Probabilistic Study of Peak Floor Acceleration Demands in Nonlinear Structures

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SUMMARY:

In this paper, a nonlinear MDOF model has been developed to study the effect of nonlinearity of main structure on the seismic demand of acceleration sensitive components. To achieve this, a set of MDOF structural models with different number of stories, lateral resisting systems as well as fundamental period of vibration have been modeled and subjected to a set of ground motions at different intensity levels. Results of this study shows while in linear structures floor acceleration tends to amplify ground accelerations, in nonlinear structures, acceleration may be smaller than ground acceleration. Comparing the response of rigid and flexible models with the same number of stories and same lateral stiffness ratio shows that flexible models are more dependent of ground motion intensity and nonlinearity of structures. Also peak floor acceleration show lognormal distribution and therefore median and coefficient of variation can be used to represent the full distribution function.

Keywords: floor acceleration, probabilistic analysis, continuum beam model, non-structural component

1. INTRODUCTION

A vast amount of studies have been dedicated to develop rational methods of analysis of acceleration sensitive components in buildings in the past few decades. Excellent summaries of the state-of-art have been reported by Chen and Soong (1988), Singh (1988), Hirosawa et al. (1992), Soong (1995) and Villaverde (1996a, 1997). Although considerable progress has been made to date to improve our understanding of the seismic response of acceleration sensitive components, for the most part, previous studies have been aimed at the seismic design of equipment and heavy piping systems in nuclear power plants. Therefore, not all of this previous research is applicable or practical for application to the design and construction of non-structural components commonly installed in regular buildings. For example, lots of the studies previously mentioned considered that the primary system had a linear elastic behaviour. While this assumption is adequate for the design of nuclear power plants, which are typically designed to remain elastic or at most to exhibit very small levels of nonlinearity during severe earthquake ground motions, it is not applicable to most conventional buildings that may undergo significant inelastic deformations during strong and severe earthquakes. There have been only a very limited number of studies that have considered inelastic behaviour in the structure that support the secondary system. Most of the studies that have considered nonlinear behaviour, have been limited to single-degree-of freedom (SDOF) secondary systems mounted on a nonlinear SDOF secondary system (Kawakatsu, T. et al., 1979; Lin and Mahin, 1985; Chen et al. 1989; Igusa, 1990; Zhu, 1994).

Another reason that makes many of the previous studies not applicable to the design and installation of non-structural components commonly installed in regular buildings is that the large majority of previous studies require the use of complicated dynamic models for both the primary and the secondary systems. While developing and using complicated dynamic models for the seismic analysis of nuclear power plants is justified, this is usually too complicated and not suitable for the routine
design and installation of non-structural buildings commonly installed in ordinary buildings (Poroush, 1990, Villaverde, 1996b, Singh, 1998). Hence, there is a need to develop accurate, yet simple, demand estimation procedures that can be incorporated into codes and standards. Notable exceptions are the recent work of Soong et al. (1993) and of Singh et al. (1993). In particular, the study by Soong et al. (1993) proposed three alternative recommendations to improve seismic provisions in codes. A simplified version of his first alternative provides the basis for design recommendations in current design provisions in the U.S. for non-structural components (Bachman et al. 1998; Drake et al. 2000). A recent study concluded that these design-oriented methods still do not account for all the factors that significantly affect the response of non-structural components (Villaverde 1996b). In order to overcome these limitations, Villaverde (1997) proposed a design-oriented method that takes into account, in an approximate way, the interaction between the primary and secondary systems. This method provides a significant improvement over current code recommendations, however, it neglects the nonlinearity in the structure and it also neglects the contribution of higher modes of vibration in the structure, which often contribute significantly to floor acceleration demands in buildings. Improved simplified methods have more recently been developed (Singh et al., 1998, Villaverde, 2000). Villaverde’s new method is similar to his previous method but it now incorporates, in an approximate manner, the effect of structure nonlinearity.

