses checked both the gravity condition and the seismic vulnerability (mainly in the plane of the colonnade to highlight the merit of the innovative solution on trabeation, even if a higher vulnerability can be expected in the out-of-plane direction). To evaluate the seismic performance of this structure, a fully nonlinear analysis was conducted. Nonlinearities were inserted both in the joints, allowing for frictional shear behaviour and brittle cracking in the axial direction (no tension material) and in the blocks. In any case the loads inside the structure resulted low so that only interfaces performed nonlinear while blocks remained almost in their elastic field. The response was mainly dominated by interface mechanisms, so that blocks performed as they were almost rigid.

For this reason, sensitivity analysis was performed on the stiffness of the joints and on the frictional performance. After the analysis of a benchmark situation, stiffness of the joints was heavily reduced to simulate the presence of a weaker filling material, e.g. a mortar, still having no tensile strength. Then the uncertainty on the friction angle was analysed. Two basic values were adopted, but a third reduced one was analysed to take into account potential water leakage (reducing friction performance of the overlaying planes). These analyses were extended to both the static and dynamic situation.

2.1. FEM modelling

To understand the performance of this archaeological heritage colonnade and the influence of the behaviour of the joints on the global structural behaviour, a FEM model has been used as a theoretical tool (as previously done by some of the authors to evaluate seismic vulnerability of other masterpieces in Pompeii, de Martino et al. 2006). Micro-modelling was adopted and each block was modelled reproducing accurately the effective geometry of the blocks as they are today (Fig. 2.1).

Figure 2.1. Particular of the analysed block colonnade today.
It is clear that some blocks have different dimensions especially in the columns and partially in the trabeation. Numerical two-dimensional analyses have been performed under plane-stress assumption. This assumption is based on the fact that the width of the structure is much lower than the other two dimensions, and in plane loads are considered. The paper focuses on the in-plane behaviour of the colonnade, to emphasize the presence of inclined joints, however it is a partial seismic assessment because out-of-plane weak behaviour (not involving specifically the inclined joints, but a global overturning) could jeopardize the in-plane safety assessment.

The analyses have been performed by means of the TNO DIANA v9.4.4 code. The colonnade is constituted by more than 1500 eight-node quadrilateral isoparametric plane stress elements. These elements are based on quadratic interpolation and Gauss integration. Crucial role has the special 3+3 nodes interface elements between blocks in a two-dimensional configuration. The 700 interface elements are based on quadratic interpolation and a 4-point Newton-Cotes integration scheme. Interfaces relate the forces acting on them to the relative displacement of the two sides as shown in Figure 2.2. The interface between two blocks is governed by a frictional behaviour. This behaviour is modelled with the Coulomb friction model, which has close resemblance with the Mohr-Coulomb plasticity model for continuum elements (Fig. 2.2). It is based on decomposition of total relative displacement rate into a reversible part and an irreversible part which is determined following the flow theory of plasticity (Manie and Kikstra 2012). In this case the tangent stiffness matrix is symmetric because the friction angle is equal to the dilatancy angle. In this way the nonlinear elastic bedding, e.g. 'no tension' bedding with a constant stiffness for compression and zero stiffness for tension is modelled also. Non-linear properties for blocks were inserted, namely plasticity in compression and brittle cracking in tension; however stress state was so reduced that blocks performed almost elastically.

![Topology and Displacements](image)

**Figure 2.2.** Interface nonlinear model

Few data were available for constituent materials, especially for the nonlinear post peak phase, so that minimum properties for ancient blocks and reasonable properties for frictional joints and fully brittle no tension/cohesion behaviour were assumed on safe side (see Table 2.1 for linear and nonlinear properties). In fact it was not authorized to perform mechanical tests on archaeological material.