Seismic provisions for the seismic design of non-structural components were introduced in the U.S. in the mid seventies, by considering an equivalent static lateral force proportional to the weight of the component applied at its center of gravity. Those provisions which essentially remained unchanged until the early 90’s, did not take into account the effect of soil conditions, the location of the component within the structure nor the inelastic deformation capacity of the anchorage of the component. Furthermore, they contained considerable amount of arbitrariness and subjectivity which produced ambiguous results (Soong et al. 1993). Partly based on analytical studies (Soong et al., 1993; Schroeder and Bachman, 1994) and partly based on data recorded in 28 instrumented buildings during the 1984 Morgan Hill, 1989 Loma Prieta and 1992 Landers earthquakes (Drake and Gillengarten, 1994; Bachman and Drake, 1994) improved provisions for the seismic design of non-structural components were introduced in the 1994 NEHRP recommended seismic provisions and subsequently in the 1997 UBC. In these provisions, a trapezoidal distribution of peak floor accelerations was used which varies from the peak ground acceleration at the base of the structure to four times this value at the top of the structure, which was then amplified by a factor of two for flexible components.

Recently, these provisions have seen severely criticized by some practicing structural engineers (Kehoe and Freeman, 1998; Searer and Freeman, 2002) who concluded that the intensity and distribution of floor accelerations over the height of the building appears to be influenced by the predominant period of vibration of the building and the mode shapes, and that the influence of the nonlinear response of the structure on the floors accelerations also needs to be considered. These practicing engineers recommended that research studies be conducted to investigate what parameters influence whether the floor accelerations are essentially constant over the height of the building or vary over the height of the building, and if they vary, to study how a reasonable variation can be estimated. Furthermore, they recommended that simplified procedures be developed and included in seismic provisions for determining the accelerations on non-structural components at each floor depending on the dynamic characteristics of the building.

2. MODELING ASSUMPTIONS

2.1. Structural model

The generic frames are one bay multi story two-dimensional frames consistent of linear beams connected to nonlinear columns through nonlinear rotational springs. The story height is constant along the height and equal to 12 ft while beams are 24 ft long. The overall damping ratio of the structures is considered to be 5 percent. Since the bay span of the generic frames are assumed to be 24 ft, with assumption of 175 psf as the sum of dead and live loads and 24 ft span of the beams in the
other direction of the structure, the total load on each column is about 100 kips. This load is applied as a vertical nodal load on each column of the structure in each story.

It is assumed that the stiffness ratio of beam to columns remain constant along the height of the structure. Taghavi (2006) showed that the beam to column stiffness ratio \( \rho \) which is often used in literature to represent generic models can be described in term of lateral stiffness ratio \( \alpha_0 \) (Miranda and Taghavi, 2005). Based on this study, for a building with \( N \) stories, the beam to column stiffness ratio of a single bay model can be derived as:

\[
\rho = \frac{\alpha_0^2}{12N^2}
\]  

(2.1)

Eqn. 2.1 shows that for a given lateral stiffness ratio, buildings with different heights will have equivalent single bay models with different beam to column stiffness ratio. Use of lateral stiffness ratios has some advantages over the use of beam to column stiffness. First, the modes shapes of structures with the same lateral stiffness ratio but different number of stories are the same, while structures with the same beam to column stiffness ratio but different number of floors are totally different. Also, modal participation factors and period ratios are the same between the structures with the same lateral stiffness ratio but different number of stories while this is not true for frame structures with the same beam to column stiffness ratio and different number of stories.

2.2. Ground motions

A set of 40 ground motions was used in this study. The ground motions were recorded on sites classified as class D according to recent NEHRP recommended provisions for the design of new buildings. These ground motions were then classified into two bins according to their earthquake magnitude as follows (Medina, 2002): (1) SMLR (Small Magnitude, Large Distance); (2) LMLR (Large Magnitude, Large Distance). The earthquakes with magnitude of 5.8 to 6.5 are referred as small magnitude and from 6.6 to 6.9 are referred as large magnitude. The distance of recording station to epicenter varies from 30 to 60 km. The ground motions have PGA ranging from 0.03g to 0.44g. The plots of spectra from individual ground motions scaled to PGA=1 and mean ground spectrum for 5 percent damped systems are shown in Fig. 2.1.

![Ground motion spectra](image)

**Figure 2.1.** Ground motion spectra.

2.3. Analysis parameters

In summary, the three parameters that their effects are investigated in this study are:

Number of stories, \( N \): 3, 6, 9, 12, 15, 18
Lateral stiffness ratio, $\alpha_0$: 2, 4, 8, 20
Structural stiffness: Rigid, Flexible
Base Excitation: 40 ground motions recorded on firm soils
Ground motion intensity level: $(S_a(T_1)/g)/\gamma$: 0.2, 0.8, 1.4, 2.0, 2.6, 3.2, 3.8, 4.4, 5.0
where $\gamma$ is base shear coefficient.

3. RESULTS

In the following section, summary of the results of a series of nonlinear dynamic analysis on a range of models mentioned in section 2.3 when subjected to ground motions of section 2.2 will be discussed.

3.1. Variation of peak floor acceleration with ground motion intensity level

It is shown in Fig. 3.1 that the peak ground acceleration is lower for short buildings and higher for tall buildings. This is due to the fact that the fundamental period of vibration is higher for taller buildings and therefore, the ratio of spectral acceleration at the first period to peak ground acceleration decreases and since the intensity measure used here is proportional to the spectral acceleration at the base, peak ground acceleration becomes higher for taller buildings. Except the 3-story structure that the peak floor acceleration follows the first period of the structure when subjected to low intensity ground motion ($\eta = 0.2$), in other models, peak floor acceleration demands remains unchanged through the height up to one or two top floors. In all models with 6 stories or more, the peak floor acceleration demand has a jump in top 20 percent of the height. It is seen that for moment resisting frames with ground motion intensity of $\eta = 0.2$, peak ground acceleration varies between 20 in/s$^2$ and 45 in/s$^2$ while at roof, the peak acceleration is between 35 in/s$^2$ and 75 in/s$^2$.

![Figure 3.1. PFA vs. height of a 3 to 18-story flexible structure of linear and nonlinear structures with shear deformation](image)

When structures are subjected to ground motions with intensity of $\eta = 5.0$, the peak floor acceleration profile entirely changes such that the maximum acceleration is at the base and roof acceleration is the least. Except the 3-story structure, where the nonlinearity is not as significant as other models, the peak floor acceleration demand is the same for all the structure from the second floor up to the roof level. A similar behaviour was observed in shear wall models where the roof floor acceleration in highly nonlinear models is independent of the number of stories. In moment resisting frames, peak floor acceleration in 80 percent of the height is about the same in all the structures regardless of the height. Peak ground acceleration varies between 450 in/s$^2$ to 1050 in/s$^2$ while peak roof acceleration is remains between 250 in/s$^2$ and 300 in/s$^2$. Comparing the peak floor acceleration of structures when subjected to ground motions with intensity of 0.2 and 5.0 shows that while peak ground acceleration increases 25 times, peak roof acceleration is amplified about 8 times in 3-story structure and 3.5 times
in 18-story structure.

Figure 3.2. PFA vs. intensity level of 3 to 18-story buildings

Fig. 3.2 shows the variation of peak floor acceleration versus the ground motion intensity level ($\eta$) at roof level in 3, 6, 9, 12, 15 and 18-story structures with lateral stiffness ratio of 2 and 20 with flexible structure. Results show there is less signs of nonlinearity at $z/H = 0.33$ compared to $z/H = 0.67$ and likewise, $z/H = 0.67$ compared to $z/H = 1.0$. In shear wall buildings with flexible structure, at $z/H = 0.33$, there is not that much nonlinearity and therefore, the peak floor acceleration is proportional to ground motion intensity. At roof level, the stiffness of the structures decreases gradually. As can be seen, the stiffness of the structures decreases as the intensity of ground motion increases. All the structures with different number of stories have similar rate of decrease of stiffness. It is seen that when these models are subjected to large intensity ground motions, the peak floor acceleration at roof level is not very sensitive to the ground acceleration intensity and in other words, the acceleration is saturated. In moment resisting model, at roof level, it is seen that peak floor acceleration increases more or less linearly up to $\eta = 0.8$ and as structures are subjected to stronger ground motions, the slope of the PFA-$\eta$ curve drops quickly and eventually converges to zero and peak roof acceleration reaches to its limit very soon. Fig. 3.3 shows similar information to Fig 3.2 but normalized to the peak floor acceleration of a linear structure.