<table>
<thead>
<tr>
<th>Property</th>
<th>Block</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young modulus</td>
<td>40 GPa</td>
<td>n.a.</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.25</td>
<td>n.a.</td>
</tr>
<tr>
<td>Density (mass)</td>
<td>2.68 gr/cm$^3$</td>
<td>n.a.</td>
</tr>
<tr>
<td>Normal Stiffness</td>
<td>n.a.</td>
<td>(0.4) 4 kN/mm</td>
</tr>
<tr>
<td>Transverse Stiffness</td>
<td>n.a.</td>
<td>(0.2) 2 kN/mm</td>
</tr>
</tbody>
</table>
Table 2.1b. Non-linear mechanical properties of materials (in brackets, values used for sensitivity analysis)

<table>
<thead>
<tr>
<th>Property</th>
<th>Block</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength</td>
<td>40 MPa</td>
<td>40 MPa</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>4 MPa</td>
<td>0 MPa</td>
</tr>
<tr>
<td>Cohesion</td>
<td>n.a.</td>
<td>0 MPa</td>
</tr>
<tr>
<td>Friction Angle (Tangent)</td>
<td>n.a.</td>
<td>(0.25) 0.50-0.75</td>
</tr>
</tbody>
</table>

In particular a reduced stiffness was considered also (one tenth of the benchmark value assuming direct contact of blocks) and also a tangent of friction angle of 0.25, compared to the basic benchmark value ranging between 0.5-0.75. All the analyses were performed under force control, applying generalized diffused accelerations (vertical to simulate static conditions and increasing horizontal to simulate seismic response) measuring in-plane displacements and evolution of stresses in the joints (mainly shear failures and axial openings).

2.1.1. Static Behaviour

Static behaviour was analysed by means of deformed shape of the structure under gravity loads, evaluation of opening of the joints, contour maps of tensile and compression stresses. Under gravity loads, the structural scheme provided by inclined joints suffers due to the collapse of a wide part of the colonnade. The right portion, almost intact, representing the end of the colonnade, is almost stable and the horizontal thrust, induced by the inclined joints in the main blocks of the trabeation is well counteracted by the last two columns on the right. The main issues arise in the left part where columns and blocks collapsed and nowadays the horizontal thrust is not correctly counteracted by other symmetric portions of the structure and the overturning moment is carried by the first column on the left which tends to turn and widens the opening of the first blocks in the trabeation. Figure 2.3 shows the deformed shape of the colonnade under gravity loads; coloured arrows represent the only portions of the inclined joints still in contact, while a gap formed in other portions of inclined joints. It is remarked that the deformed shape has an amplification factor of 10,000x and contact forces are shown in inclined joints only. In particular, the enlargements inside the main deformed plot of the structure highlight the block displacements. Stability is surely still granted, however the opening of the joints allows water and other polluting agents to attack the hidden surfaces of the marble elements.

![Figure 2.3. Deformed shape under gravity loads and enlargements of joint openings, with contact force arrows.](image)

Even considering a lower friction due to water lubrication or the filling of joint by means of weaker material, the behaviour is similar. Wider displacements are evaluated, but the safety conditions are almost comparable; also the stress values are surely within ample safety margins, as shown in the
contour plot in Fig. 2.4. The principal compression directions show a clear arch mechanism between columns, where the upper smaller blocks of the trabeation are widely involved, while lower blocks with inclined joints result almost supported by the geometrical shape.

**Figure 2.4a.** Contour map of compression stresses (units are Pa), static condition

**Figure 2.4b.** Contour map of tensile stresses (units are Pa), static condition
Compression stresses are lower than 0.6 MPa, so not causing concern. Similarly tensile stresses are rather low and lower than 0.13 MPa. The contour of the tensile stresses is really explicative, in fact, tensions arise in the middle of the blocks whose behaviour is similar to a beam simply supported, or in other cases in the middle of blocks that are supported in their centre, so they behave like a swing and they are almost fixed in their axis of symmetry and hang on the two sides as cantilevers. This response is totally different compared to a trabeation made of longer blocks each one supported on two consecutive columns; in that case clear tractions are expected in the lower part and compressions in the upper region of each “beam”.

2.1.1. Seismic in-plane vulnerability

Seismic vulnerability is evaluated by means of a pushover analysis and horizontal accelerations, $a_g$, leading to collapse of the colonnade (in-plane loss of stability due to sliding failure of the blocks). Peak ground accelerations, PGAs, are than expressed in terms of return period or probability of occurrence in a reference 50 years period (according to NTC’08), based on ITACA database seismic data. Preliminary modal analyses gave an estimation of the dynamic response of the colonnade. The structure is clearly massive and very stiff, so that a uniform distribution of horizontal accelerations was applied. Acceleration was applied increasingly up to failure, following nonlinear behaviour of the structure. The nonlinearity involved mainly joints rather than blocks. Seismic vulnerability was analysed by means of horizontal acceleration compared to displacement of a control point on top columns on the upper row. The structure is not symmetric so that both positive and negative lateral accelerations were considered, however comparable performance was found.

Under horizontal actions, the structural issues due to unbalanced horizontal thrust emphasize. However the stability issues arise at almost high (in-plane) acceleration, almost higher than 0.2g. Simply speaking, in any case, the tangent of friction angle represent a theoretical upper bound, representing the acceleration leading to shear plasticity in joints, however, the interaction of structural blocks, alter this basic idea.

Even considering a lower friction due to water lubrication the behaviour is almost similar, while the filling of joints by means of weaker material yields to lower seismic performance, but still acceptable being acceleration at failure almost equal to 0.15g. Figure 2.5 reports the pushover curves for different situations, three thin lines report response of the benchmark joint stiffness and changing the three values of friction angle. Thick line represents the case of reduced joint stiffness (accounting for a filling material). In any case the structure is able to resist an expected earthquake with a return period of about 1000 years, or 5% in 50 years, but only the benchmark joint stiffness with the highest friction capacity is able to satisfy a return period of more than 4000 years, or about 1% in 50 years.

![Figure 2.5. Pushover analysis in positive and negative directions and comparison with expected seismic actions](image-url)
The effect of seismic accelerations, having an $a_g=0.22 \, g$, is shown in the contour plot in Fig. 2.6. Deformed shape clearly shows the evolution versus the mechanism (still highly magnified). Blocks partially lose their geometrical support and sliding occurs. Even if the expected displacement is not relevant, it is considered as a potential in-plane failure condition. From a stress point of view: compression stresses are lower than 2 MPa; even if higher than under static conditions, they are still not causing particular concern. Similarly tensile stresses are lower than 0.7 MPa. This means that no internal blocks failure is expected, but sliding each other.

**Figure 2.6a.** Contour map of compression stresses (units are Pa), dynamic condition

**Figure 2.6b.** Contour map of tensile stresses (units are Pa), dynamic condition
3. CONCLUSIONS

In the ancient city of Pompeii in Italy, masonry buildings were surrounded by temples and public places presenting slender columns and typical shapes of marble overlaying blocks. In the case of the colonnade of the forum, the main square of the town, an “innovative solution” was adopted for the trabeation. To avoid long span beams over the columns, short segments were built up providing opposing inclined patterned edges. This solution was conceived to simplify construction phases; however the blocks mutually supporting each other over inclined surfaces induced horizontal thrust in the horizontal structures to carry loads without any tensile capacity.

The paper assesses the seismic vulnerability of this block based frame. The paper focuses also on the effect of joint cracking and opening. Even if stability is granted, for gravity only, so under static conditions, open joint promote meteoric water leakage and income of pollutants. The degradation and lubrication effect has reduced consequences on the static and dynamic response, however the filling of joints with weak materials may increase the seismic vulnerability. The structural evaluation goes over the mere evaluation of the innovation of the building system in use in the ancient town of Pompeii, but provides also a quantitative assessment of the seismic vulnerability to improve the knowledge for conservation of an UNESCO World cultural heritage site.

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