Figure 3.3. Nonlinear to linear PFA vs. intensity level of 3 to 18-story structures

3.2. Probability distribution function of peak floor acceleration

In a fully probabilistic analysis of buildings, it is necessary to know the probability distribution of any response parameter. A log-normal distribution of structural response has been used in literature for displacement responses and here the same measure is going to be tested for peak floor acceleration. In Fig. 3.4, cumulative distribution function of peak floor acceleration demand of the 18-story moment
resisting frame with flexible structure and 5 percent modal damping is shown for 9 levels of ground motion intensity at 7th and 12th levels of an 18-story model. For each cumulative distribution function, a log-normal probability distribution function is fitted based on the median and coefficient of variation of the response obtained from the analysis of the structure when subjected to the 40 ground motions. It can be seen that log-normal probability distribution functions fits the data very well. This behaviour was observed for other structures with different number of stories and lateral stiffness ratio and therefore, it is concluded that the knowledge of median and coefficient of variation of peak floor acceleration is enough to define the full probability distribution function of the response and be used in probabilistic structural analysis.

![Cumulative probability distribution function of peak floor acceleration and fitted distribution at different floors.](image)

**3.3. Variation of peak roof acceleration with number of stories**

Fig. 3.5 shows what is the effect of ground motion intensity on peak roof acceleration and presents the variation of the ratio of nonlinear to linear peak floor acceleration versus number of stories at \( \eta = 1.4, 2.6, 3.8 \) and 5 for rigid and flexible structures with lateral stiffness ratio of 2. It is seen that in early stages of loading (\( \eta = 1.4 \)), the 3-story model is least affected by the nonlinearity where the ratio of nonlinear to linear response is about 0.9 and the 18-story model is the most affected with the ratio of about 0.65. As the intensity of ground motion increases to \( \eta = 2.6 \), the ratio of nonlinear to linear peak floor acceleration of 3-story model drops to 0.55 (0.35 decrease) while in the 18-story model, the ratio is dropped to 0.5 (0.15 decrease) meaning that the nonlinearity level of the 3-story model is getting closer to that of the 18-story model. For ground motions with intensity of \( \eta = 3.8 \) and 5, the nonlinear to linear peak roof acceleration ratio is almost constant for all the structures.

The effect of number of stories on nonlinear to linear peak roof acceleration ratio in flexible shear wall models is higher compared to rigid models. It is seen that at \( \eta = 1.4 \), this ratio is about 0.9, 0.8, 0.6, 0.55, 0.45 and 0.5 for 3, 6, 9, 12, 15 and 18-story models, respectively. There is a factor of almost two between the nonlinear to linear peak roof acceleration ratio of 3 and 18-story structures for not a very strong ground motion meaning that taller buildings experience nonlinearity in lower ground motion intensities. Unlike rigid models, even at high ground motion intensity levels, taller buildings are more affected by the nonlinear response such that for \( \eta = 5 \), nonlinear response is about 35 percent of the linear response in a 3-story model while in the 18-story model, this ratio is about 30 percent. This ratio for structures with other number of stories varies more or less linearly between the two limits of 3 and 18-story structures.
The dependency of the ratio of nonlinear to linear peak floor acceleration to the number of stories at different ground motion intensity levels for moment resisting frames with rigid and flexible models is shown in Fig. 3.6. It is seen that the general trend is that in rigid models, the ratio is highest for 3-story models, decreases at the number of stories increases up to 9 stories, remain the same up to 15-story model and drops more in 18-story model. In flexible models, this ratio decreases as the number of stories increases. Comparing rigid and flexible models show that the nonlinear to linear peak roof acceleration ratio is the same for rigid and flexible 3-story models but as the number of stories increases, flexible models have less ratio of nonlinear to linear response compared to rigid models.

3.4. Variation of peak roof acceleration demands with ground motion intensity level and lateral stiffness ratio

Fig. 3.7 shows the variation of peak roof acceleration of 3, 6, 12 and 18-story buildings with rigid structure with the ground motion intensity. Each graph compares the peak floor acceleration of 4 models with lateral stiffness ratio of 2, 4, 8 and 20. It is seen that in 3-story structures, the effect of ground motion intensity on peak roof acceleration is about the same for all lateral stiffness ratios meaning that the lateral resisting system does not have that much of an effect on acceleration demand of roof level. As number of stories increases, lateral stiffness ratio shows more effect on peak roof acceleration. In 6-story structures, all models with different lateral resisting systems have the same peak roof acceleration when responding linearly to the ground motions but as the intensity of ground motion increases, peak roof acceleration of structures with larger lateral stiffness ratio tends to be generally smaller than that of the structure with smaller lateral stiffness ratio. At $\eta = 5$, peak roof acceleration of a structure with lateral stiffness ratio of 2 is about 450 in/s$^2$ while that of the structure...
with lateral stiffness ratio of 20 is about 350 in/s². For 18-story structures, there is a factor 2 in peak roof acceleration of the shear wall model ($\alpha_0 = 2$) compared to moment resisting model ($\alpha_0 = 20$). It can also be seen that as the lateral stiffness ratio increases, the peak roof acceleration saturates faster and therefore to a smaller value.

**Figure 3.7.** Effect of lateral stiffness ratio on variation of peak floor acceleration vs. ground motion intensity in 3, 6, 12, and 18-story rigid structures.

Fig. 3.8 shows the variation of peak roof acceleration with ground motion intensity for flexible structures with different number of stories and lateral stiffness ratio of 2, 4, 8 and 20. It can be seen that similar to rigid models, peak roof acceleration of flexible 3-story models have small dependency on lateral stiffness ratio such that at $\eta = 0.2$, all have the same peak roof acceleration (40 in/s²) and at $\eta = 5.0$, peak roof acceleration of shear wall model is about 375 in/s² while that of the moment resisting frame is 325 in/s². In 6-story structures, peak roof acceleration of moment resisting frames is about 65 percent of that of shear wall models, 60 percent in 12-story structures and 50 percent in 18-story structures when subjected to ground motion intensity of $\eta = 5.0$.

**Figure 3.8.** Effect of lateral stiffness ratio on variation of peak floor acceleration vs. ground motion intensity in 3, 6, 12, and 18-story flexible structures.

### 4. SUMMARY

It was shown in this paper that peak floor acceleration heavily depends on the level of ground motion intensity as well as the dynamic properties of the structure and location of the floor. The general observation is that floor acceleration in structures responding nonlinearily to the ground motion increase slower than the ground motion intensity level and there will be a limit that the peak floor acceleration cannot increase more than that regardless of the ground motion intensity levels. This fact helps to keep the seismic demand of acceleration sensitive component within a limit. Due to this
behaviour, in a particular model that the peak floor acceleration increases from base to the top, in the same structure, the peak roof acceleration can be well below the peak ground acceleration as a result of saturation of floor acceleration. It was also shown that peak floor acceleration has a log normal distribution and therefore, estimates of median peak floor acceleration and coefficient of variation can be used to utilize the full probability distribution to be used in a probabilistic structural analysis.

Study of peak floor acceleration of nonlinear structures and comparing with their linear response showed that the effect of ground motion intensity in taller buildings with longer period is larger than the effect on shorter buildings. Also comparing the response of rigid and flexible models with the same number of stories and same lateral stiffness ratios showed that flexible models are more dependent on ground motion intensity and nonlinearity of structures. It was also shown that peak floor acceleration generally saturates earlier in upper floor of the structure while lower floors still show increase in peak floor acceleration. Although it was seen that in some cases, around the node of second mode shape where the response is dominated by the second mode, the saturation of floor acceleration is slower compared to other adjacent floors.

It was shown that in shear wall models, peak roof acceleration of taller buildings is higher than that of the shorter ones at all ground motion intensity levels. In moment resisting models, in linear structure, similar behaviour is observed but as the intensity of ground motion increases, the peak roof acceleration of taller structures becomes smaller than that of the shorter structure. This is valid for both rigid and flexible models. Also comparing the ratio of nonlinear peak floor acceleration to the linear peak floor acceleration at roof level of structures with different lateral stiffness ratio and rigid and flexible models showed that taller buildings have smaller ratio of nonlinear to linear response compared to shorter buildings or in other word, taller models are more affected by the nonlinear behaviour of the structure.

Comparison of peak roof acceleration of structures with different lateral stiffness ratio showed that the peak acceleration converges to a smaller bound in structures with larger lateral stiffness ratio compare to those with smaller lateral stiffness ratio. This behaviour is more evident in taller structures. Study of the ratio of peak roof acceleration of nonlinear models to that of the linear models showed that with some exceptions, structures with different lateral stiffness ratio have similar ratio of nonlinear peak roof acceleration to linear peak roof acceleration.

REFERENCES


